A Global Approach to New Keynesian Macroeconomics

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Paper 1. Self-fulfilling Volatility and a New Monetary Policy

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What we do

Standard non-linear New Keynesian model

$\exists$a price of risk coming from

$$\underbrace{\text{Aggregate volatility}\ \uparrow}_{\text{Endogenous}} \Longrightarrow \underbrace{\text{precautionary saving}}_{\text{Endogenous}}\uparrow \Longrightarrow \underbrace{\text{aggregate demand}\ \downarrow}_{\text{Endogenous}}$$

Takeaway (Self-fulfilling volatility)

In macroeconomic models with nominal rigidities, $\exists$global solution where:

- Taylor rules (targeting inflation and output) $\longrightarrow$ $\exists$self-fulfilling apparition of aggregate volatility
- Only direct volatility (e.g., risk premium) targeting can restore determinacy

A textbook New Keynesian model with rigid price

The representative household's problem (given B_0):

$$\Gamma_t \equiv \max_{\{B_t\}_{t>0}, \{C_t, L_t\}_{t\geq 0}} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left[\log C_t - \frac{L_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right] dt \text{ s.t. } \dot{B}_t = i_t B_t - \bar{p} C_t + w_t L_t + D_t$$

where

- B_t: nominal bond holding, D_t includes fiscal transfer + profits
- Rigid price: $p_t = \bar{p}$ for $\forall t$ (i.e., purely demand-determined)

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- B_t: nominal bond holding, D_t includes fiscal transfer + profits
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1. A log-linearized Euler equation (textbook one)

Monetary policy

$$\mathbb{E}_t \left(\frac{dC_t}{C_t} \right) = (i_t - \rho) dt$$

- Forward-looking consumption process (monetary policy i_t stabilizes C_t)

Endogenous drift

Caveat: Taylor rules with the Taylor principle is enough

A textbook New Keynesian model with rigid price

The representative household's problem (given B_0):

$$\Gamma_t \equiv \max_{\{B_t\}_{t>0},\{C_t,L_t\}_{t\geq 0}} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left[\log C_t - \frac{L_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right] dt \quad \text{s.t.} \quad \dot{B}_t = i_t B_t - \bar{p} C_t + w_t L_t + D_t$$

where

- B_t: nominal bond holding, D_t includes fiscal transfer + profits
- Rigid price: $p_t = \bar{p}$ for $\forall t$ (i.e., purely demand-determined)

Endogenous volatility

2. A non-linear Euler equation (in contrast to log-linearized one)

$$\mathbb{E}_t\left(\frac{dC_t}{C_t}\right) = (i_t - \rho)dt + \underbrace{\mathrm{Var}_t\left(\frac{dC_t}{C_t}\right)}_{\text{Precautionary premium}}$$

Endogenous drift

- Aggregate volatility↑ $\Longrightarrow$ precautionary saving↑ $\Longrightarrow$ recession (the drift↑)

Problem: both **variance** and **drift** are endogenous, is Taylor rule enough?

A textbook New Keynesian model with rigid price

Intra-temporal optimality:

$$\frac{1}{\bar{p}C_t} = \frac{L_t^{\frac{1}{\eta}}}{w_t}$$

Transversality condition:

$$\lim_{t\to\infty} \mathbb{E}_0 \left[e^{-\rho t} \Gamma_t \right] = 0 \tag{1}$$

A textbook New Keynesian model with rigid price

Firm i: face monopolistic competition à la Dixit-Stiglitz with $Y_t^i = A_t L_t^i$ and

$$\frac{dA_t}{A_t} = g dt + \underbrace{\sigma}_{\text{Fundamental risk}} dZ_t$$

- dZ_t: aggregate Brownian motion (i.e., only risk source)
- (g, σ) are exogenous

Flexible price economy as benchmark: the 'natural' output Y_t^n follows

$$\frac{dY_t^n}{Y_t^n} = \left(r^n - \rho + \sigma^2\right) dt + \sigma dZ_t$$
$$= g dt + \sigma dZ_t = \frac{dA_t}{A_t}$$

where $r^n = \rho + g - \sigma^2$ is the 'natural' rate of interest

Non-linear IS equation

$$\hat{Y}_t = \ln \frac{Y_t}{Y_t^n}, \quad \underbrace{(\,\sigma\,)^2\, dt = \mathrm{Var}_t\left(\frac{dY_t^n}{Y_t^n}\right)}_{\text{Benchmark volatility}}, \quad \underbrace{\left(\sigma + \sigma_t^s\right)^2 dt = \mathrm{Var}_t\left(\frac{dY_t}{Y_t}\right)}_{\text{Actual volatility}}$$

Exogenous

Endogenous

Non-linear IS equation

$$\hat{Y}_t = \ln \frac{Y_t}{Y_t^n}, \quad \underbrace{\left(\sigma \right)^2 dt = \mathrm{Var}_t \left(\frac{dY_t^n}{Y_t^n} \right)}_{\text{Benchmark volatility}}, \quad \underbrace{\left(\sigma + \sigma_t^s \right)^2 dt = \mathrm{Var}_t \left(\frac{dY_t}{Y_t} \right)}_{\text{Actual volatility}}$$

Exogenous

Endogenous

1. A log-linearized IS equation (textbook one)

$$d\hat{Y}_t = (i_t - r^n)\, dt + \sigma_t^s dZ_t \tag{2}$$

which becomes

$$\mathbb{E}_t \left(d\hat{Y}_t \right) = (i_t - r^n)\, dt,$$

or

$$\hat{Y}_t = \mathbb{E}_t \left(\hat{Y}_{t+dt} \right) - (i_t - r^n)\, dt$$

Non-linear IS equation

$$\hat{Y}_t = \ln \frac{Y_t}{Y_t^n}, \quad \underbrace{(\sigma)^2 dt = \mathrm{Var}_t\left(\frac{dY_t^n}{Y_t^n}\right)}_{\text{Benchmark volatility}}, \quad \underbrace{(\sigma + \sigma_t^s)^2 dt = \mathrm{Var}_t\left(\frac{dY_t}{Y_t}\right)}_{\text{Actual volatility}}$$

Exogenous

Endogenous

2. A non-linear IS equation (in contrast to textbook linearized one)

$$d\hat{Y}_t = \left(i_t - \underbrace{\left(r^n \overbrace{-\frac{1}{2}(\sigma + \sigma_t^s)^2 + \frac{1}{2}\sigma^2}^{\text{New terms}} \right)}_{\equiv r_t^T} \right) dt + \sigma_t^s \, dZ_t \qquad (3)$$

What is r_t^T?: a risk-adjusted natural rate of interest ($\sigma_t^s \uparrow \Longrightarrow r_t^T \downarrow$)

$$r_t^T \equiv r^n - \frac{1}{2} \underbrace{(\sigma + \sigma_t^s)^2}_{\text{Precautionary premium}} + \frac{1}{2}\sigma^2$$

Non-linear IS equation

Big Question

Taylor rule $i_t = r^n + \phi_y \hat{Y}_t$ for $\phi_y > 0 \implies$ perfect stabilization?

Answer (Up to a First Order)

$\phi_y > 0$ **(Taylor principle)** guarantees

- Unique equilibrium with perfect stabilization, i.e., $\hat{Y}_t = 0$ with $\sigma_t^s = 0$ for $\forall t$

Why? (recap): without the volatility feedback:

$$d\hat{Y}_t = (i_t - r^n)\, dt + \sigma_t^s dZ_t \underbrace{=}_{\substack{\text{Under} \\ \text{Taylor rule}}} \phi_y \hat{Y}_t dt + \sigma_t^s dZ_t$$

Then,

$$\mathbb{E}_t \left(d\hat{Y}_t \right) = \phi_y \hat{Y}_t .$$

If $\hat{Y}_t \neq 0$,

$$\lim_{s \to \infty} \mathbb{E}_t \left(\hat{Y}_s \right) \to \pm\infty$$

- Foundation of modern central banking

Now, with the non-linear effects in (3)

Proposition (Fundamental Indeterminacy)

For any $\phi_y > 0$, $\exists$an equilibrium supporting a volatility $\sigma_0^s > 0$ satisfying:

1. $\mathbb{E}_t\left(d\hat{Y}_t\right) = 0$ for $\forall t$ (i.e., local martingale)
2. $\sigma_t^s \xrightarrow{a.s} \sigma_\infty^s = 0$ and $\hat{Y}_t \xrightarrow{a.s} 0$ (i.e., almost sure stabilization)
3. 0^+-possibility divergence or non-uniform integrability given by

$$\mathbb{E}_0\left(\sup_{t\geq 0}\left(\sigma + \sigma_t^s\right)^2\right) = \infty$$

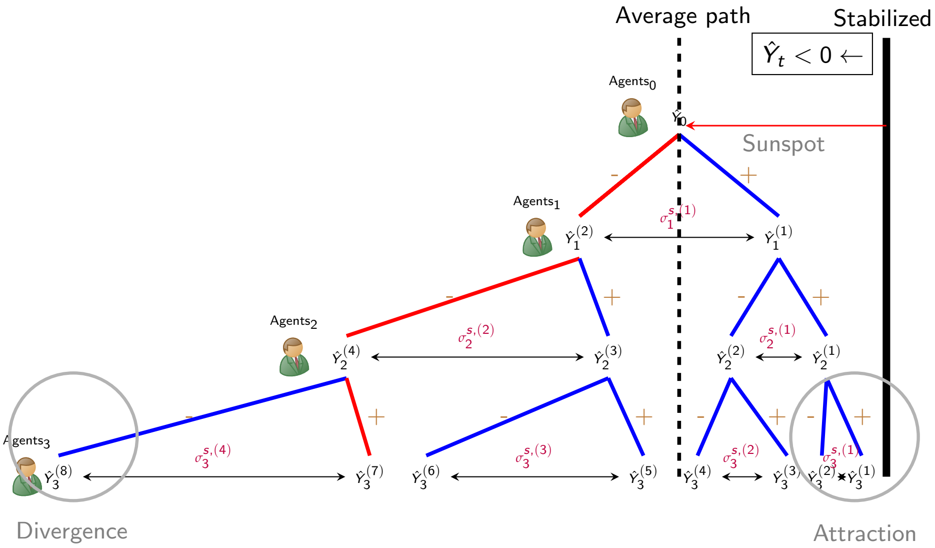
with

$$\lim_{K\to\infty}\sup_{t\geq 0}\left(\mathbb{E}_0\left(\sigma + \sigma_t^s\right)^2 \mathbb{1}_{\{(\sigma+\sigma_t^s)^2\geq K\}}\right) > 0.$$

Aggregate volatility↑ possible through the intertemporal coordination of agents

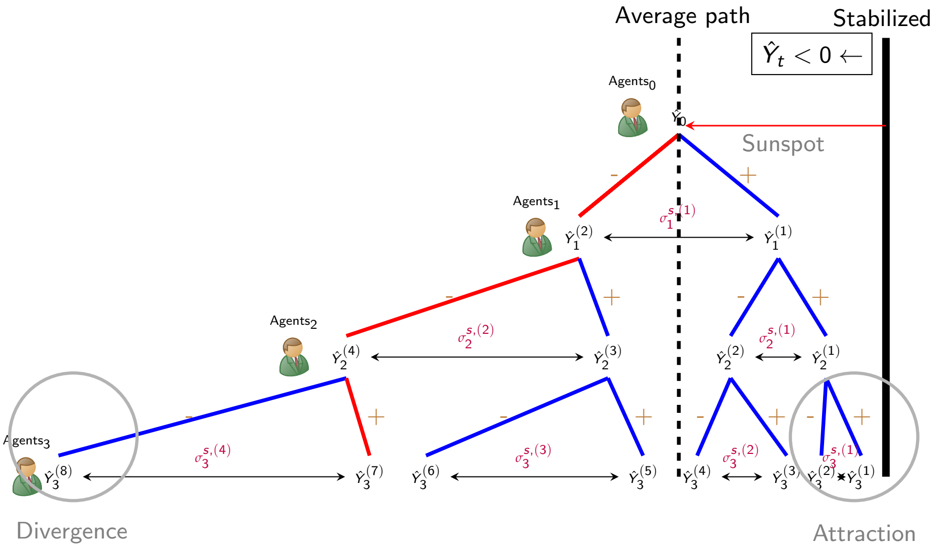
- Called a "martingale equilibrium" - non-stationary equilibrium
- Satisfies the transversality condition (1)

Key: a path-dependent intertemporal aggregate demand strategy



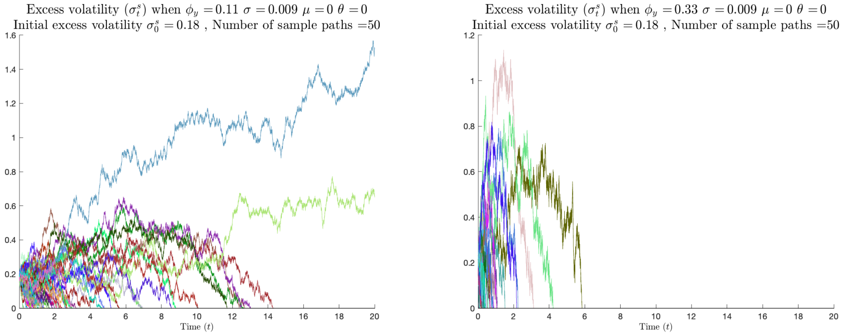
Stabilized as **attractor**: $\sigma_t^s \xrightarrow{a.s} \sigma_\infty^s = 0$ and $\hat{Y}_t \xrightarrow{a.s} 0$

Key: a path-dependent intertemporal aggregate demand strategy



But divergence with 0^+-probability: $\mathbb{E}_0\left(\sup_{t\geq 0}(\sigma + \sigma_t^s)^2\right) = \infty$

Simulation results - martingale equilibrium



(a) With Taylor coefficient $\phi_y = 0.11$

(b) With Taylor coefficient $\phi_y = 0.33$

Figure: Martingale equilibrium: with $\phi_y = 0.11$ (Figure 1a) and $\phi_y = 0.33$ (Figure 1b)

Stationary equilibria

A new monetary policy with volatility targeting

New monetary policy:

$$i_t = r^n + \phi_y \hat{Y}_t - \boxed{\frac{1}{2}\left(\underbrace{(\sigma + \sigma_t^s)^2}_{\equiv \mathrm{pp}_t} - \underbrace{\sigma^2}_{\equiv \mathrm{pp}^n}\right)}$$

Aggregate volatility targeting?

- Restores a **determinacy** and **stabilization**, but what does it mean?

A new monetary policy with volatility targeting

Leading to:

Ito term Ito term

$$\boxed{i_t + \mathrm{pp}_t - \frac{1}{2}\mathrm{pp}_t} = \boxed{r^n + \mathrm{pp}^n - \frac{1}{2}\mathrm{pp}^n} + \underbrace{\phi_y \hat{Y}_t}_{\text{Business cycle targeting}}$$

$$i_t + \mathrm{pp}_t - \frac{1}{2}\mathrm{pp}_t = \rho + \frac{\mathbb{E}_t(d\log Y_t)}{dt} \qquad r^n + \mathrm{pp}^n - \frac{1}{2}\mathrm{pp}^n = \rho + \frac{\mathbb{E}_t(d\log Y_t^n)}{dt}$$

- A % change of (i.e., return on) aggregate output (i.e., demand), not just the policy rate, follows Taylor rules

Key issue: monetary policy tool available $\neq$ objective

▸▸ Model with sticky prices

Paper 2. Higher-Order Forward Guidance

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Simplified version based on Lee and Dordal i Carreras (2025):
"Self-fulfilling Volatility and a New Monetary Policy"

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Motivation

Big Question

Forward guidance — How does it work, exactly?

- First-order effects (level): "Interest rates will stay low" $\longrightarrow$ intertemporal substitution channel (aggregate demand$\uparrow$)
- Second-order effects (volatility): reduce uncertainty, avoid worst-case scenarios, "whatever it takes" $\longrightarrow$ precautionary savings channel (aggregate demand$\uparrow$)

This paper: focus on central bank's strategic uncertainty management and coordination. Possible for central banks to pick an equilibrium where:

- During the ZLB (now): reduce aggregate volatility. Then aggregate demand$\uparrow$
- But central banks now create uncertainty about where the economy ends up after the ZLB (future): commit less stabilization after the ZLB
- Welfare-enhancing overall

Again, the non-linear IS equation in a standard New Keynesian model

Fundamental volatility

$$d\hat{Y}_t = \left(i_t - \underbrace{\left(r^n - \frac{1}{2} \boxed{(\sigma + \sigma_t^s)^2} + \frac{1}{2} \boxed{\sigma^2} \right)}_{\text{pp}_t \equiv r_t^T \qquad\qquad \text{pp}^n} \right) dt + \sigma_t^s \, dZ_t$$

$$= (i_t - r_t^T) dt + \sigma_t^s dZ_t$$

$$\sigma_t^s \uparrow \longrightarrow \text{pp}_t \uparrow \longrightarrow \hat{Y}_t \downarrow$$

ZLB from fundamental volatility shock

Thought experiment: fundamental volatility $\sigma\uparrow$: $\bar{\sigma}$ on $[0, T]$ (e.g., Werning (2012)) and comes back to $\underline{\sigma}$ with $\bar{\sigma} > \underline{\sigma}$

- $\bar{r} \equiv r^n(\underline{\sigma}) = \rho + g - \underline{\sigma}^2 > 0$: no ZLB before, $t < 0$, or after, $t > T$
- $\underline{r} \equiv r^n(\bar{\sigma}) = \rho + g - \bar{\sigma}^2 < 0$: ZLB binds for $0 \leq t \leq T$

Assume: perfect stabilization (i.e., $\hat{Y}_t = 0$) is achievable outside ZLB, i.e.,

$$i_t = \bar{r} + \phi_y \hat{Y}_t - \frac{1}{2}\left(\underbrace{\mathrm{pp}_t - \mathrm{pp}_t^n}_{\text{Variance gap}}\right), \quad \text{with } \phi_y > 0$$

Result: perfect stabilization of variance gap (i.e., excess uncertainty) inside the ZLB

- Recursive argument: full stabilization at T implies $\hat{Y}_T = 0 \longrightarrow \sigma^s_{T-\mathrm{d}t} = 0$, and keeps going on (so $\mathrm{pp}_t = \mathrm{pp}_t^n = \bar{\sigma}^2$ for $\forall t$)

ZLB path (full stabilization after T)

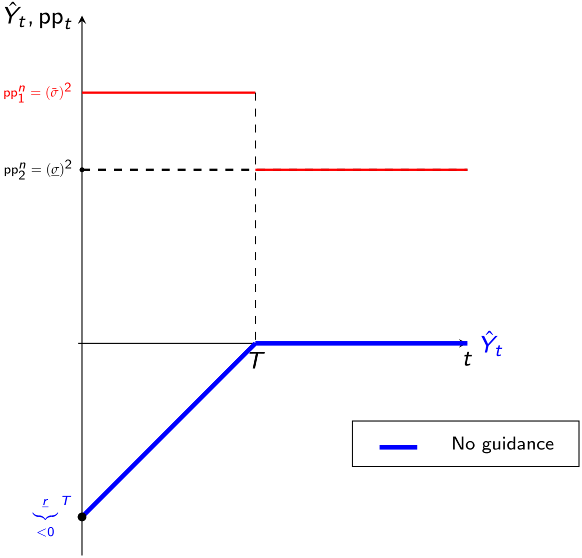


Figure: ZLB dynamics (Benchmark)

Traditional forward guidance (keep $i_t = 0$ until $\hat{T}^{\text{TFG}} > T$)

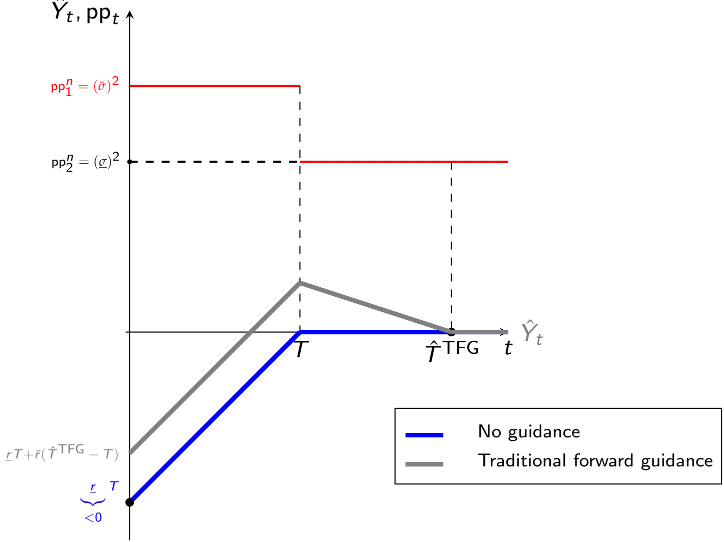


Figure: ZLB dynamics with forward guidance until $\hat{T}^{\text{TFG}} > T$

Details

Alternative forward guidance policies

Big Question

Can we do even better than the traditional forward guidance?

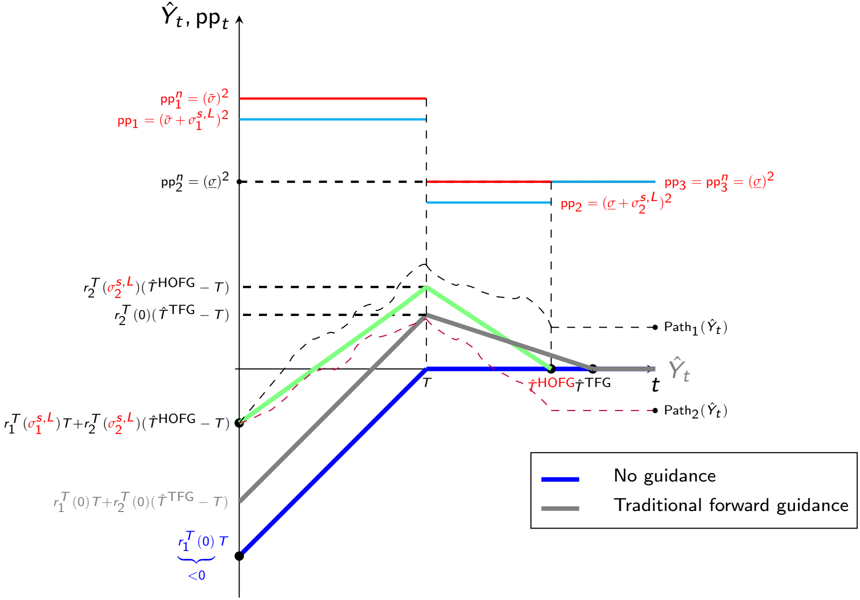
What if we reduce aggregate uncertainty via $\sigma_t^s < 0$?

- Then $\text{pp}_t = (\bar{\sigma} + \sigma_t^s)^2 < \text{pp}_t^n$, raising aggregate demand and $\hat{Y}_t$

But how?

- Nominal rigidities $\longrightarrow$ demand-determined production
- Policy challenge: the central bank *must convince* households to "coordinate" on this particular equilibrium $\longrightarrow$ *higher-order forward guidance*
- Give up perfect stabilization in the future (no stabilization at all)
- Imagine the central bank pegs the policy rate at $i_t = \bar{r}$ after zero rate periods

Central bank picks $\hat{T}^{HOFG}$ and $\{\sigma^s_t < 0\}_{t<\hat{T}^{HOFG}}$



At optimum, $\sigma^{s,L}_1 < 0 = \sigma^{s,n}_1$, $\sigma^{s,L}_2 < 0 = \sigma^{s,n}_2$, and $\hat{T}^{HOFG} < \hat{T}^{TFG}$ Details

Optimal policy

Proposition (Optimal forward guidance policy)

Optimal higher-order forward guidance (HOFG) always results in an equal or lower expected quadratic loss than the traditional guidance policy

Proof.

With $(\sigma_1^{s,L}, \sigma_2^{s,L}, \hat{T}^{\text{HOFG}}) = (0, 0, \hat{T}^{\text{TFG}})$, solutions coincide $\square$

Remarks:

- Alternative higher-order forward guidance policy implementations **are** possible
- This paper shows HOFG dominates TFG in a simple setting

Optimal policy: stochastic stabilization

Extension: still higher-order forward guidance policy, now with stochastic stabilization after $\hat{T}^{\text{HOFG}}$. Return to stabilization with νdt probability after $\hat{T}^{\text{HOFG}}$

- Central bank commits to stabilizing the economy after $\hat{T}^{HOFG}$ with some probability. Expected stabilization after $1/\nu$ quarters
- $\nu = 0$: the above higher-order forward guidance
- $\nu = \infty$: the traditional forward guidance policy

Big discontinuity:

$$\lim_{\nu \to +\infty^-} \mathbb{L}^{Y,*}\left(\{\hat{Y}_t\}_{t\geq 0}, \nu\right) < \underbrace{\mathbb{L}^{Y,*}\left(\{\hat{Y}_t\}_{t\geq 0}, \nu = \infty\right)}_{\text{Traditional forward guidance}}$$

- Slight probability that stabilization might not happen $\longrightarrow$ HOFG possible

Details

Policy implication

Real World Example (Covid-19 and the Federal Reserve)

Flexible Average Inflation Targeting (FAIT) (2020)

- Commitment to delaying stabilization – by allowing inflation to "moderately" overshoot its target after periods of persistent undershooting at the ZLB
- "Moderate" overshooting of the business cycle now is allowed: nudging agents toward a favorable equilibrium with lower volatility

HOFG equilibrium $\longrightarrow$ can be supported by fiscal policy as a unique equilibrium

- Zero transfer along the equilibrium path (out-of-equilibrium threat)
- Draghi's "whatever it takes" speech $\longrightarrow$ lower periphery yields without actual expenditures, coordinating agents to an equilibrium with lower risk premium (Acharya et al., 2019)

Actual paper: based on a Two-Agent New Keynesian (TANK) model with capitalists and workers to illustrate these aspects

Thank you very much!
(Appendix)

Potential stationary equilibria?

Conjecture: Ornstein-Uhlenbeck process with endogenous volatility $\{\sigma_t^s\}$

$$d\hat{Y}_t = \left(i_t - \underbrace{\left(r^n \overbrace{- \frac{1}{2}(\sigma + \sigma_t^s)^2 + \frac{1}{2}\sigma^2}^{\text{New terms}} \right)}_{\equiv r_t^T} \right) dt + \sigma_t^s \, dZ_t \tag{4}$$

$$= \underbrace{\theta}_{>0} \cdot \left[\underbrace{\mu}_{\gtreqless 0} - \hat{Y}_t \right] dt + \sigma_t^s dZ_t$$

- μ as an *approximate* average of $\hat{Y}_t$
- θ as a speed of mean reversion
- $i_t = r^n + \phi_y \hat{Y}_t$ (i.e., Taylor rule) stays the same

Go back

Proposition (Fundamental Indeterminacy)

For $\theta > 0$, $\mu < \frac{\sigma^2}{2\phi_y}$ with $\mu \neq 0$:

1. $\{\sigma_t^s\}$ process satisfying (4) is stable, and admits a unique stationary distribution: with $\sigma \to 0$ and $\mu < 0$, the stationary distribution coincides with the "generalized gamma distribution" $GGD(a, d, p)$, given by

$$a = \sqrt{\frac{2(\theta + \phi_y)^2}{\theta}}, \quad d = -\frac{2\theta\mu\phi_y}{(\theta + \phi_y)^2}, \quad \text{and} \quad p = 2, \tag{5}$$

   where a is the scale parameter, d is the power-law shape parameter, p is the exponential shape parameter.

2. For $\theta > 0$ and $\mu = 0$, the σ_t^s process is again non-stationary (degenerate distribution at $\sigma_\infty^s = 0$).

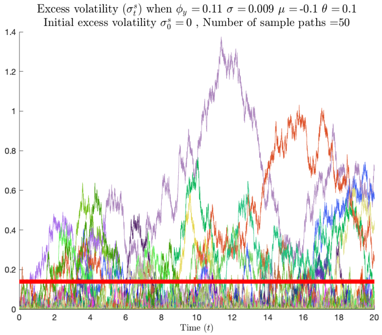
3. The long-run expectations of the output gap $\hat{Y}_t$ and excess variance $(\sigma + \sigma_t^s)^2 - \sigma^2$ are given by

$$\lim_{t\to\infty} \mathbb{E}_0\left[\hat{Y}_t\right] = \mu, \quad \text{and} \quad \lim_{t\to\infty} \mathbb{E}_0\left[(\sigma + \sigma_t^s)^2 - \sigma^2\right] = -2\mu\phi_y.$$

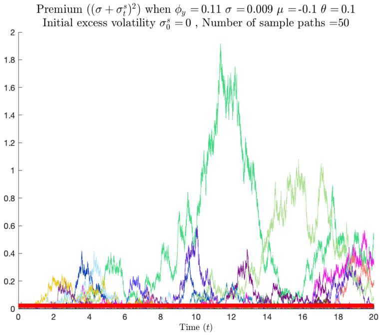
Go back

Simulation results - Ornstein-Uhlenbeck equilibrium

With $\theta > 0$, $\mu < 0$



(a) Endogenous volatility σ_t^s



(b) Precautionary premium $(\sigma + \sigma_t^s)^2$

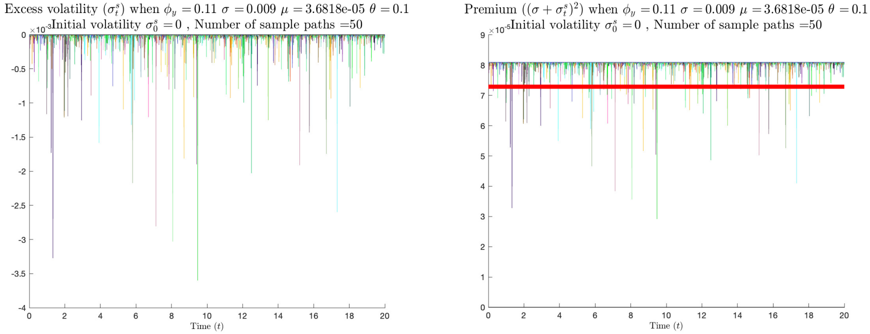
Figure: Ornstein-Uhlenbeck equilibrium: endogenous volatility $\{\sigma_t^s\}$ (Figure 4a) and the precautionary premium$\{(\sigma + \sigma_t^s)^2\}$ (Figure 4b)

- Even with $\sigma_0^s = 0$ (no initial volatility) $\Longrightarrow$ stationary $\{\sigma_t^s\}$ process

Go back

Simulation results - Ornstein-Uhlenbeck equilibrium

With $0 < \mu < \dfrac{\sigma^2}{2\phi_y}$



(a) Endogenous volatility σ_t^s

(b) Precautionary premium $(\sigma + \sigma_t^s)^2$

Figure: Ornstein-Uhlenbeck equilibrium: endogenous volatility $\{\sigma_t^s\}$ (Figure 5a) and the precautionary premium$\{(\sigma + \sigma_t^s)^2\}$ (Figure 5b)

- Even with $\sigma_0^s = 0$ (no initial volatility) $\Longrightarrow$ stationary $\{\sigma_t^s\}$ process

Go back

Simulation results - Ornstein-Uhlenbeck equilibrium

With $\theta > 0$, $\mu = 0$

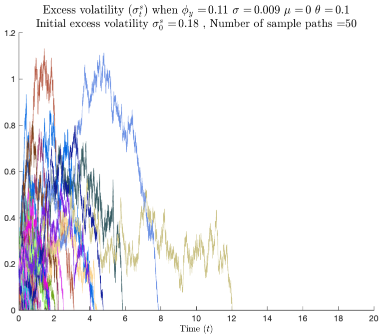


Figure: Endogenous volatility σ_t^s

- Again, degenerate distribution at $\sigma_\infty^s = 0$
- Faster convergence than the martingale equilibrium $(\theta = 0)$

Go back

Model with inflation

Nominal rigidities à la Rotemberg (1982)

$$dp_t^i = \pi_t^i p_t^i \, dt,$$

with adjustment cost of inflation rate π_t^i:

$$\Theta(\pi_t^i) = \frac{\tau}{2}(\pi_t^i)^2 p_t Y_t$$

Volatility of inflation growth

New Keynesian Phillips curve (NKPC):

$$d\pi_t = \left[\left[2(\rho + \pi_t) - i_t - (\sigma + \sigma_t^s)(\sigma + \sigma_t^s + \sigma_t^\pi)\right]\pi_t - \left(\frac{\epsilon - 1}{\tau}\right)\left(e^{\left(\frac{\eta+1}{\eta}\right)\hat{Y}_t} - 1\right)\right] dt$$
$$+ \sigma_t^\pi \, \pi_t \, dZ_t$$

The IS equation then becomes:

$$d\hat{Y}_t = \left[i_t - \pi_t - r_t^T\right] dt + \sigma_t^s \, dZ_t \tag{6}$$

Taylor rule: $i_t = r^n + \phi_y \hat{Y}_t$

- Transversality given by the same equation (1)

Go back

Model with inflation

Proposition (Fundamental Indeterminacy)

The model with sticky prices à la Rotemberg (1982) admits an alternative solution to the benchmark equilibrium given by:

$$\begin{aligned} d\hat{Y}_t &= \theta \left[\mu - \hat{Y}_t\right] \, dt + \sigma_t^s \, dZ_t, \\ \pi_t &= f(\sigma_t^s), \end{aligned} \tag{7}$$

where $f(\cdot)$ is a smooth function of excess volatility σ_t^s. This alternative equilibrium solution exists for any positive degree of price stickiness, as captured by the adjustment rate parameter $\tau > 0$.

- Similar structure to the Ornstein-Uhlenbeck equilibrium, with π_t as a smooth function of σ_t^s
- Similar in the case of pricing à la Calvo (1983): see Online Appendix G

Go back

Traditional forward guidance

Assume:

- Central bank commits to keep $i_t = 0$ until $\hat{T}^{\text{TFG}} \geq T$ (i.e., Odyssean guidance)
- Perfect stabilization (i.e., $\hat{Y}_t = 0$) afterwards, i.e., for $t > \hat{T}^{\text{TFG}}$
- By the same arguments, volatility gap stabilization beforehand, $t \leq \hat{T}^{\text{TFG}}$ (no excess volatility while $i_t = 0$)

Problem: minimize smooth quadratic welfare loss

$$\min_{\hat{T}^{\text{TFG}}} \mathbb{L}^Y\left(\{\hat{Y}\}_{t\geq 0}\right) \equiv \mathbb{E}_0 \int_0^\infty e^{-\rho t}\left(\hat{Y}_t\right)^2 dt$$

$$\text{s.t. } \hat{Y}_0 = \underbrace{\underline{r}}_{<0} T + \underbrace{\bar{r}}_{>0}\left(\hat{T}^{\text{TFG}} - T\right)$$

- Smoothing the ZLB costs over time (i.e., welfare enhancing)

Go back

Higher-order forward guidance

Assume:

- Central bank can commit to keep $i_t = 0$ until $\hat{T}^{HOFG} \geq T$
- No stabilization (i.e., $\hat{Y}_t = \hat{Y}_{\hat{T}^{HOFG}}$) guaranteed afterwards, $t \geq \hat{T}^{HOFG}$
- Pick $\{\sigma_t^s\}$ for $t < \hat{T}^{HOFG}$

Problem: minimize smooth quadratic welfare loss

$$\min_{\sigma_1^{s,L}, \sigma_2^{s,L}, \hat{T}^{HOFG}} \mathbb{L}^Y\left(\{\hat{Y}\}_{t\geq 0}\right) \equiv \mathbb{E}_0 \int_0^\infty e^{-\rho t}\left(\hat{Y}_t\right)^2 dt,$$

$$\text{s.t.} \begin{cases} d\hat{Y}_t = -\underbrace{r_1^T\left(\sigma_1^{s,L}\right)}_{<0} dt + \sigma_1^{s,L} dZ_t, & \text{for } t < T, \\ d\hat{Y}_t = -\underbrace{r_2^T\left(\sigma_2^{s,L}\right)}_{>0} dt + \sigma_2^{s,L} dZ_t, & \text{for } T \leq t < \hat{T}^{HOFG}, \\ d\hat{Y}_t = 0, & \text{for } t \geq \hat{T}^{HOFG}, \end{cases}$$

with

$$\hat{Y}_0 = \underbrace{r_1^T\left(\sigma_1^{s,L}\right)}_{<0} T + \underbrace{r_2^T\left(\sigma_2^{s,L}\right)}_{>0}\left(\hat{T}^{HOFG} - T\right)$$

Higher-order forward guidance with stochastic stabilization

Change:

- Central bank commits to stabilizing the economy after $\hat{T}^{HOFG}$ with Poisson probability ν: at each point after $\hat{T}^{HOFG}$, $\hat{Y}_t$ becomes 0 with probability νdt

Problem: minimize smooth quadratic welfare loss

$$\min_{\sigma_1^{s,L},\,\sigma_2^{s,L},\,\hat{T}^{\text{HOFG}}} \quad \mathbb{E}_0\left[\int_0^{\hat{T}^{\text{HOFG}}} e^{-\rho t}\,\hat{Y}_t^2\,dt + \int_{\hat{T}^{\text{HOFG}}}^{\infty} e^{-\rho t}\, e^{-\nu\left(t-\hat{T}^{\text{HOFG}}\right)}\,\hat{Y}_t^2\,dt\right],$$

$$\text{s.t.} \quad \begin{cases} d\hat{Y}_t = -\underbrace{r_1^T\left(\sigma_1^{s,L}\right)}_{<0} dt + \sigma_1^{s,L} dZ_t, & \text{for } t < T, \\ d\hat{Y}_t = -\underbrace{r_2^T\left(\sigma_2^{s,L}\right)}_{>0} dt + \sigma_2^{s,L} dZ_t, & \text{for } T \le t < \hat{T}^{HOFG}, \\ d\hat{Y}_t = 0, & \text{for } t \ge \hat{T}^{HOFG}, \end{cases}$$

with

$$\hat{Y}_0 = \underbrace{r_1^T\left(\sigma_1^{s,L}\right)}_{<0} T + \underbrace{r_2^T\left(\sigma_2^{s,L}\right)}_{>0}\left(\hat{T}^{HOFG} - T\right)$$

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