Higher-Order Forward Guidance

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Motivation

Big Question

Forward guidance — How does it work, exactly?

- First-order effects (level): "Interest rates will stay low" $\longrightarrow$ intertemporal substitution channel (aggregate demand$\uparrow$)
- Second-order effects (volatility): reduce uncertainty, avoid worst-case scenarios, "whatever it takes" $\longrightarrow$ precautionary savings channel (aggregate demand$\uparrow$)

This paper: focus on central bank's strategic uncertainty management and coordination. Possible for central banks to pick an equilibrium where:

- During the ZLB (now): reduce aggregate volatility. Then aggregate demand$\uparrow$
- But central banks now create uncertainty about where the economy ends up after the ZLB (future): commit less stabilization after the ZLB
- Welfare-enhancing overall

A textbook New Keynesian model with rigid price

- The representative household's problem (given B_0) is

$$\Gamma_t \equiv \max_{\{B_t\}_{t>0},\{C_t,L_t\}_{t\geq 0}} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left[\log C_t - \frac{L_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right] dt \quad \text{s.t.} \quad \dot{B}_t = i_t B_t - \bar{p} C_t + w_t L_t + D_t$$

where

- B_t: nominal bond holding, D_t includes fiscal transfer + profits
- Rigid price: $p_t = \bar{p}$ for $\forall t$ (i.e., purely demand-determined)

Endogenous volatility

A non-linear Euler equation (in contrast to log-linearized one)

$$\mathbb{E}_t\left(\frac{dC_t}{C_t}\right) = (i_t - \rho)dt + \underbrace{\mathrm{Var}_t\left(\frac{dC_t}{C_t}\right)}_{\text{Precautionary premium}}$$

Endogenous drift

- Aggregate volatility↑ $\Longrightarrow$ precautionary saving↑ $\Longrightarrow$ recession (the drift↑)

A textbook New Keynesian model with rigid price

The remaining equilibrium conditions are as usual

Intratemporal optimality condition:

$$\frac{1}{\bar{p}C_t} = \frac{L_t^{\frac{1}{\eta}}}{w_t}$$

Transversality condition:

$$\lim_{t\to\infty} \mathbb{E}_0 \left[e^{-\rho t}\Gamma_t\right] = 0$$

Firms: monopolistic competition à la Dixit-Stiglitz with $Y_t^i = A_t L_t^i$ and

$$\frac{dA_t}{A_t} = g\,dt + \underbrace{\sigma}_{\text{Fundamental risk}} dZ_t$$

- dZ_t: aggregate Brownian motion (i.e., only risk source)
- (g, σ) are exogenous

Non-linear IS equation

Defining output gap and excess volatility:

$$\hat{Y}_t = \ln \frac{Y_t}{Y_t^n}, \quad \underbrace{(\sigma)^2 dt = \mathrm{Var}_t\left(\frac{dY_t^n}{Y_t^n}\right)}_{\text{Benchmark volatility}}, \quad \underbrace{(\sigma + \sigma_t^s)^2 dt = \mathrm{Var}_t\left(\frac{dY_t}{Y_t}\right)}_{\text{Actual volatility}}$$

A non-linear IS equation in output gap:

Fundamental volatility

$$d\hat{Y}_t = \left(i_t - \underbrace{\left(r^n - \frac{1}{2}(\sigma + \sigma_t^s)^2 + \frac{1}{2}\sigma^2\right)}_{\mathrm{rp}_t \equiv r_t^T \qquad \mathrm{rp}^n}\right) dt + \sigma_t^s \, dZ_t$$

$$= (i_t - r_t^T)dt + \sigma_t^s dZ_t$$

$$\sigma_t^s \uparrow \longrightarrow \mathrm{rp}_t \uparrow \longrightarrow \hat{Y}_t \downarrow$$

ZLB from fundamental volatility shock

Thought experiment: fundamental volatility $\sigma\uparrow$: $\bar{\sigma}$ on $[0, T]$ (e.g., Werning (2012)) and comes back to $\underline{\sigma}$ with $\bar{\sigma} > \underline{\sigma}$

- $\bar{r} \equiv r^n(\underline{\sigma}) = \rho + g - \underline{\sigma}^2 > 0$: no ZLB before, $t < 0$, or after, $t > T$
- $\underline{r} \equiv r^n(\bar{\sigma}) = \rho + g - \bar{\sigma}^2 < 0$: ZLB binds for $0 \leq t \leq T$

Assume: perfect stabilization (i.e., $\hat{Y}_t = 0$) is achievable outside ZLB, i.e.,

$$i_t = \bar{r} + \phi_y \hat{Y}_t - \frac{1}{2}\underbrace{\left(\mathrm{rp}_t - \mathrm{rp}_t^n\right)}_{\text{Variance gap}}, \quad \text{with } \phi_y > 0$$

Result: perfect stabilization of variance gap (i.e., excess uncertainty) inside the ZLB

- Recursive argument: full stabilization at T implies $\hat{Y}_T = 0 \longrightarrow \sigma^s_{T-\mathrm{d}t} = 0$, and keeps going on (so $\mathrm{rp}_t = \mathrm{rp}_t^n = \bar{\sigma}^2$ for $\forall t$)

ZLB path (full stabilization after T)

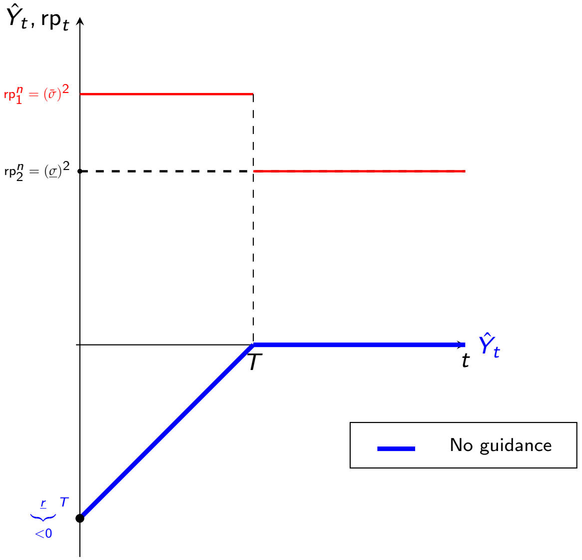


Figure: ZLB dynamics (Benchmark)

Traditional forward guidance (keep $i_t = 0$ until $\hat{T}^{\text{TFG}} > T$)

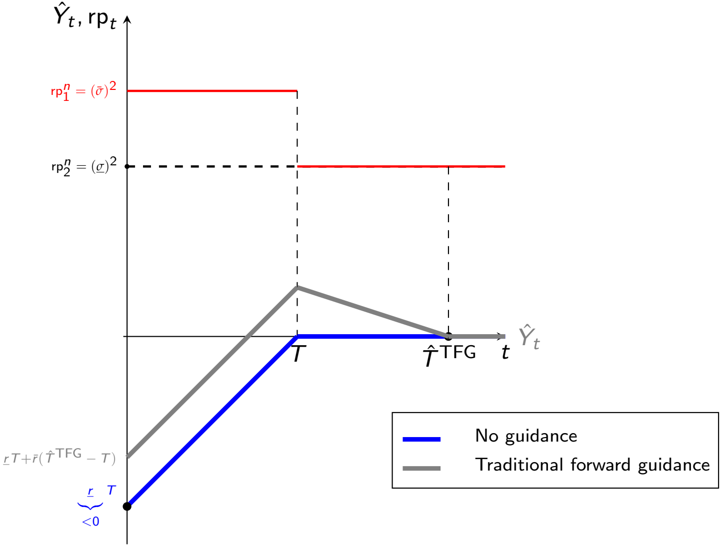


Figure: ZLB dynamics with forward guidance until $\hat{T}^{\text{TFG}} > T$

Details

Alternative forward guidance policies

Big Question

Can we do even better than the traditional forward guidance?

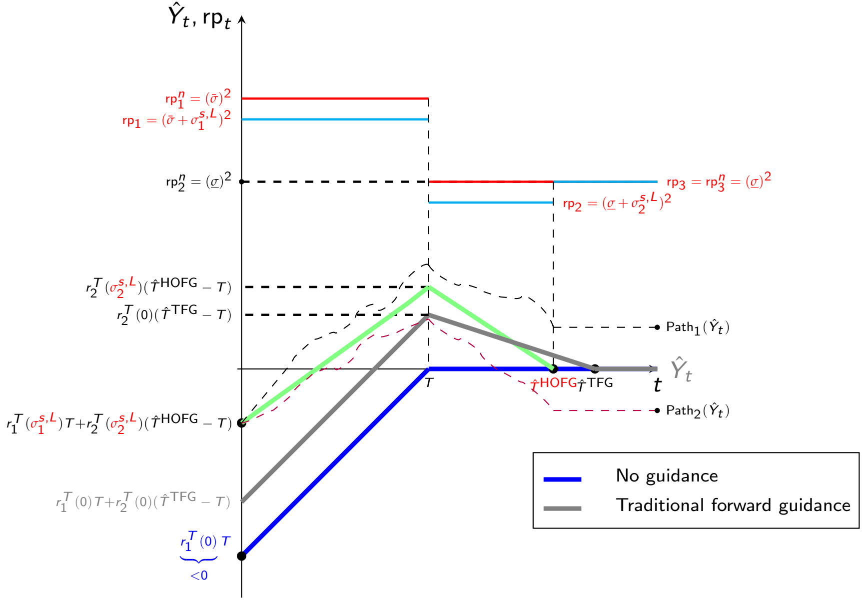
What if we reduce aggregate uncertainty via $\sigma_t^s < 0$?

- Then $\mathrm{rp}_t = (\bar{\sigma} + \sigma_t^s)^2 < \mathrm{rp}_t^n$, raising aggregate demand and $\hat{Y}_t$

But how?

- Nominal rigidities $\longrightarrow$ demand-determined production
- Policy challenge: the central bank *must convince* households to "coordinate" on this particular equilibrium $\longrightarrow$ *higher-order forward guidance*
- Give up perfect stabilization in the future (no stabilization at all)
- Imagine the central bank pegs the policy rate at $i_t = \bar{r}$ after ZLB ends

Central bank picks $\hat{T}^{HOFG}$ and $\{\sigma_t^s < 0\}_{t<\hat{T}^{HOFG}}$



At optimum, $\sigma_1^{s,L} < 0 = \sigma_1^{s,n}$, $\sigma_2^{s,L} < 0 = \sigma_2^{s,n}$, and $\hat{T}^{HOFG} < \hat{T}^{TFG}$

Details

Optimal policy

Proposition (Optimal forward guidance policy)

Optimal higher-order forward guidance (HOFG) always results in an equal or lower expected quadratic loss than the traditional guidance policy

Proof.

With $(\sigma_1^{s,L}, \sigma_2^{s,L}, \hat{T}^{\text{HOFG}}) = (0, 0, \hat{T}^{\text{TFG}})$, solutions coincide □

Remarks:

- Alternative higher-order forward guidance policy implementations **are** possible
- This paper shows HOFG dominates TFG in a simple setting

Optimal policy: stochastic stabilization

Extension: still higher-order forward guidance policy, now with stochastic stabilization after $\hat{T}^{\text{HOFG}}$. Return to stabilization with νdt probability after $\hat{T}^{\text{HOFG}}$

- Central bank commits to stabilizing the economy after $\hat{T}^{HOFG}$ with some probability. Expected stabilization after $1/\nu$ quarters
- $\nu = 0$: the above higher-order forward guidance
- $\nu = \infty$: the traditional forward guidance policy

Big discontinuity:

$$\lim_{\nu \to +\infty^-} \mathbb{L}^{Y,*}\left(\{\hat{Y}_t\}_{t \geq 0}, \nu\right) < \underbrace{\mathbb{L}^{Y,*}\left(\{\hat{Y}_t\}_{t \geq 0}, \nu = \infty\right)}_{\text{Traditional forward guidance}}$$

- Slight probability that stabilization might not happen $\longrightarrow$ HOFG possible

Details

Policy implication

Real World Example (Covid-19 and the Federal Reserve)

Flexible Average Inflation Targeting (FAIT) (2020)

- Commitment to delaying stabilization – by allowing inflation to "moderately" overshoot its target after periods of persistent undershooting at the ZLB
- "Moderate" overshooting of the business cycle now is allowed: nudging agents toward a favorable equilibrium with lower volatility

HOFG equilibrium $\longrightarrow$ can be supported by fiscal policy as a unique equilibrium

- Zero transfer along the equilibrium path (out-of-equilibrium threat) Details
- Draghi's "whatever it takes" speech $\longrightarrow$ lower periphery yields without actual expenditures, coordinating agents to an equilibrium with lower risk premium (Acharya et al., 2019)

Welfare comparisons

$T = 20$ quarters ZLB spell

Loss function $\mathbb{L}$ as the (conditional) quadratic output loss per quarter:

$$\mathbb{L}^{Y}_{\text{Per-period}} \equiv \rho \int_0^\infty e^{-\rho t} \mathbb{E}_0 \hat{Y}_t^2$$

Policy	**No guidance**	**Traditional**	**Higher-Order** (no stochastic stabilization)	**Higher-Order** (*with* stoch. stab., $\nu = 1$)
$\sigma_1^{s,L}$	0	0	-1.27%	-4.13%
$\sigma_2^{s,L}$	0	0	-0.24%	-3.79%
$\hat{T}^{\text{HOFG}}$	20	25.27	25.09	24.68
$\mathbb{L}^Y$	1.14%	0.32%	0.29%	0.27%

- Still, traditional forward guidance too strong: e.g., McKay et al. (2016)
- HOFG with $\nu \to \infty$ but $\nu \neq \infty$ most effective

Takeaways

1. **Higher-order forward guidance:** manage intertemporal uncertainty via central-bank *equilibrium selection*.

2. Traditional forward guidance raises welfare, **but HOFG can do better.**

3. **Stabilization trade-off:** stabilize today by credibly being "irresponsible" later.

4. **Credibility remains necessary.**

Thank you very much!
(Appendix)

Traditional forward guidance

Assume:

- Central bank commits to keep $i_t = 0$ until $\hat{T}^{\text{TFG}} \geq T$ (i.e., Odyssean guidance)
- Perfect stabilization (i.e., $\hat{Y}_t = 0$) afterwards, i.e., for $t > \hat{T}^{\text{TFG}}$
- By the same arguments, volatility gap stabilization beforehand, $t \leq \hat{T}^{\text{TFG}}$ (no excess volatility while $i_t = 0$)

Problem: minimize smooth quadratic welfare loss

$$\min_{\hat{T}^{\text{TFG}}} \mathbb{L}^Y\left(\{\hat{Y}\}_{t\geq 0}\right) \equiv \mathbb{E}_0 \int_0^\infty e^{-\rho t}\left(\hat{Y}_t\right)^2 dt$$

$$\text{s.t. } \hat{Y}_0 = \underbrace{\underline{r}}_{<0} T + \underbrace{\bar{r}}_{>0}\left(\hat{T}^{\text{TFG}} - T\right)$$

- Smoothing the ZLB costs over time (i.e., welfare enhancing)

Go back

Higher-order forward guidance

Assume:

- Central bank can commit to keep $i_t = 0$ until $\hat{T}^{HOFG} \geq T$
- No stabilization (i.e., $\hat{Y}_t = \hat{Y}_{\hat{T}^{HOFG}}$) guaranteed afterwards, $t \geq \hat{T}^{HOFG}$
- Pick $\{\sigma_t^s\}$ for $t < \hat{T}^{HOFG}$

Problem: minimize smooth quadratic welfare loss

$$\min_{\sigma_1^{s,L}, \sigma_2^{s,L}, \hat{T}^{HOFG}} \mathbb{L}^Y \left(\{\hat{Y}\}_{t \geq 0}\right) \equiv \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left(\hat{Y}_t\right)^2 dt,$$

$$\text{s.t.} \begin{cases} d\hat{Y}_t = -\underbrace{r_1^T\left(\sigma_1^{s,L}\right)}_{<0} dt + \sigma_1^{s,L} dZ_t, & \text{for } t < T, \\ d\hat{Y}_t = -\underbrace{r_2^T\left(\sigma_2^{s,L}\right)}_{>0} dt + \sigma_2^{s,L} dZ_t, & \text{for } T \leq t < \hat{T}^{HOFG}, \\ d\hat{Y}_t = 0, & \text{for } t \geq \hat{T}^{HOFG}, \end{cases}$$

with

$$\hat{Y}_0 = \underbrace{r_1^T\left(\sigma_1^{s,L}\right)}_{<0} T + \underbrace{r_2^T\left(\sigma_2^{s,L}\right)}_{>0} \left(\hat{T}^{HOFG} - T\right)$$

Go back

Higher-order forward guidance with stochastic stabilization

Change:

- Central bank commits to stabilizing the economy after $\hat{T}^{HOFG}$ with Poisson probability ν: at each point after $\hat{T}^{HOFG}$, $\hat{Y}_t$ becomes 0 with probability νdt

Problem: minimize smooth quadratic welfare loss

$$\min_{\sigma_1^{s,L},\sigma_2^{s,L},\hat{T}^{\text{HOFG}}} \quad \mathbb{E}_0\left[\int_0^{\hat{T}^{\text{HOFG}}} e^{-\rho t}\,\hat{Y}_t^2\,dt + \int_{\hat{T}^{\text{HOFG}}}^{\infty} e^{-\rho t}\,e^{-\nu\left(t-\hat{T}^{\text{HOFG}}\right)}\,\hat{Y}_t^2\,dt\right],$$

$$\text{s.t.}\quad \begin{cases} d\hat{Y}_t = -\underbrace{r_1^T\left(\sigma_1^{s,L}\right)}_{<0}dt + \sigma_1^{s,L}dZ_t, & \text{for } t < T, \\ d\hat{Y}_t = -\underbrace{r_2^T\left(\sigma_2^{s,L}\right)}_{>0}dt + \sigma_2^{s,L}dZ_t, & \text{for } T \leq t < \hat{T}^{HOFG}, \\ d\hat{Y}_t = 0, & \text{for } t \geq \hat{T}^{HOFG}, \end{cases}$$

with

$$\hat{Y}_0 = \underbrace{r_1^T\left(\sigma_1^{s,L}\right)}_{<0}T + \underbrace{r_2^T\left(\sigma_2^{s,L}\right)}_{>0}\left(\hat{T}^{HOFG} - T\right)$$

Go back

Fiscal policy coordination

Go back

Fiscal authority's monetary reserves F_t

$$dF_t = -\tau_t^D dZ_t, \quad \text{with:} \quad F_0 = F_{0-} - \underbrace{\chi^D}_{\substack{\text{Instant}\\ \text{transfer}}} ,$$

Household transfers D_t consist of firm profits plus an exogenous transfer from government reserves (not financed by taxes):

$$\Delta D_0 = \bar{p}\Delta Y_0 - \Delta(w_0 L_0) + \chi^D$$
$$D_t = \bar{p} Y_t - w_t L_t + \tau_t^D$$

Proposition

HOFG equilibrium (with $\sigma_t^{s,*}$) becomes a unique equilibrium under the following rule:

$$\tau_t^D = \bar{p}(Y_t^* - Y_t) \, , \quad \text{and} \quad \chi^D = \bar{p}(Y_0^* - Y_0) \, ,$$

In this case, $\tau_t^D = 0$, and $\chi^D = 0$ on the equilibrium path