Do Cost-of-Living Shocks Pass Through to Wages?: An Analytic Two-Period Model

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Abstract

We provide a simplified two-period version of Bloesch, Lee and Weber (2024) and analytically prove the main propositions.

1 Two-Period Model

In this section, we build a simple two-period general equilibrium model that illustrates the following two features in a sharper way:

1. When the employed and unemployed share consumption risks according to $\frac{C_t^e}{C_t^u} = \xi$, i.e., the unemployed receives the consumption expenditure that is ξ^{-1} times that of employed workers, the cost of living shock does not affect wage and labor market outcomes in general.

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2. When the unemployed benefit b_t is in real terms, which workers compare with real wage $\frac{W_t}{P_t}$ in deciding whether to join the workforce, a cost of living shock generates a positive wage response. This pass-through to wages becomes more muted as λ_{EE}, the on-the-job search probability, increases.

We consider 3 different points in time: $t = 0, 1, 2$. At $t = 0$, the economy is at its steady-state: the number of employed is $\bar{N}$, that of unemployed is $\bar{U} = 1 - \bar{N}$. For simplicity, we assume that at $t = 2$, the economy gets back to the steady state, regardless of what happens at the interim period, $t = 1$.

Demand block The policy rate is given by $i_t = \rho$ for $t = 0, 1$ (i.e., pegged) so the households' Euler equation under log-preference implies the intertemporal equalization of consumption expenditures, given by

$$P_0 C_0 = P_1 C_1 = P_2 C_2, \tag{1}$$

where P_t is the price aggregator (of endowment good X_t and service good Y_t which is produced by firms) at time t, and C_t is the corresponding consumption aggregator. Under the unit elasticity of substitution between goods X_t and Y_t, i.e., $\eta = 1$ in our dynamic general equilibrium model, the households' expenditures on X_t and Y_t goods become proportional, implying

$$\frac{P_{X,t} X_t}{\alpha_X} = \frac{P_{Y,t} Y_t}{\alpha_Y} = P_t C_t \tag{2}$$

for all $t = 0, 1, 2$. From (1) and (2), we obtain:

$$P_{Y,0} Y_0 = P_{Y,1} Y_1 = P_{Y,2} Y_2 \tag{3}$$

in equilibrium. We assume the perfect price rigidity for the service good sector for tractability purposes: so $P_{Y,0} = P_{Y,1} = P_{Y,2} = \bar{P_Y}$,[1] which implies $Y_0 = Y_1 = Y_2 = \bar{Y}$ where $\bar{Y}$ is the steady-state level of service output. Therefore, the service output Y_1 at the interim period $t = 1$ is always at the steady state level $\bar{Y}$, regardless of shocks realized at $t = 1$. It is because the economy is demand-determined, and the household always insures their perfect consumption smoothing under pegged monetary policy.

[1] We will characterize the flexible price case later as a separate case.

Firm's problem Firm i, with its production function $Y_t^i = N_t^i$,[2] solves the following optimization at $t = 1$, with its number of workers $N_0 = \bar{N}$ inherited from the previous period:

$$J(\bar{N}) = \max_{V_1^i . W_1^i} \bar{P}_Y N_1^i - W_1^i N_1^i - \kappa(W_1) \cdot V_1^i + \frac{1}{1+\rho} J(N_1^i) \tag{4}$$

subject to

$$N_1^i = \bar{N} = (1 - S(W_1^i|W_1))\bar{N} + R(W_1^i|W_1)V_1^i, \tag{5}$$

where $\kappa(W_1)V_1^i$ is a vacancy-creation cost, where $\kappa(W_1)$ is a function of aggregate wage W_1. We will later consider two cases: $\kappa(W_1) = \kappa$ (i.e., constant) and $\kappa(W_1) = \kappa W_1$ (i.e., linear function). $S(W_1^i|W_1)$ and $R(W_1^i|W_1)$ are separation and retaining probabilities, respectively, that depend on the firm's individual wage W_1^i and the aggregate wage W_1. We will use the same functional form as in our dynamic general equilibrium model of Section **??**. Note that in (4), we do not incorporate wage nominal wage rigidities for now. Note that due to demand-determined nature, $N_1 = \bar{N}$ is taken as given by each firm.

Solving (4) and (5) with μ_1^i as the Lagrange multiplier to (5) yields the followings:

- For vacancy V_1^i:

$$\mu_1^i = \frac{\kappa(W_1)}{R(W_1^i|W_1)} \tag{6}$$

  which implies: the value of each worker is equal to the expected cost of hiring the worker. The creation of one vacancy costs $\kappa(W_1)$ but each vacancy is filled with probability $R(W_1^i|W_1)$. This interpretation is provided in de la Barrera i Bardalet (2023) as well.

- Wage W_1^i:

$$\begin{aligned} N_1^i &= \frac{\kappa(W_1)}{R(W_1^i|W_1)} \left[R'(W_1^i|W_1)V_1^i - S'(W_1^i|W_1)\bar{N} \right] \\ &= \frac{\kappa(W_1)}{R(W_1^i|W_1)} \left[\frac{R(W_1^i|W_1)}{W_1^i} \underbrace{\frac{R'(W_1^i|W_1)W_1^i}{R(W_1^i|W_1)}}_{=\varepsilon_{R,1}} V_1^i - \underbrace{\frac{S'(W_1^i|W_1)W_1^i}{S(W_1^i|W_1)}}_{=\varepsilon_{S,1}} \cdot \frac{S(W_1^i|W_1)}{W_1^i} \bar{N} \right] \end{aligned} \tag{7}$$

which becomes

$$N_1^i = \frac{\kappa(W_1)}{W_1^i} \left[\varepsilon_{R,1} \cdot V_1^i - \varepsilon_{S,1} \cdot \frac{S(W_1^i|W_1)}{R(W_1^i|W_1)} \bar{N} \right]. \tag{8}$$

[2] With production function $Y_t^i = N_t^i$, from (3), we obtain that $N_0 = N_1 = N_2 = \bar{N}$.

Envelope condition:

$$J'(\bar{N}) = (1 - S(W_1^i|W_1))\mu_1^i = (1 - S(W_1^i|W_1))\frac{\kappa(W_1)}{R(W_1^i|W_1)}. \tag{9}$$

Later, we will impose the (symmetric) equilibrium condition: $W_1^i = W_1$ and $N_1^i = N_1 = \bar{N}$.

Search and matching process For now, we use the same functional forms for $R(W_1^i|W_1)$ and $S(W_1^i|W_1)$ as in our dynamic general equilibrium model in Bloesch, Lee and Weber (2024). As we stated, we assume employed and unemployed share consumption risks according to $\frac{C_t^e}{C_t^u} = \xi$. Therefore, under the equilibrium condition with equal decisions across firms, i.e., $W_1^i = W_1$, $N_1^i = N_1$, $V_1^i = V_1$, the following definitions can be introduced:

- Labor market tightness θ_1:

$$\theta_1 = \frac{V_1}{\lambda_{EE}\bar{N} + 1 - \bar{N}} \tag{10}$$

  where λ_{EE} is the on-the-job search intensity, and we use $N_0 = \bar{N}$.

- Retaining probability $R(W_1^i = W_1|W_1)$:

$$R(W_1|W_1) = g(\theta_1)\left(\frac{1}{2}\phi_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}\right) \tag{11}$$

  where $\phi_{E,1}$ and $\phi_{U,1} \equiv 1 - \phi_{E,1}$ are fractions of employed (i.e., on-the-job searchers) and unemployed among job seekers, given by

$$\phi_{E,1} = \frac{\lambda_{EE}\bar{N}}{\lambda_{EE}\bar{N} + 1 - \bar{N}}. \tag{12}$$

- Separation probability $S(W_1^i = W_1|W_1)$:

$$S(W_1|W_1) = \frac{1}{2}\lambda_{EE} f(\theta_1) + \frac{1}{1+\xi^\gamma}\lambda_{EU} \tag{13}$$

  where we assume zero automatic separation (i.e., $s = 0$ in our dynamic general equilibrium model), and λ_{EU} is the exogenous job-quitting probability.

- Elasticity $\varepsilon_{R,1}$ and $\varepsilon_{S,1}$: from (11) and (13), we obtain

$$\varepsilon_{R,1} = \gamma \cdot \left(\frac{\frac{1}{4}\phi_{E,1} + \phi_{U,1}\left(\frac{\xi^\gamma}{(1+\xi^\gamma)^2}\right)}{\frac{1}{2}\phi_{E,1} + \left(\frac{\xi^\gamma}{1+\xi^\gamma}\right)\phi_{U,1}} \right) \simeq \gamma \cdot \left(\frac{\frac{1}{4}\phi_{E,1}}{\frac{1}{2}\phi_{E,1} + \left(\frac{\xi^\gamma}{1+\xi^\gamma}\right)\phi_{U,1}} \right), \tag{14}$$

and

$$\varepsilon_{S,1} = -\gamma \cdot \left(\frac{f(\theta_1)\lambda_{EE}\frac{1}{4} + \lambda_{EU}\frac{\xi^r}{(1+\xi^r)^2}}{0.5\lambda_{EE}f(\theta_1) + \left(\frac{1}{1+\xi^\gamma}\right)\lambda_{EU}} \right) \simeq -\frac{\gamma}{2}. \tag{15}$$

where we approximate $\frac{\lambda_{EU}}{1+\xi^\gamma} \simeq 0$ and $\frac{\phi_{U,1}\xi^\gamma}{(1+\xi^\gamma)^2} \simeq 0$, which hold well under our calibration. In (14), our approximation is based on that the effect of higher wages in making currently unemployed people choose to work at a firm is small compared with the effect on attracting on-the-job searchers from other firms.

Equilibrium characterization Since every firm i chooses the same decisions in equilibrium, i.e., $W_1^i = W_1$, $V_1^i = V_1$, and $N_1^i = N_1 = \bar{N}$, from (11) and (13), we obtain

$$\frac{S(W_1|W_1)\bar{N}}{R(W_1|W_1)} = \frac{\frac{1}{2}\lambda_{EE}\underbrace{f(\theta_1)}_{=\theta_1 g(\theta_1)}\bar{N} + \frac{1}{1+\xi^\gamma}\lambda_{EU}\bar{N}}{g(\theta_1)\left(\frac{1}{2}\phi_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}\right)} = \frac{\frac{1}{2}\phi_{E,1}g(\theta_1)V_1 + \frac{1}{1+\xi^\gamma}\lambda_{EU}\bar{N}}{g(\theta_1)\left(\frac{1}{2}\phi_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}\right)}. \tag{16}$$

We then plug in (14), (15), and (16) to (8) to obtain

$$\bar{N} = N_1 = \frac{\kappa(W_1)}{W_1}\left\{ \underbrace{V_1\left[\gamma\left(\frac{\frac{1}{2}\phi_{E,1}}{\frac{1}{2}\phi_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}}\right)\right]}_{\equiv\varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2}\frac{1}{1+\xi^\gamma}\lambda_{EU}\bar{N}}{\left(\frac{1}{2}\phi_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}\right)g(\theta_1)}}_{\equiv\varepsilon_{21}} \right\}, \tag{17}$$

where $\varepsilon_{11} + \varepsilon_{21}$ in (17) becomes the 'effective' labor supply elasticity each firm faces. ε_{11} is about the elasticity due to those who are on-the-job search: an increase in wage attracts more on-the-job searchers from other firms and reduce the endogenous separation of current workers, and given other variables, this effect becomes more pronounced with higher measure of on-the-job searchers among job seekers, i.e., higher $\phi_{E,1}$ (thereby decrease in $\phi_{U,1}$). Eventually in equilibrium, every firm sets the same wage: $W_1^i = W_1$ for $\forall i$.

ε_{21} is the elasticity attributed to those who quit their jobs to be unemployed: a higher wage deters workers from going to be unemployed. The proportion of those who exit the labor force becomes smaller under a bigger and more competitive job market with higher λ_{EE}, i.e., higher λ_{EE} lowers ε_{21} and raises ε_{11}.

From (5), (11), and (13), we obtain the labor dynamics as follows:

$$\begin{aligned}\bar{N} = N_1 &= \left[1 - \left(\frac{1}{2}\lambda_{EE} \underbrace{f(\theta_1)}_{=\theta_1 g(\theta_1)} + \frac{1}{1+\xi^\gamma}\lambda_{EU}\right)\right]\bar{N} + g(\theta_1)\left(\frac{1}{2}\phi_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}\right)V_1 \\ &= \bar{N} - \bar{N}\frac{1}{1+\xi^\gamma}\lambda_{EU} + g(\theta_1)V_1\left[\left\{\cancel{\frac{1}{2}\phi_{E,1}} + \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}\right\} - \left\{\cancel{\frac{1}{2}\phi_{E,1}}\right\}\right] \\ &= \bar{N} - \bar{N}\frac{1}{1+\xi^\gamma}\lambda_{EU} + g(\theta_1)V_1\frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1},\end{aligned} \tag{18}$$

which implies

$$\frac{\bar{N}\frac{1}{1+\xi^\gamma}\lambda_{EU}}{\lambda_{EE}\bar{N} + 1 - \bar{N}} = f(\theta_1)\frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}. \tag{19}$$

Equations (17) and (19) constitute our equilibrium, with the condition $N_1 = Y_1 = \bar{N}$. We can theoretically elicit equilibrium W_1 and V_1 from those two equations.

Cost-of-living shock As we assume in Bloesch, Lee and Weber (2024), the endowment good X_t drops from its steady state level $\bar{X}$ to $X_1 < \bar{X}$ at $t = 1$ in an unanticipated manner. From (17) and (19), a sudden drop in X_1 from $\bar{X}$ does not affect the equilibrium levels of V_1 and W_1, and from the household's Euler equation (3), $N_1 = \bar{N}$ remains the same. From (2), the only change is the price of endowment good X_t, and $P_{X,1}$ rises satisfying $P_{X,1}X_1 = \bar{P_X}\bar{X}$. The following Proposition 1 summarizes this finding.

Proposition 1 *A cost-of-living shock, i.e., a sudden drop in X_1 from $\bar{X}$, does not affect equilibrium labor market outcomes: $N_1 = \bar{N}$, $W_1 = \bar{W}$, and $V_1 = \bar{V}$. The price $P_{X,1}$ of endowment good X_1 rises so that the expenditure stays the same, i.e., $P_{X,1}X_1 = \bar{P_X}\bar{X}$.*

Flexible price case The irrelevance result of cost-of-living shocks in Proposition 1 holds even if firms set their prices fully flexibly. As assumed in Bloesch, Lee and Weber (2024), firms are in monopolistic competition, represented by Dixit-Stiglitz aggregator with elasticity of substitution ϵ. Then

$$Y_1^i = Y_1\left(\frac{P_{Y,1}^i}{P_{Y,1}}\right)^{-\epsilon}. \tag{20}$$

Each firm i solves instead the following problem:

$$J(\bar{N}) = \max_{P^i_{Y,1}, N^i_1, V^i_1. W^i_1} P^i_{Y,1} N^i_1 - W^i_1 N^i_1 - \kappa(W_1) \cdot V^i_1 + \frac{1}{1+\rho} J(N^i_1) \tag{21}$$

subject to (20) and

$$Y^i_1 = N^i_1 = (1 - S(W^i_1|W_1))\bar{N} + R(W^i_1|W_1)V^i_1. \tag{22}$$

The solution to (21), with $W^i_1 = W_1$, will be given by

$$\begin{aligned} P^i_{Y,1} = P_{Y,1} &= \frac{\epsilon}{\epsilon - 1}\left(W_1 + \frac{\kappa(W_1)}{R(W_1|W_1)} - \frac{1}{1+\rho} J'(N^i_1)\right) \\ &= \frac{\epsilon}{\epsilon - 1}\left(W_1 + \frac{\kappa(W_1)}{R(W_1|W_1)} - \frac{1}{1+\rho}(1 - S(W_2|W_2))\frac{\kappa(W_2)}{R(W_2|W_2)}\right) \end{aligned} \tag{23}$$

where $W_2 = \bar{W}$ as the economy gets back to its steady state at $t = 2$. The term $\frac{\kappa(W_1)}{R(W_1|W_1)}$ is a cost of hiring through additional vacancy. If a firm hires at $t = 1$, it can reduce hiring at $t = 2$ by one. The last term $\frac{1}{1+\rho}(1 - S(W_2|W_2))\frac{\kappa(W_2)}{R(W_2|W_2)}$ represents this reduction in future hiring costs.[3] From (3), (17), and (23), we obtain

$$\begin{aligned} \underbrace{P_{Y,0}}_{=\bar{P}_Y} \bar{Y} = P_{Y,1} Y_1 = &\frac{\epsilon}{\epsilon - 1}\left[W_1 + \frac{\kappa(W_1)}{R(W_1|W_1)} - \frac{1}{1+\rho}(1 - S(W_2|W_2))\frac{\kappa(W_2)}{R(W_2|W_2)}\right] \\ &\cdot \frac{\kappa(W_1)}{W_1}\left\{ \underbrace{V_1\left[\gamma\left(\frac{\frac{1}{2}\phi_{E,1}}{\frac{1}{2}\phi_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}}\right)\right]}_{\equiv \varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2}\frac{1}{1+\xi^\gamma}\lambda_{EU}\bar{N}}{\left(\frac{1}{2}\phi_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}\right) g(\theta_1)}}_{\equiv \varepsilon_{21}} \right\}, \end{aligned} \tag{24}$$

which, with (19), constitute the flexible price equilibrium. Since (19) and (24) do not depend on X_1 or $P_{X,1}$, a cost-of-living shock, i.e., reduction in X_1 from $\bar{X}$, does not affect the labor market equilibrium outcome as in the rigid price case.

Corollary 1 *Even if the price-setting of firms is fully flexible, a cost-of-living shock defined as a sudden drop in X_1 from $\bar{X}$, does not affect the equilibrium labor market outcomes:*

[3] The decomposition of marginal costs in equation (23) is similarly given in de la Barrera i Bardalet (2023).

$N_1 = \bar{N}$, $W_1 = \bar{W}$, and $V_1 = \bar{V}$. The price $P_{X,1}$ of endowment good X_1 rises so that the expenditure stays the same, i.e., $P_{X,1}X_1 = \bar{P_X}\bar{X}$.

1.1 Quits rate and wage growth under demand shocks

In this section, we show analytically that a positive demand shock generates positive responses in both on-the-job switching rate $\frac{1}{2}\lambda_{EE}f(\theta_1)$[4] and wage growth. As $f(\cdot)$ is increasing, it is equivalent to a positive correlation between market tightness θ_1 and wage growth under a demand shock.

We first define a positive demand shock that raises N_1 from $\bar{N}$, e.g., a reduction in the policy rate at $t = 1$ will result in a consumption boom, thereby leading to firms' higher labor demand level at $t = 1$. We start from our equilibrium conditions: instead of $\bar{N}$, we use $N_1 > \bar{N}$ there:

$$\bar{N} < N_1 = \frac{\kappa(W_1)}{W_1}\left\{\underbrace{V_1\left[\gamma\left(\frac{\frac{1}{2}\phi_{E,1}}{\frac{1}{2}\phi_{E,1}+\frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}}\right)\right]}_{\equiv\varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2}\frac{1}{1+\xi^\gamma}\lambda_{EU}\bar{N}}{\left(\frac{1}{2}\phi_{E,1}+\frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}\right)g(\theta_1)}}_{\equiv\varepsilon_{21}}\right\}, \tag{25}$$

and

$$\bar{N} < N_1 = \bar{N} - \bar{N}\frac{1}{1+\xi^\gamma}\lambda_{EU} + g(\theta_1)V_1\frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}. \tag{26}$$

We divide into two cases according to different functional forms of $\kappa(W_1)$: (i) $\kappa(W_1) = \kappa$ (i.e., constant), and (ii) $\kappa(W_1) = \kappa W_1$ (i.e., linear) with nominal wage rigidity.[5]

Case 1: $\kappa(W_1) = \kappa$ In this case, (25) becomes:

$$\bar{N} < N_1 = \frac{\kappa}{W_1}\left\{\underbrace{V_1\left[\gamma\left(\frac{\frac{1}{2}\phi_{E,1}}{\frac{1}{2}\phi_{E,1}+\frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}}\right)\right]}_{\equiv\varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2}\frac{1}{1+\xi^\gamma}\lambda_{EU}\bar{N}}{\left(\frac{1}{2}\phi_{E,1}+\frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}\right)g(\theta_1)}}_{\equiv\varepsilon_{21}}\right\}. \tag{27}$$

[4]Quits rate includes those who voluntarily quit to unemployed as well, which is a small margin compared to the on-the-job switching part.

[5]The case of $\kappa(W_1) = \kappa W_1$ corresponds to our model in Section **??** with no convexity in the vacancy creation cost function, i.e., $\chi = 0$.

In order to get a sharper results, we log-linearize (26) and obtain[6]

$$0 < \check{N}_1 = \frac{1}{1+\xi^{\gamma}}\lambda_{EU}\left(\underbrace{\frac{g'(\bar{\theta}_1)\bar{\theta}_1}{g(\bar{\theta}_1)}}_{\equiv -\varepsilon_{g,\theta}}\check{\theta}_1 + \check{\theta}_1\right) = \frac{1}{1+\xi^{\gamma}}\lambda_{EU}\left(1 - \underbrace{\varepsilon_{g,\theta}}_{<1}\right)\check{\theta}_1, \tag{28}$$

where we assume the firm's matching elasticity $\varepsilon_{g,\theta} \geqslant 0$ of $g(\theta_1)$ is less than 1, which holds under various specification.[7] Therefore, from (28), $\check{\theta}_1 > 0$ when $\check{N}_1 > 0$, i.e., labor market gets tighter at $t = 1$.

We then log-linearize (27) and use (28) to obtain

$$\underbrace{\frac{1}{1+\xi^{\gamma}}\lambda_{EU}\left(1 - \underbrace{\varepsilon_{g,\theta}}_{<1}\right)\check{\theta}_1}_{=\check{N}_1} + \check{W}_1 = \left[\frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}} + \frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}}\varepsilon_{g,\theta}\right]\check{\theta}_1. \tag{29}$$

Since $\frac{1}{1+\xi^{\gamma}}\lambda_{EU}$ is small under our calibration, $\check{\theta}_1 > 0$ from (28) implies $\check{W}_1 > 0$ in (29). Thus, we generate a positive correlation between movements in wage and market tightness (on-the-job switching rate), which is summarized in the following Proposition 2.

Proposition 2 *When $\kappa(W_1) = \kappa$, i.e., $\kappa(W_1)$ is a constant function, both market tightness θ_1 (on-the-job switching rate $0.5\lambda_{EE}f(\theta_1)$) and wage W_1 rises in response to a positive demand shock.*

Case 2: $\kappa(W_1) = \kappa W_1$ with nominal wage stickiness Now we assume $\kappa(W_1) = \kappa W_1$ (i.e., linear function) but incorporate nominal wage rigidity à la Rotemberg (1982). Firm i solves:

$$J(\bar{N}) = \max_{V_1^i, W_1^i} \bar{P}_Y N_1^i - W_1^i N_1^i - \underbrace{\kappa(W_1)}_{\equiv \kappa W_1} \cdot V_1^i - \underbrace{\frac{\psi^W}{2}\left(\frac{W_1^i}{\bar{W}} - 1\right)^2 \bar{W} N_1^i}_{\text{Wage changing cost}} + \frac{1}{1+\rho}J(N_1^i) \tag{30}$$

subject to

$$N_1^i = (1 - S(W_1^i|W_1))\bar{N} + R(W_1^i|W_1)V_1^i. \tag{31}$$

[6]We use $\check{\theta}_1 = \check{V}_1$ as θ_1 and V_1 are proportional and $\lambda_{EE}\bar{N} + 1 - \bar{N}$ is constant.

[7]Since $f(\theta_1) = \theta_1 g(\theta_1)$, $\varepsilon_{f,\theta} \equiv \frac{g'(\bar{\theta}_1)\bar{\theta}_1}{g(\bar{\theta}_1)} = 1 - \varepsilon_{g,\theta} > 0$ under our specification, as $f(\theta_1)$ is increasing in θ_1.

Solving (30) subject to (31) with $W_1^i = W_1$ and $N_1^i = N_1$ yields

$$N_1\left(1+\psi^W\frac{W_1-\bar{W}}{\bar{W}}\right) = \frac{\kappa \cancel{W_1}}{\cancel{W_1}}\left\{\underbrace{V_1\left[\gamma\left(\frac{\frac{1}{2}\phi_{E,1}}{\frac{1}{2}\phi_{E,1}+\frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}}\right)\right]}_{\equiv\varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2}\left(\frac{1}{1+\xi^\gamma}\right)\lambda_{EU}\bar{N}}{\left(\frac{1}{2}\phi_{E,1}+\frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}\right)g(\theta_1)}}_{\equiv\varepsilon_{21}}\right\}, \tag{32}$$

We log-linearize (32) and use (28) to obtain

$$\frac{1}{1+\xi^\gamma}\lambda_{EU}\left(1-\underbrace{\varepsilon_{g,\theta}}_{<1}\right)\check{\theta}_1+\psi^W\check{W}_1 = \left[\frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}}+\frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}}\varepsilon_{g,\theta}\right]\check{\theta}_1. \tag{33}$$

Since $\frac{1}{1+\xi^\gamma}\lambda_{EU}$ is small under our calibration, $\check{\theta}_1 > 0$ from (28) implies $\check{W}_1 > 0$ in (33) as well. Therefore, we generate a positive correlation between movements in wage and market tightness (on-the-job switching rate), which is summarized in the following Proposition 3.[8] Finally, note that **Case 2** (which is the case of $\chi = 0$ in Bloesch, Lee and Weber (2024)) generate similar results to **Case 1**, where $\kappa(\cdot)$ is a constant function.

Proposition 3 *When $\kappa(W_1) = \kappa W_1$ and firms face nominal wage rigidities à la Rotemberg (1982), both market tightness θ_1 (on-the-job switching rate $0.5\lambda_{EE}f(\theta_1)$) and wage W_1 rises in response to a positive demand shock.*

Therefore, our simple model generates the benchmark results in Bloesch, Lee and Weber (2024).

1.2 With real benefits of unemployment

In this section, we assume that unemployed workers some inflation-indexed quantity of consumption b_1 at $t = 1$. In those cases, all the above equilibrium conditions, i.e., (10), (11), (12), (13), (14), (15), (17), (19), hold, with

$$c(P_1, W_1) \equiv \frac{\left(\frac{W_1}{P_1}\right)^\gamma}{b_1^\gamma+\left(\frac{W_1}{P_1}\right)^\gamma}.$$

[8] Note that without wage nominal rigidities, i.e., $\psi^W = 0$, equilibrium might not exist.

in the position of $\frac{\xi^\gamma}{1+\xi^\gamma}$. Here b_1 is the consumption-equivalent during unemployment, which an unemployed person compares with real wage $\frac{W_1}{P_1}$ in deciding whether to be back at work.[9]

Note that $c(P_1, W_1)$ is increasing in W_1 and decreasing in P_1, where P_1 is total price aggregator of endowment good X_1 and service good Y_1. Under the rigid service prices, i.e., $P_{Y,1} = \bar{P}_Y$, a cost-of-living shock as described above increases P_1 and lower $c(P_1, W_1)$. We ask how the economy's responses to a cost-of-living shock under this specification would differ from the above case where $c(P_1, W_1) \equiv \frac{\xi^\gamma}{1+\xi^\gamma}$. Intuitively, a rise in cost-of-living reduces the relative attractiveness of working compared with being unemployed, resulting in a lower $c(P_1, W_1)$. The equilibrium will be represented by

$$\frac{\bar{N}\left(1 - c(P_1, W_1)\right)\lambda_{EU}}{\lambda_{EE}\bar{N} + 1 - \bar{N}} = f(\theta_1)c(P_1, W_1)\phi_{U,1}. \tag{34}$$

and

$$\bar{N} = N_1 = \frac{\kappa(W_1)}{W_1}\left\{\underbrace{V_1\left[\gamma\left(\frac{\frac{1}{2}\phi_{E,1}}{\frac{1}{2}\phi_{E,1} + c(P_1, W_1)\phi_{U,1}}\right)\right]}_{\equiv\varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2}\left(1 - c(P_1, W_1)\right)\lambda_{EU}\bar{N}}{\left(\frac{1}{2}\phi_{E,1} + c(P_1, W_1)\phi_{U,1}\right)g(\theta_1)}}_{\equiv\varepsilon_{21}}\right\}, \tag{35}$$

where we use the fact that output (and labor) remains at the steady state level due to households' perfect consumption smoothing. We assume that at the steady state, $c(\bar{P}_1, \bar{W}_1) = \bar{c} = \frac{\xi^\gamma}{1+\xi^\gamma}$.

We divide into three cases according to different functional forms of $\kappa(W_1)$: (i) $\kappa(W_1) = \kappa \cdot W_1$ (i.e., linear); (ii) $\kappa(W_1) = \kappa$ (i.e., constant), and (iii) whether we introduce nominal wage rigidity.

Case 1: $\kappa(W_1) = \kappa \cdot W_1$ In this case, (35) becomes:

$$\bar{N} = \kappa\left\{\underbrace{V_1\left[\gamma\left(\frac{\frac{1}{2}\phi_{E,1}}{\frac{1}{2}\phi_{E,1} + c(P_1, W_1)\phi_{U,1}}\right)\right]}_{\equiv\varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2}\left(1 - c(P_1, W_1)\right)\lambda_{EU}\bar{N}}{\left(\frac{1}{2}\phi_{E,1} + c(P_1, W_1)\phi_{U,1}\right)g(\theta_1)}}_{\equiv\varepsilon_{21}}\right\}. \tag{36}$$

Since (34) and (36) constitute the equilibrium, an increase in P_1 will lead to an increase

[9] Here we assume that the monetary authority sticks to pegging its policy rate.

in W_1 so that $c(P_1, W_1) = \bar{c}$. Then other labor market variables, e.g., V_1, θ_1, remain the same. Therefore, in this case, wage rises to compensate higher costs of living so that real wage stays constant, and real wage rigidity naturally arises as optimal decisions of firms.

Proposition 4 **(**$\kappa(W_1) = \kappa \cdot W_1$**)** *A rise in cost-of-living is exactly compensated by the same rate of increase in wage in equilibrium, and labor market equilibrium outcomes remain the same. The result does not depend on* λ_{EE}*, the on-the-job search intensity. Real wage rigidity naturally arises as optimal decisions of firms.*

Case 2: $\kappa(W_1) = \kappa$ In this case, (35) becomes

$$\bar{N} = \frac{\kappa}{W_1} \left\{ \underbrace{V_1 \left[\gamma \left(\frac{\frac{1}{2}\phi_{E,1}}{\frac{1}{2}\phi_{E,1} + c(P_1, W_1)\phi_{U,1}} \right) \right]}_{\equiv \varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2}\left(1 - c(P_1, W_1)\right)\lambda_{EU}\bar{N}}{\left(\frac{1}{2}\phi_{E,1} + c(P_1, W_1)\phi_{U,1}\right) g(\theta_1)}}_{\equiv \varepsilon_{21}} \right\}. \quad (37)$$

If, as in the above case, W_1 rises at the same rate as P_1 so that $c(P_1, W_1)$ does not change, then (37) is not satisfied as its left hand side becomes smaller than $\bar{N}$. Thus, we can infer that in this case, the wage response would be generically smaller than the price increase. In order to obtain sharper results, we log-linearize (34) and obtain

$$-\frac{\bar{c}}{1 - \bar{c}}\check{c} = \underbrace{\frac{f'(\bar{\theta}_1)\bar{\theta}_1}{f(\bar{\theta}_1)}}_{\equiv \varepsilon_{f,\theta}} \check{\theta}_1 + \check{c} \quad (38)$$

with

$$\check{c} = \frac{\bar{c}_P \bar{P}_1}{\bar{c}}\check{P}_1 + \frac{\bar{c}_W \bar{W}_1}{\bar{c}}\check{W}_1. \quad (39)$$

Equations (38) and (39) yield

$$\check{\theta}_1 = -\frac{1}{(1 - \bar{c})\varepsilon_{f,\theta}} \left(\frac{\bar{c}_P \bar{P}_1}{\bar{c}}\check{P}_1 + \frac{\bar{c}_W \bar{W}_1}{\bar{c}}\check{W}_1 \right). \quad (40)$$

We also log-linearize (37) and obtain[10]

$$0=-\check{W}_1+\frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}}\left[\check{\theta}_1-\frac{\bar{c}\phi_{U,1}}{\frac{1}{2}\phi_{E,1}+\bar{c}\phi_{U,1}}\check{c}\right]+\frac{\bar{\varepsilon}_{21}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}}\left[-\frac{\bar{c}}{1-\bar{c}}\check{c}-\frac{\bar{c}\phi_{U,1}}{\frac{1}{2}\phi_{E,1}+\bar{c}\phi_{U,1}}\check{c}-\underbrace{\frac{g'(\bar{\theta}_1)\bar{\theta}_1}{g(\bar{\theta}_1)}}_{\equiv\varepsilon_{g,\theta}>0}\check{\theta}_1\right]. \tag{41}$$

If we define

$$\begin{aligned} d_W &\equiv \frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}}\left(\frac{\bar{c}\phi_{U,1}}{\frac{1}{2}\phi_{E,1}+\bar{c}\phi_{U,1}}+\frac{1}{(1-\bar{c})\varepsilon_{f,\theta}}\right)+\frac{\bar{\varepsilon}_{21}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}}\left(\frac{\bar{c}}{1-\bar{c}}+\frac{\bar{c}\phi_{U,1}}{\frac{1}{2}\phi_{E,1}+\bar{c}\phi_{U,1}}+\frac{\varepsilon_{g,\theta}}{(1-\bar{c})\varepsilon_{f,\theta}}\right) \\ &= \underbrace{\frac{\bar{c}\phi_{U,1}}{\frac{1}{2}\phi_{E,1}+\bar{c}\phi_{U,1}}}_{\equiv d_{W,1}}+\underbrace{\frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}}\frac{1}{(1-\bar{c})\varepsilon_{f,\theta}}+\frac{\bar{\varepsilon}_{21}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}}\left(\frac{\bar{c}}{1-\bar{c}}+\frac{\varepsilon_{g,\theta}}{(1-\bar{c})\varepsilon_{f,\theta}}\right)}_{\equiv d_{W,2}}>0 \end{aligned} \tag{42}$$

then because at the steady state we have[11]

$$\frac{\bar{c}_W\bar{W}_1}{\bar{c}}=-\frac{\bar{c}_P\bar{P}_1}{\bar{c}}=\frac{\gamma}{1+\xi^\gamma}=\gamma(1-\bar{c}),$$

the wage response $\check{W}_1$ is given by

$$\check{W}_1=\frac{d_W}{\frac{1}{\gamma(1-\bar{c})}+d_W}\check{P}_1<\check{P}_1, \tag{43}$$

which is increasing in d_W. From (39) and (43), $\check{\theta}_1>0$ follows, i.e., labor market becomes tighter. This result is summarized in the following Proposition 5.

Proposition 5 *When $\kappa(W_1)=\kappa$, i.e., $\kappa(W_1)$ is a constant function, wage W_1 rises in response to a cost-of-living shock, but the rate of wage increase is lower than that of price aggregator, i.e., $\check{W}_1<\check{P}_1$. As a result, labor market becomes tighter, i.e., $\check{\theta}_1>0$.*

[10] Again, we use $\check{\theta}_1=\check{V}_1$ as θ_1 and V_1 are proportional and $\lambda_{EE}\bar{N}+1-\bar{N}$ is constant.
[11] We assume that at the steady state, $c(\bar{P}_1,\bar{W}_1)=\bar{c}=\frac{\xi^\gamma}{1+\xi^\gamma}$.

Role of on-the-job search intensity λ_{EE} At the steady state, $\frac{1}{1+\xi^{\gamma}}\lambda_{EU} \simeq 0$ under our calibration, and $\frac{\bar{\varepsilon}_{21}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}} \simeq 0$ with $\frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}} \simeq 1$. Then from (42),

$$d_W \simeq \underbrace{\frac{\bar{c}\phi_{U,1}}{\frac{1}{2}\phi_{E,1} + \bar{c}\phi_{U,1}}}_{\equiv d_{W,1}} + \underbrace{\frac{1}{(1-\bar{c})\varepsilon_{f,\theta}}}_{\equiv d_{W,2}},$$

which is decreasing in λ_{EE} as $\phi_{E,1}$ increases and $\phi_{U,1}$ falls.[12] Therefore, we can see from (43) that wage rises less under higher λ_{EE}. This result is summarized by the next Proposition 6.

Proposition 6 *Wage rises less in response to a cost-of-living shock, under higher on-the-job search intensity λ_{EE}.*

Case 3: $\kappa(W_1) = \kappa W_1$ with nominal wage stickiness Now we return to the first **Case 1** where $\kappa(W_1)$ is linear in W_1, but incorporate nominal wage rigidity à la Rotemberg (1982). Firm i solves:

$$J(\bar{N}) = \max_{V_1^i . W_1^i} \bar{P}_Y N_1^i - W_1^i N_1^i - \underbrace{\kappa(W_1)}_{\equiv \kappa W_1} \cdot V_1^i - \underbrace{\frac{\psi^W}{2}\left(\frac{W_1^i}{\bar{W}} - 1\right)^2 \bar{W} N_1^i}_{\text{Wage changing cost}} + \frac{1}{1+\rho} J(N_1^i) \quad (45)$$

subject to

$$N_1^i = (1 - S(W_1^i|W_1))\bar{N} + R(W_1^i|W_1)V_1^i. \quad (46)$$

Solving (45) subject to (46) with $W_1^i = W_1$ and $N_1^i = \bar{N}$ yields

$$\bar{N}\left(1 + \psi^W \frac{W_1 - \bar{W}}{\bar{W}}\right) = \frac{\kappa \cancel{W_1}}{\cancel{W_1}} \left\{ \underbrace{V_1\left[\gamma\left(\frac{\frac{1}{2}\phi_{E,1}}{\frac{1}{2}\phi_{E,1} + c(P_1, W_1)\phi_{U,1}}\right)\right]}_{\equiv \varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2}(1 - c(P_1, W_1))\lambda_{EU}\bar{N}}{\left(\frac{1}{2}\phi_{E,1} + c(P_1, W_1)\phi_{U,1}\right) g(\theta_1)}}_{\equiv \varepsilon_{21}} \right\}, \quad (47)$$

---

[12] From (19), at the steady state we have

$$\bar{N}\frac{1}{1+\xi^{\gamma}}\lambda_{EU} = f(\theta)\frac{\xi^{\gamma}}{1+\xi^{\gamma}}(1-\bar{N}), \quad (44)$$

from which we deduce that steady state θ does not depend on λ_{EE}: therefore, $d_{W,2}$ does not change with λ_{EE}.

which in log-linear form becomes

$$\psi^W \check{W}_1 = \frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}}\left[\check{\theta}_1 - \frac{\bar{c}\phi_{U,1}}{\frac{1}{2}\phi_{E,1}+\bar{c}\phi_{U,1}}\check{c}\right] + \frac{\bar{\varepsilon}_{21}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}}\left[-\frac{\bar{c}}{1-\bar{c}}\check{c} - \frac{\bar{c}\phi_{U,1}}{\frac{1}{2}\phi_{E,1}+\bar{c}\phi_{U,1}}\check{c} - \underbrace{\frac{g'(\bar{\theta}_1)\bar{\theta}_1}{g(\bar{\theta}_1)}}_{\equiv \varepsilon_{g,\theta}}\check{\theta}_1\right]. \tag{48}$$

With (40) and (48), in equilibrium, the equilibrium wage $\check{W}_1$ is given by

$$\check{W}_1 = \frac{d_W}{\psi^W \frac{1}{\gamma(1-\bar{c})} + d_W}\check{P}_1 < \check{P}_1, \tag{49}$$

and Propositions 5 and 6 holds as well in this case. Again, note that **Case 3** (which is the case in Section Bloesch, Lee and Weber (2024) with $\chi = 0$) generate similar results to **Case 2**, where $\kappa(\cdot)$ is a constant function.

1.3 Variable On-the-Job Search Intensity

Following Pilossoph and Ryngaert (2023), we now assume that on-the-job probability λ_{EE} at $t = 1$ is following

$$\lambda_{EE}(P_1, W_1) \equiv \bar{\lambda}_{EE}\left(\frac{\bar{W}_1}{\bar{P}_1}\right)^m \left(\frac{W_1}{P_1}\right)^{-m} \tag{50}$$

with $m = 4$. A cost-of-living shock raises $\lambda_{EE,1}$. Now from

$$\phi_{E,1} = \frac{\lambda_{EE}\bar{N}}{\lambda_{EE}\bar{N}+1-\bar{N}}, \quad \phi_{U,1} = \frac{1-\bar{N}}{\lambda_{EE}\bar{N}+1-\bar{N}}, \quad \theta_1 = \frac{V_1}{\lambda_{EE}\bar{N}+1-\bar{N}}, \tag{51}$$

we see that higher $\lambda_{EE,1}$ raises $\phi_{E,1}$ and lowers $\phi_{U,1}$, i.e., more job seekers are on-the-job searchers. We start from the equilibrium conditions with $\kappa(W_1) = \kappa$:

$$N_1 = \frac{\kappa}{W_1}\left\{\underbrace{\underbrace{\left(\lambda_{EE}\bar{N}+1-\bar{N}\right)\theta_1}_{=V_1}\left[\gamma\left(\frac{\frac{1}{2}\phi_{E,1}}{\frac{1}{2}\phi_{E,1}+\frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}}\right)\right]}_{\equiv \varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2}\frac{1}{1+\xi^\gamma}\lambda_{EU}\bar{N}}{\left(\frac{1}{2}\phi_{E,1}+\frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}\right)g(\theta_1)}}_{\equiv \varepsilon_{21}}\right\}, \tag{52}$$

and

$$
\begin{aligned}
N_1 &= \bar{N} - \bar{N}\frac{1}{1+\xi^\gamma}\lambda_{EU} + g(\theta_1)V_1\frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1} \\
&= \bar{N} - \bar{N}\frac{1}{1+\xi^\gamma}\lambda_{EU} + f(\theta_1)\frac{\xi^\gamma}{1+\xi^\gamma}\left(1-\bar{N}\right).
\end{aligned}
\tag{53}
$$

Price stickiness In contrast to Section 1.1 and Section 1.2 where we assume fully rigid prices, we assume a flexible form of price stickiness: in contrast to increase in W_1, service price $P_{Y,1}$ increases to some degree. More specifically, we assume $\check{P}_{Y,1} = d_P \check{W}_1$, with $d_P > 0$, where $\check{P}_{Y,1}$ and $\check{W}_1$ are log-deviations from their own steady state levels. $d_P = 0$ corresponds to the case of rigid prices.

Since $P_{Y,1}N_1 = \bar{P}_Y\bar{Y}$ holds due to the household's equal expenditure under pegged monetary policy, we know

$$
\check{N}_1 = -\check{P}_{Y,1} = -d_P\check{W}_1 = \frac{1}{1+\xi^\gamma}\lambda_{EU}\underbrace{\varepsilon_{f,\theta}}_{>0}\check{\theta}_1 \tag{54}
$$

where the last equality is derived from (53). From (54), we can see that if we have $\check{W}_1 > 0$ in equilibrium in response to a cost-of-living shock, i.e., $\check{P}_1 > 0$, then we need to have $\check{\theta}_1 < 0$, i.e., labor market becomes less tight. With lower θ_1, wage $\check{W}_1$ rises less in response to $\check{P}_1 > 0$ in (52), as θ_1 appears in ε_{11} and $g(\theta_1)$ is decreasing in θ_1: less tight labor market means that firms need not raise wage as much to attract job seekers and potential leavers.

By log-linearizing (51), we obtain

$$
\check{\phi}_{E,1} = \bar{\phi}_{U,1}\check{\lambda}_{EE}, \quad \check{\phi}_{U,1} = -\bar{\phi}_{E,1}\check{\lambda}_{EE} \tag{55}
$$

with $\check{\lambda}_{EE} = -m\left(\check{W}_1 - \check{P}_1\right)$. Linearizing (52) yields:

$$
\check{N}_1 = -\check{W}_1 + \frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}}\left[\bar{\phi}_{E,1}\check{\lambda}_{EE} + \check{\theta}_1 + (1-\chi)\check{\lambda}_{EE}\right] - \frac{\bar{\varepsilon}_{21}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}}\left[\chi\check{\phi}_{E,1} + (1-\chi)\check{\phi}_{U,1} - \varepsilon_{g,\theta}\check{\theta}_1\right], \tag{56}
$$

where

$$
\chi \equiv \frac{\frac{1}{2}\bar{\phi}_{E,1}}{\frac{1}{2}\bar{\phi}_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma}\bar{\phi}_{U,1}}.
$$

Combining equations (54), (55), and (56) with $\check{\lambda}_{EE} = -m(\check{W}_1 - \check{P}_1)$ and approximating

$\frac{\bar{\varepsilon}_{21}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}} \simeq 0$ with $\frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}} \simeq 1$ as before, we obtain

$$\check{W}_1 = \frac{m\left(\bar{\phi}_{EE}+1-\chi\right)}{1-d_P+m\left(\bar{\phi}_{EE}+1-\chi\right)+\frac{d_P}{\frac{\lambda_{EU}}{1+\xi^{\gamma}}\varepsilon_{f,\theta}}}\check{P}_1 > 0. \tag{57}$$

Interpretation Under fully rigid prices, i.e., $d_P = 0$, then we would have

$$\check{W}_1 = \frac{m\left(\bar{\phi}_{EE}+1-\chi\right)}{1+m\left(\bar{\phi}_{EE}+1-\chi\right)}\check{P}_1 > 0.$$

with $\check{\theta}_1 = 0$: no change in tightness. When employees engage in intensified on-the-job searches, firms offer more vacancies so that labor market tightness θ_1 remains the same: it is because under fully rigid prices, labor demand remains unchanged in response to a cost-of-living shock.

Under sticky prices following (54), $\check{\theta}_1 < 0$ and $\check{W}_1 > 0$ hold from equation (57). In equilibrium, firms raise service price in response to a cost-of-living shock, leading to lower service and labor demand. Since workers show a higher probability of on-the-job search, it reduces the market tightness θ_1. It in turn lowers the incentive of firms to raise wage to attract job seekers, resulting in muted wage responses: this effect is represented by $\frac{d_P}{\frac{\lambda_{EU}}{1+\xi^{\gamma}}\varepsilon_{f,\theta}}$.

On the other hand, a lower level of labor demand by service firms implies the marginal cost of wage increase in terms of wage bills (e.g., $ increase in wage implies that all workers, new hires and incumbents, benefit from it) is lower from each firm's perspective, and raises firms' incentive to raise wage: this effect is represented by d_P term in (57). In effect, the first effect dominates the second effect,[13] and wage increase under endogenous on-the-job search intensity is muted following (50).

Finally, even when $d = 0$, we see that $\check{W}_1 < \check{P}_1$ under $m > 0$, where $\check{W}_1$ is increasing in m, which confirms our intuition in Bloesch, Lee and Weber (2024).

[13] Remember $\frac{\lambda_{EU}}{1+\xi^{\gamma}}$ is small.

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