Beliefs and the Net Worth Trap

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Resilience: Brunnermeier (2024)



Figure 1. Panel A depicts the log level of U.S. GDP from 1900 to 2023, while Panel B zooms in level from 1996 onwards. Shaded areas show recession periods. (Color figure can be viewed at wileyonlinelibrary.com)

• The US economy has been resilient to shocks in most previous crises, except after the global financial crisis (GFC) and the great recession in 2008.

Resilience: Brunnermeier (2024)

Resilience is a dynamic concept (as opposed to risk) which can be intuitively modeled using a stochastic process.



What we do

Big Question

What is the role of belief distortions in undermining the economic resiliency?

Contribution:

- Build a tractable heterogeneous agent general equilibrium model with financial frictions in which experts hold dogmatic distorted beliefs over long-run output growth
- Analyze the role of intermediary's (or expert's) **distorted beliefs** about the long-term growth prospects on the creation of net worth trap, i.e., perennial crisis
- Net worth trap: experts never *recapitalize* due to their expectation error, generating extremely slow-moving capital crisis and zero resiliency

Usually, fast recapitalization in the model due to high risk premium during crises:

- hard to generate slow-moving capital

Model Setup

Two types of agents: experts (more productive) who hold dogmatic beliefs about long-run output growth, and rational households (less productive)

• Experts and households hold risky capital, subject to aggregate shock, and can borrow against their net worth.

Financial friction:

- Experts cannot issue outside equity: incomplete market, leading to occasionally binding capital misallocation.
- In Markov equilibrium, the wealth share of experts is the sole state variable.

A standard setting: based on Basak (2000) and Brunnermeier and Sannikov (2014)

 Budding literature on the interactions between financial frictions and investors' beliefs (Maxted, 2023; Camous and Van der Ghote, 2023; Krishnamurthy and Li, 2024)

Mechanisms

Dynamics:

- At the stochastic steady state, the economy is in a "normal" regime where all capital is held by experts, and beliefs have little impact.
- Series of negative shocks: wealth share of experts, and the economy enters a "crisis" regime (with higher volatility and risk premium). Beliefs matter a lot.

Two competing forces governing resilience: (i) risk premium channel; (ii) the expectation error channel

Resilience is determined by the relative strength of these two forces.

- $\bullet\,$ For small belief distortions, risk premium channel dominates $\longrightarrow\,$ economy is resilient
- For large belief distortions, expectation error channel dominates → economy enters a net worth trap with zero resiliency.

The Model

Setting: experts

Single capital: owned by experts and (rational) households

Experts: produces $y_t^{O} = \gamma_t^{O} k_t^{O}$, $\forall t \in [0, \infty)$ where

$$\frac{dk_t^O}{k_t^O} = \left(\Lambda^O(t_t^O) - \delta^O\right) dt, \quad \forall t \in [0, \infty)$$
Investment ratio
Their investment = $t_t^O y_t^O$

with technological growth:

$$\frac{d\gamma_t^O}{\gamma_t^O} = \alpha \ dt + \sigma \underbrace{dZ_t}_{\text{Brownian motion}}, \quad \forall t \in [0, \infty)$$
True (expected) growth

Setting: rational households

Households: produces $y_t^H = \gamma_t^H k_t^H$, $\forall t \in [0, \infty)$ where

$$\frac{dk_t^H}{k_t^H} = \left(\Lambda^H(t_t^H) - \delta^H\right) dt, \quad \forall t \in [0, \infty)$$
Investment ratio
Their investment = $t_t^H y_t^H$

with the same technological growth:



- $\longrightarrow \textbf{Level difference: } \gamma^H_t = I \cdot \gamma^O_t, \ \Lambda^H(\cdot) = I \cdot \Lambda^O(\cdot), \text{ with } I \leq 1$
 - Efficiency in both production and capital formation \downarrow

Capital return

- Endogenous volatility

Capital price process: (endogenous) pt follows

$$\frac{dp_t}{p_t} = \mu_t^p dt + \sigma_t^p dZ_t$$

Capital return process:

• Experts' total return on capital:

$$dr_{t}^{Ok} = \underbrace{\frac{\gamma_{t}^{O}k_{t}^{O} - \iota_{t}^{O}\gamma_{t}^{O}k_{t}^{O}}{p_{t}k_{t}^{O}}}_{\text{Dividend yield}} dt + \underbrace{\left(\Lambda^{O}(\iota_{t}^{O}) - \delta^{O} + \mu_{t}^{p}\right)dt + \sigma_{t}^{p}dZ_{t}}_{\text{Capital gain}}$$

$$= \frac{1 - \iota_{t}^{O}}{q_{t}}dt + \left(\Lambda^{O}(\iota_{t}^{O}) - \delta^{O} + \mu_{t}^{p}\right)dt + \sigma_{t}^{p}dZ_{t}$$
Price-earnings ratio
(experts)

Capital return for households

Beliefs of experts

Experts dogmatically believe γ_t^O follows

$$\frac{d\gamma_t^O}{\gamma_t^O} = \alpha^O dt + \sigma \underbrace{\frac{dZ_t^O}{Experts'}}_{\text{Brownian Motion}}, \quad \forall t \in [0, \infty)$$

where $\alpha^{O} > \alpha$ corresponds to optimism and $\alpha^{O} < \alpha$ corresponds to pessimism

while the true process is given as

$$\frac{d\gamma_t^O}{\gamma_t^O} = \alpha dt + \sigma \underbrace{\frac{dZ_t}{True}}_{\text{Brownian Motion}}$$



Optimization

Financial market: capital and risk-free (zero net-supplied)

Experts: consumption-portfolio problem (price-taker)

$$\max_{\substack{t_t^O, x_t^O \ge 0, c_t^O \ge 0 \\ w_t^O, x_t^O \ge 0, c_t^O \ge 0 \\ w_t^O = x_t^O w_t^O dr_t^{Ok} + (1 - x_t^O) r_t w_t^O dt - c_t^O dt, \text{ and } \underbrace{w_t^O \ge 0}_{\text{Solvency constraint}}$$

Rational households: solve the similar problem with \mathbb{E}_0 ($\neq \mathbb{E}_0^O$)

• Correctly understanding that dZ_t is the Brownian motion









(d) Perceived-true risk-premium

Ergodic distribution of the state variable η_t (optimism)



Figure: Stationary distribution of η_t and the net worth trap

Net worth trap: perennial crisis

Two countervailing forces:

- \bullet Once crisis hits, higher optimism of experts \longrightarrow higher risk premium helping them to recapitalize faster
- Expectation error of experts preventing them from recapitalizing (stronger)

Proposition (Net Worth Trap)

There exists a threshold level of belief beyond which the economy is trapped at $\eta = 0$, and the probability of recapitalization for experts converges to zero. For the **optimistic** case, i.e., $\alpha^O > \alpha$, the threshold is determined by

$$\alpha^{O} - \alpha > \sigma \sqrt{\Gamma_0^2 \sigma^2 + 2\Delta_0},\tag{1}$$

and for the **pessimistic** case, i.e., $\alpha^O < \alpha$, the threshold is given by

$$\alpha^{O} - \alpha < -\min\left\{\sigma\sqrt{\Gamma_{0}^{2}\sigma^{2} + 2\Delta_{0}}, \max\left\{\sigma^{2}\left(1+\Gamma_{0}\right), \Delta_{0}+\frac{1}{2}\sigma^{2}\right\}\right\}.$$
(2)

[▶] Without short-sale constraint and complete markets

Net worth trap: perennial crisis

Around $\eta \sim 0$:



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From dogmatic to swinging beliefs

Now, the log-run growth rate perceived by experts

$$O_t = \mathbf{1}_{\psi_t < 1} \cdot \alpha^P + \mathbf{1}_{\psi_t = 1} \cdot \alpha^O$$

• Experts are optimistic at the stochastic steady state, but become pessimistic in crisis (similar to diagnostic expectations)



Initially stabilizing (e.g., Maxted (2023)), but stronger pessimism in a crisis becomes desta-

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Thank you very much! (Appendix)

Capital return

Capital return process:

• Households' total return on capital:



Experts' total return on capital:

$$dr_{t}^{Ok} = \underbrace{\frac{\gamma_{t}^{O}k_{t}^{O} - \iota_{t}^{O}\gamma_{t}^{O}k_{t}^{O}}{p_{t}k_{t}^{O}}}_{\text{Dividend yield}} dt + \underbrace{\left(\Lambda^{O}(\iota_{t}^{O}) - \delta^{O} + \mu_{t}^{P}\right)dt + \sigma_{t}^{P}dZ_{t}}_{\text{Capital gain}} \xrightarrow{\text{Perceived}}_{\text{Brownian motion}} dt = \frac{\gamma_{t}^{O} - \iota_{t}^{O}\gamma_{t}^{O}}{p_{t}}dt + \left(\Lambda^{O}(\iota_{t}^{O}) - \delta^{O} + \mu_{t}^{P} + \frac{\alpha^{O} - \alpha}{\sigma}\sigma_{t}^{P}\right)dt + \sigma_{t}^{P}dZ_{t}^{O}$$

Portfolio decisions under belief distortions

Experts' optimal portfolio decision (e.g., Merton (1971))



If $\alpha^O > \alpha$ (optimism)

 Given the risk-free r^{*}_t and the endogenous volatility σ^p_t, optimism raises the leverage[↑] and capital demand[↑]

 σ_t^p affects leverage x_t^O in two different ways:

- $\sigma_t^{p\uparrow}$ lowers x_t^{O} as the required risk-premium level[†]
- σ_t^{P} raises x_t^{O} as it raises the degree of belief premium on capital returns

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Market clearing

Total capital $K_t = k_t^O + k_t^H$ evolves with

$$\frac{dK_{t}}{dt} = \underbrace{\left(\Lambda^{O}\left(\iota_{t}^{O}\right) - \delta^{O}\right)k_{t}^{O}}_{\text{From experts}} + \underbrace{\left(\Lambda^{H}\left(\iota_{t}^{H}\right) - \delta^{H}\right)k_{t}^{H}}_{\text{From households}}, \quad \forall t \in [0, \infty)$$

Debt: zero net-supplied



Good market equilibrium:

$$\underbrace{\frac{x_t^O w_t^O}{p_t} \left(\gamma_t^O - \iota_t^O \gamma_t^O\right)}_{\substack{\text{Experts'} \\ \text{production} \\ \text{net of investment}}} + \underbrace{\frac{x_t^H w_t^H}{p_t} \left(\gamma_t^H - \iota_t^H \gamma_t^H\right)}_{\substack{\text{Households'} \\ \text{production} \\ \text{net of investment}}} = c_t^O + c_t^H$$

Markov equilibrium: experts' wealth share η_t as state variable



Markov equilibrium

Wealth share of experts as state variable, as in Brunnermeier and Sannikov (2014):

$$\eta_t \equiv \frac{W_t^O}{W_t^O + W_t^H} = \frac{W_t^O}{p_t K_t}$$

which leads to:

$$x_t^O \leq \frac{1}{\eta_t}$$

- When it binds: "normal" (i.e., all capital is owned by experts)
- When it does not bind: "crisis" (i.e., less productive households hold some capital)

Under Markov equilibrium: normalized variables depend only on η_t $q_t = q(\eta_t), x_t^O = x(\eta_t), \underbrace{\psi_t}_{\substack{Capital share \\ (experts)}} = \psi(\eta_t)$



Specification and calibration

Investment function

$$\Lambda^{O}(\iota_{t}^{O}) = \frac{1}{k} \left(\sqrt{1 + 2k\iota_{t}^{O}} - 1 \right), \quad \forall t \in [0, \infty)$$

with

$$\Lambda^{P}(\iota_{t}) = I \cdot \Lambda^{O}(\iota_{t}), \quad \forall \iota_{t}$$
(4)

	Parameter Description	Value	Source (target)
ρ	Discount rate	0.03	Standard: e.g., Brunnermeier and
			Sannikov (2014).
α	Productivity growth	0.02	2% growth in the long run.
σ	Exogenous TFP volatility	0.0256	
			Schimitt-Grohé and Uribe (2007)
δ	Depreciation rate (δ^{H} , δ^{O})	0	2% capital growth in the long run
			(2.5% in the stochastic steady state)
k	Investment function	851.6	Consumption-to-output ratio at 69%
1	Productivity gap	0.7	Most severe recessions: the average
			output drop from the trend in the
			Great Depression was \sim 30% accord-
			ing to Romer (1993) > Go back

Endogenous volatility: two channels

Capital price volatility σ_t^p is given by

$$\sigma_t^p \left(1 - \left(x_t^O - 1 \right) \frac{\frac{dq(\eta_t)}{q(\eta_t)}}{\frac{d\eta_t}{\eta_t}} \right) \equiv \sigma_t^p \left(1 - \left(x_t^O - 1 \right) \varepsilon_{q,\eta} \right) = \underbrace{\sigma}_{\text{Exogenous volatility}}^{\mathcal{O}}$$

• $\varepsilon_{q,\eta}$ is the elasticity of the price-earnings ratio (i.e., normalized capital price) with respect to the experts' wealth share η_t

With optimism, volatility σ_t^p is amplified in a crisis through:

• "Elasticity" effect: optimism $\alpha^{O}\uparrow \longrightarrow \varepsilon_{q,\eta}\uparrow \longrightarrow \sigma_{t}^{P}\uparrow$

• "Leverage" effect:
$$\alpha^{O} \uparrow \longrightarrow x_t^{O} \uparrow \longrightarrow \sigma_t^{p} \uparrow$$

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(b) Capital share ψ_t



Behavior of wealth share $\eta_t \sim 0$

Lemma

In the limit $\eta \to 0^+$, the drift $\mu^{\eta}(0^+)$ and diffusion $\sigma^{\eta}(0^+)$ of the wealth share of experts is given by

$$\mu^{\eta}(\mathbf{0}^{+}) \equiv \lim_{\eta \to 0} \mu^{\eta}(\eta) = \Gamma_{0}(\alpha^{O} - \alpha) + \Gamma_{0}^{2}\sigma^{2} + \Delta_{0}$$
$$\sigma^{\eta}(\mathbf{0}^{+}) \equiv \lim_{\eta \to 0} \sigma^{\eta}(\eta) = \frac{\alpha^{O} - \alpha}{\sigma} + \Gamma_{0}\sigma.$$

where

$$\begin{split} \Gamma_{0} &= \frac{1}{\sigma^{2}} \left[(1-l) \frac{1-\iota_{0}}{q_{0}} + (\delta^{H} - \delta^{O}) + (1-l) \Lambda^{O}(\iota_{0}) \right] \\ \Delta_{0} &= \frac{1-\iota_{0}}{q_{0}} + (\delta^{H} - \delta^{O}) + (1-l) \Lambda^{O}(\iota_{0}) - \rho \end{split}$$

and the quantities $\iota_0 = \lim_{\eta \to 0} \iota(\eta)$ and $q_0 = \lim_{\eta \to 0} q(\eta)$ are given in Appendix B.2.

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Drift and volatility of the wealth share



Figure: Wealth share dynamics: drift and volatility

With higher α^O↑, the wealth share drift μ_η(η_t)η_t↓ in stochastic steady states: more likely to enter crises

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Other cases

Corollary (Without short-sale constraint)

The threshold level of belief that determines the net worth trap in an economy without a short-selling constraint is given by

$$\left|\alpha^{O} - \alpha\right| > \sigma \sqrt{\Gamma_{0}^{2} \sigma^{2} + 2\Delta_{0}},\tag{5}$$

Proposition (Complete markets)

Under complete markets with I = 1 and $\delta^H = \delta^O$, if $\alpha^O \neq \alpha$, experts lose the entire wealth in the long run and the economy features a net worth trap.

- In this case, experts earn the same risk premium as less productive agents. Only the expectation error channel is there and drags η_t to zero
- Similar to "market selection hypothesis" à la Blume and Easley (2006) and Borovička (2020)



Does optimism hurt the household's welfare?

Welfare Loss =
$$\mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \log c_t^H dt \right] - \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \log c_t^{H,REE} dt \right]$$

• $c_t^{H,REE}$: household's consumption in the rational expectations benchmark

Decomposition:

$$\mathbb{E}_{0}\left[\int_{0}^{\infty} e^{-\rho t} \log c_{t}^{H} dt\right] = \underbrace{\mathbb{E}_{0}\left[\int_{0}^{\infty} e^{-\rho t} \log(1-\eta_{t}) dt\right]}_{\text{Wealth effect}_{+}} + \underbrace{\mathbb{E}_{0}\left[\int_{0}^{\infty} e^{-\rho t} \log K_{t} dt\right]}_{\text{Capital effect}_{-}} + \underbrace{\mathbb{E}_{0}\left[\int_{0}^{\infty} e^{-\rho t} \log A(\psi) dt\right]}_{\text{Misallocation effect}_{-}} + \underbrace{\underbrace{\text{t.i.e.}}_{\text{Terms independent of equilibria}}\right]$$

• $A(\psi) = \psi_t + l(1-\psi_t)$: productivity-adjusted aggregate capital share

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Figure: Decomposition of the rational household's welfare loss

• Overall, optimism reduces welfare of households

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