Ignorance is Bliss: Ex-Ante vs. Ex-Post Information Systems in an Agency Model

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Starting from the canonical principal-agent framework:

- Usually, the agent's utility $u(s)$ depends only on s, the monetary payment
- There might be other factors, t, that affect the agent's satisfaction and also are beyond the principal and the agent's controls
- Maybe $u(s, t)$ instead of $u(s)$
- t is eventually revealed in the end, so can be used in contracts: $s(x, t)$

Big Question (Main Topic)

If the principal designs (i) the timing of when t is revealed to the agent; and (ii) the amount of information about t revealed to the agent:

What is her optimal strategy?

[Timing] Principal designs when t is revealed

1. Before the agent signs on the contract?:

   Pre-Contract Information System

2. After the agent signs on the contract, but before takes an action

   Post-Contract Ex-Ante System

3. After takes an action

   Post-Contract Ex-Post System

[Amount] Principal designs how much of t is revealed

Finer vs. Coarser Partitions

Scenario 1 (Clients)

A bank hires a manager who provides his clients the personalized private banking service on a one-to-one basis

- The character, 't', of a client the manager takes charge of affects the manager's job satisfaction: $u(s, t)$

The bank's CEO knows about its high-profile clients well, and is deciding whether and when she should reveal information about clients to the new manager:

- Before he signs on the contract;
- After he signs on the contract but before starts his job
- Never reveals the information ex-ante and let the manager figure out through learning by doing (after he takes an action)

Eventually, the new hire, as well as the CEO, will get to know about his clients:

- Final monetary payments to the manager $s(x, t)$ rely on the revealed information about clients, 't'.

Scenario 2 (Match)

A worker can infer about his personal match to the new workplace through on-site visits and initial induction programs offered by the company.

- Fitness to the culture, aptitude for the work's nature, a sense of belonging to different peer groups, and a feeling of accomplishment from the job, 't', affect the new hire's satisfaction: $u(s, t)$

The firm's hiring manager is deciding whether and when to holds the events:

- Before he signs on the contract;
- After he signs on the contract but before starts his job
- Never offers those programs for new hires ex-ante and let the worker figure out (after he takes an action)

Eventually, both the hiring manger and the worker get to know about the degree of their match

- Final monetary payments to the new hire $s(x, t)$ rely on the revealed information about match, 't'.

Scenario 3 (Pension)

A company provides its workers with various pension plans

- Payouts from the pension plans, 't', are determined by a pension manager's portfolio decisions and the state of market
- Pension payouts, 't', affect workers' job satisfaction: $u(s, t)$

The company's CEO knows the track records of the pension manager's performance during the past few years, and is considering whether and when she should reveal those data to her new employee

- Before he signs on the contract;
- After he signs on the contract but before starts his job
- Never offers the information ex-ante and let the worker figure out (after he takes an action)

Eventually, the pension performance will be revealed

- Final monetary payments to the worker $s(x, t)$ rely on the revealed pension performance, 't'.

Findings

Big Question (Main Topic)

1. Which information system yields the best efficiency among the followings?
   - Pre-contract system
   - Post-contract ex-ante system
   - Post-contract ex-post system
2. Should principal offer the information as precise as possible ex-ante?
   - An amount of information about 't'

Answer

1. In terms of efficiency:

$$\text{Pre-contract} < \text{Post-contract ex-ante} < \text{Post-contract ex-post}$$

2. More precise ex-ante information$\uparrow$ $\rightarrow$ efficiency$\downarrow$
   - Agent uses the given ex-ante information in pursuit of his own interests, not on the principal's behalf

Our contribution:

- Prove the results in a very general agency framework

The Formulation

Single period agency setting: principal and agent

Actions: a action, and θ state of nature with $\mathbb{E}(\theta) = 0$

$$\text{Output } x = \underbrace{a}_{\text{Expected output}} + \underbrace{\theta}_{\text{State of nature}} \tag{1}$$

Preference of the agent:

$$U(s, t, a) = u(s, t) - v(a) \tag{2}$$

where

$$v'(a) > 0, v''(a) > 0, \underbrace{v'''(a) > 0}_{\text{Convexity}\uparrow\uparrow} \tag{3}$$

Contract: t is revealed in the end (contractible), thus s(x,t)

The agent knows the value of t even before signing on the contract

Principal designs $(a^p(t), s^p(x,t))$ to maximize:

$$\max_{a(t)\in A, s(x,t)\in S} \underbrace{\int_t \int_x (x - s(x,t)) f(x|a(t)) h(t) dx dt}_{\text{Expected welfare of the principal}} \quad \text{s.t:}$$

$$(i) \quad \boxed{\int_x u(s(x,t),t) f(x|a(t)) dx - v(a(t)) \geq \overline{U}}, \ \forall t \in T,$$

$$(ii) \quad \boxed{a(t) \in \arg\max_{a'} \int_x u(s(x,t),t) f(x|a') dx - v(a'), \ \forall a' \in A}, \ \forall t \in T,$$

$$(iii) \quad s(x,t) \geq 0, \quad \forall (x,t) \in X \times T. \tag{4}$$

Participation for $\forall t$

Incentive Compatibility for $\forall t$

Optimal contract:

$$\frac{1}{u_s(s^p(x,t),t)} = \lambda^p(t) + \mu^p(t)\frac{f_a}{f}(x|a^p(t)), \qquad (5)$$

where (5) has a solution $s^p(x,t) \geq 0$ and otherwise $s^p(x,t) = 0$

- Note: $(\lambda^p(t), \mu^p(t))$ are endogenous

Principal's welfare:

$$PW^p \equiv \int_t \int_x (x - s^p(x,t)) f(x|a^p(t))h(t)dxdt. \qquad (6)$$

Post-Contract Ex-Ante System

The agent knows t after signing on the contract but before taking action

Principal designs $(a^*(t), s^*(x,t))$ to maximize:

$$\max_{a(t)\in A, s(x,t)\in S} \underbrace{\int_t \int_x (x - s(x,t))\, f(x|a(t)) h(t) dx dt}_{\text{Expected welfare of the principal}} \quad \text{s.t:}$$

$$(i) \quad \boxed{\int_t \left(\int_x u(s(x,t),t) f(x|a(t)) dx - v(a(t)) \right) h(t) dt \geq \overline{U}}, \ \forall t \in T,$$

$$(ii) \quad \boxed{a(t) \ \in \arg\max_{a'} \int_x u(s(x,t),t) f(x|a') dx - v(a'), \ \forall a' \in A}, \ \forall t \in T,$$

$$(iii) \quad s(x,t) \geq 0, \quad \forall (x,t) \in X \times T. \tag{7}$$

Participation on average t

Incentive Compatibility for $\forall t$

Optimal contract:

$$\frac{1}{u_s(s^*(x,t),t)} = \lambda^* + \mu^*(t)\frac{f_a}{f}(x|a^*(t)) \tag{8}$$

where (8) has a solution $s^*(x,t) \geq 0$ and otherwise $s^*(x,t) = 0$

- Note: $(\lambda^*, \mu^*(t))$ are endogenous

Principal's welfare:

$$PW^* \equiv \int_t \int_x (x - s^*(x,t))\, f(x|a^*(t))h(t)dxdt \tag{9}$$

The agent knows t after signing on the contract and after taking action

Principal designs $(a^o, s^o(x,t))$ to maximize:

$$\max_{a(t)\in A, s(x,t)\in S} \underbrace{\int_t \int_x (x - s(x,t)) f(x|a(t)) h(t) dxdt}_{\text{Expected welfare of the principal}} \quad \text{s.t:}$$

$$(i) \quad \boxed{\int_t \left(\int_x u(s(x,t),t) f(x|a(t)) dx - v(a(t)) \right) h(t) dt \geq \overline{U}},$$

$$(ii) \quad \boxed{a \in \arg\max_{a'} \int_t \int_x u(s(x,t),t) f(x|a') h(t) dxdt - v(a'), \ \forall a' \in A},$$

$$(iii) \quad s(x,t) \geq 0, \quad \forall (x,t) \in X \times T. \tag{10}$$

Participation on average t

Incentive Compatibility on average t

Optimal contract:

$$\frac{1}{u_s(s^o(x,t),t)} = \lambda^o + \mu^o \frac{f_a}{f}(x|a^o) \qquad (11)$$

where (11) has a solution $s^o(x,t) \geq 0$ and otherwise $s^o(x,t) = 0$

- Note: (λ^o, μ^o) are endogenous

Principal's welfare:

$$PW^o \equiv \int_t \int_x (x - s^o(x,t))\, f(x|a^o) h(t) dxdt. \qquad (12)$$

Comparison

Lemma (Pre-Contract vs. Post-Contract Ex-Ante)

$$PW^p \leq PW^*$$

1. Compared with the post-contract ex-ante system:
   - Under the pre-contract system, the agent's (PC) binds for $\forall t$
   - Under the post-contract ex-ante system, the agent's (PC) binds on average across t, which is easier
   - Similar to Sobel (1993)
2. Therefore, in terms of efficiency:

$$\text{Pre-Contract} \leq \text{Post-Contract Ex-Ante}$$

Comparison 2

Lemma (Post-Contract Ex-Ante vs. Post-Contract Ex-Post)

$$PW^* \leq PW^o$$

Under the post-contract ex-ante system:

- The agent's action $a(t)$ depends on t, so action is a random variable
- Under the post-contract ex-post system, $a(t) = a^o$ is uniform across t

$v''(a) > 0$

- Makes it harder to satisfy the agent's (IR) on average across t
- Why? The average $\mathbb{E}\left(v(a(t))\right) \uparrow$

$v'''(a) > 0$

- Makes it harder to satisfy the agent's (IC) on average across t
- Why? The average $\mathbb{E}\left(v'(a(t))\right) \uparrow$

Proof is heavy, based on Kim (1995)

Comparison 3

Lemma (Special case 1)

If the agent's utility function satisfies $\frac{u_{ss}}{u_{sss}} = \frac{u_{st}}{u_{sst}}$, and if the agent's limited liability constraint is not binding for any (x, t), then

$$\frac{\partial^2}{\partial x \partial t} s^o(x, t) = 0. \tag{13}$$

In this case, the optimal contract $s^o(x, t)$ under the post-contract ex-post features the same incentive (sensitivity to x) for $\forall t$

- So ex-ante $\equiv$ ex-post

Proposition (Equivalence between ex-post and ex-ante)

If $\frac{u_{ss}}{u_{sss}} = \frac{u_{st}}{u_{sst}}$, and the limited liability constraint is not binding for any (x, t), then the principal is indifferent between the post-contract ex-ante information system and the post-contract ex-post information system, i.e.,

$$PW^p \leq PW^* = PW^o. \tag{14}$$

Furthermore, in this case,

$$a^*(t) = a^o, \quad \forall t \in T. \tag{15}$$

Proposition (Special case 2)

If $\frac{u_s}{u_{ss}} = \frac{u_t}{u_{st}}$ and the limited liability constraint is not binding for any (x, t), then the principal is indifferent among the three information systems, i.e.,

$$PW^p = PW^* = PW^o. \tag{16}$$

Example: Special case 1

$$u(s, t) = u(s + k(t)) + l(t) \tag{17}$$

Example: Special case 2

$$u(s, t) = u(s + k(t)) \tag{18}$$

Information Partition

Still t is revealed in the end: so contractible s(x,t)

Information system N: partitions on $T = [\underline{t}, \overline{t}]$, $\{T_1, T_2, ..., T_j, ..., T_N\}$

- Agent ex-ante knows which partition $i = 1, 2, ..., N$ the true t belongs

Information system N^+: partitions $\{T_1, T_2, ..., T_j^-, T_j^+, ..., T_N\}$

- Now T_j is decomposed into T_j^- and T_j^+
- Finer (more precise) information system

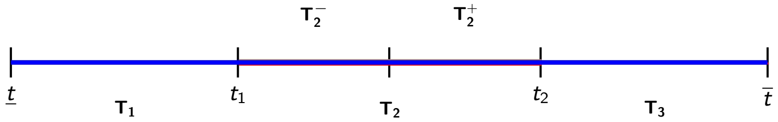

Figure: An Example of Information Systems N and N^+

The agent's action $a(T_i)$ depends on a partition T_i

Principal designs $\left(a^N(T_i), s^N(x,t)\right)$ to maximize:

$$\max_{\substack{\{a(T_i)\in A\}_{1\le i\le N} \\ s(x,t)\in S}} \sum_{i=1}^{N} p_i \left(\int_{t\in T_i} \int_x (x - s(x,t))\, f\,(x|a(T_i))\, h(t|T_i) dxdt \right) \qquad \text{s.t.}$$

$$(i) \quad \sum_{i=1}^{N} p_i \left(\int_{t\in T_i} \int_x u(s(x,t),t) f\,(x|a(T_i))\, h(t|T_i) dxdt - v(a(T_i)) \right) \ge \overline{U},$$

$$(ii) \quad \int_{t\in T_i} \int_x u(s(x,t),t) f_a\,(x|a(T_i))\, h(t|T_i) dxdt = v'(a(T_i)), \quad \forall T_i \in T$$

$$(iii) \quad s(x,t) \ge 0, \quad \forall (x,t) \in X \times T. \qquad (19)$$

Participation on average T_i

Incentive Compatibility for given T_i

Information system N^+ is similar

Principal's welfare under N:

$$PW^N \equiv \sum_{i=1}^{N} p_i \left(\int_{t \in T_i} \int_x \left(x - s^N(x,t) \right) f\left(x | a^N(T_i) \right) h(t | T_i) dx dt \right) \quad (20)$$

Principal's welfare under N^+:

$$PW^{N^+} \equiv \sum_{i=1}^{N^+} p_i \left(\int_{t \in T_i} \int_x \left(x - s^{N^+}(x,t) \right) f\left(x | a^{N^+}(T_i) \right) h(t | T_i) dx dt \right) \quad (21)$$

Proposition (N vs. N^+)

$$PW^{N^+} \leq PW^N$$

- More ex-ante information is always bad for the principal

Illustrative Example

Now assume $u(s,t) = 2t\sqrt{s}$

Under the post-contract ex-ante system:

- Given the optimal $\{a^*(t)\}$, the optimal $s^*(x,t)$ minimizes the agency cost

Agency cost (ex-ante):

$$AC_M^*(\{a^*(t)\}) \equiv \min_{s(x,t)\in S} \int_t \int_x s(x,t) f(x|a^*(t)) h(t) dxdt \quad \text{s.t.}$$
$$(i) \quad \int_t \left(\int_x 2t\sqrt{s(x,t)} f(x|a^*(t)) dx - v(a^*(t)) \right) h(t) dt \geq \overline{U},$$
$$(ii) \quad \int_x 2t\sqrt{s(x,t)} f_a(x|a^*(t)) dx - v'(a^*(t)) = 0, \ \forall t \in T,$$
$$(iii) \quad s(x,t) \geq 0, \quad \forall (x,t) \in X \times T. \tag{22}$$

Turns out that we can express:

$$AC_M^*(\{a^*(t)\}) = \underbrace{AC_{M,\mathrm{IR}}^*(\{a^*(t)\})}_{\text{Agency cost for insuring (IR) on average } t} + \underbrace{AC_{M,\mathrm{IC}}^*(\{a^*(t)\})}_{\text{Agency cost for insuring (IC) on } \forall t} \tag{23}$$

Under the post-contract ex-post system:

- Fix $a^m = \mathbb{E}\left(a^*(t)\right)$ and think of $s^m(x,t)$ that minimizes the agency cost

Agency cost (ex-post):

$$\begin{aligned} AC_M^o(a^m) \equiv \min_{s(x,t)\in S} & \int_t \int_x s(x,t) f(x|a^m) h(t) dxdt \quad \text{s.t.} \\ (i) \quad & \int_t \int_x 2t\sqrt{s(x,t)} f(x|a^m) h(t) dxdt - v(a^m) \geq \overline{U}, \\ (ii) \quad & \int_t \int_x 2t\sqrt{s(x,t)} f_a(x|a^m) h(t) dxdt - v'(a^o) = 0, \\ (iii) \quad & s(x,t) \geq 0, \quad \forall (x,t) \in X \times T. \end{aligned} \tag{24}$$

Turns out that we can express:

$$AC_M^o(\{a^m\}) = \underbrace{AC_{M,\text{IR}}^o(\{a^m\})}_{\text{Agency cost for insuring (IR) on average } t} + \underbrace{AC_{M,\text{IC}}^o(\{a^m\})}_{\text{Agency cost for insuring (IC) on average } t} \tag{25}$$

Note: $(a^m, s^m(x,t))$ might not be optimal: $(a^m, s^m(x,t)) \neq (a^o, s^o(x,t))$

Comparison (agency cost)

Lemma (Agency cost comparison)

$$AC^{o}_{M,\text{IR}}(a^{m}) < AC^{*}_{M,\text{IR}}(\{a^{*}(t)\}) \text{ and } AC^{o}_{M,\text{IC}}(a^{m}) < AC^{*}_{M,\text{IC}}(\{a^{*}(t)\})$$

Recap: $\underline{v''(a) > 0}$

- Makes it harder to satisfy the agent's (IR) on average across t
- Why? The average $\mathbb{E}\,(v(a(t)))\uparrow$

Recap: $\underline{v'''(a) > 0}$

- Makes it harder to satisfy the agent's (IC) on average across t
- Why? The average $\mathbb{E}\,(v'(a(t)))\uparrow$