# Managerial Incentives, Financial Innovation, and Risk-Management Policies*

Son Ku Kim[†] Seung Joo Lee[‡] Sheridan Titman[§]

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**Abstract**

This paper studies the risk choices of a firm run by an effort and risk-averse manager. We show that by eliminating zero NPV risk, derivatives can be used in ways that allow firms to compensate managers in ways that induce them to more efficiently provide effort and to take on higher NPV investments. Whether a self-interested manager can in fact be induced to eliminate hedgeable risks depends on the manager's risk preferences and the real investment opportunity set. Because managers may have incentives to speculate rather than hedge, in some settings the optimal contract restricts the use of derivatives.

**Keywords:** Agency, Risk-Management, Hedging

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[†]Seoul National University (sonkukim@snu.ac.kr)

[‡]Saïd Business School, Oxford University (seung.lee@sbs.ox.ac.uk)

[§]McCombs School of Business, University of Texas at Austin (titman@mail.utexas.edu)

# 1 Introduction

Corporations spend substantial resources assessing and managing their exposures to various sources of risk. In a setting with complete information and frictionless markets, the Modigliani and Miller theorem holds, and these expenditures do not create value. However, the finance literature identifies a number of market imperfections that provide a rationale for these risk management activities.[1] Most of this literature focuses on the role of financial constraints and implicitly assumes that the risk management choices are made by value-maximizing rather than self-interested executives.[2] In contrast, the focus of this paper is on the risk management choices of large public firms, which are financially unconstrained, but managed by self-interested agents.

The manager (i.e., the agent) in our model is both risk and effort averse and makes two choices that influence the firm's risk. The manager selects from a menu of risky positive net present value real investments and chooses derivatives positions that we assume have zero net present values. Depending on the direction of these positions, the derivatives can either increase or decrease the volatility of the firm's profits. The shareholders (i.e., the principal) in this model are risk neutral, and design a compensation contract with the manager that is a function of the firm's observed profits, (net of the profits or losses from the derivatives transactions), and the realization of the hedgeable risk. For example, one can view this as a model of an oil company whose profits are determined by the efforts of its CEO, oil prices, hedging and investment choices, and random noise. To motivate the CEO to exert effort and make appropriate investments, the CEO receives a bonus that is a function of both profits and the price of oil.

The introduction of derivatives in this model can potentially create value by improving the compensation contract in two ways. The first is that by hedging, the correlation between reported earnings and managerial effort can be increased, allowing for compensation contracts that more efficiently induce managerial effort. The second is that by eliminating extraneous risk, hedging allows for contracts that more efficiently induce the manager to

[1]For previous works, see Smith and Stulz (1985), Campbell and Kracaw (1990), and DeMarzo and Duffie (1991, 1995), Froot et al. (1993), Geczy et al. (1997), Leland (1998) among others.

[2]For the role of financial (e.g., collateral) constraints in risk management activities, see Rampini and Viswanathan (2010, 2013), Rampini et al. (2014) among others. As both financing and risk management need collateral, more financially constrained firms engage in less risk management, and sometimes do not hedge at all. Our model in contrast abstracts from external financing constraints and focus on managerial incentive issues, noting that risk management policies of a firm are chosen by self-interested managers, not shareholders of the firm.

take risky real investments with higher expected payouts. Whether a self-interested manager can in fact be induced to eliminate hedgeable risks depends on the manager's risk preferences and the real investment opportunity set. As we show, in some settings it is too costly to induce the manager to take derivative positions that hedge rather than speculate. In these settings, the introduction of derivatives may not create value and their use is restricted in optimal contracts.

We start by considering a setting where the firm's real investment choice is observable and contractible. However, the principal, in this setting, does not observe the firm's exposure to an element of risk that can be hedged. This information, which is observed by the agent, cannot be credibly disclosed to the principal, and in addition, we will initially assume that there is no communication between the principal and the agent.[3] Specifically, we examine the optimal compensation contract between the agent and the principal that can be characterized as a salary that is contingent on the realizations of the firm's profits, as well as an observable state variable that represents the hedgeable element of risk. For example, the profits of an oil firm is determined by oil prices, the effort of the executive and the amount hedged, and its executive's compensation is determined by the observed profits and oil prices. The principal in this setting can either allow or prohibit hedging, but if hedging is allowed, the principal cannot dictate the amount hedged.

If the firm's risk exposure cannot be communicated and cannot be hedged the efficiency of the optimal compensation contract is reduced. However, as we show, when hedging is allowed, asymmetric information about risk exposure needs not create costs. In particular, when the optimal compensation contract, conditioned on full hedging, makes the agent's indirect utility concave in hedged output, an agent given such a contract will in fact fully hedge and exerts the same effort as in the case with symmetric information.

The agent's indirect utility, conditioned on full hedging, is not, however, necessarily concave. For example, the indirect utility function is convex if the agent is not too risk averse. When this is the case, the principal optimally imposes more risk on the agent's compensation to induce higher efforts.[4] In the power utility case, for instance, decreasing

[3] In Appendix B, we consider an alternative setting, where there can be contractible communication between the principal and the agent, i.e., the compensation contract can include the risk exposure disclosed by the agent, as well as the profits and the realization of the hedgeable source of uncertainty. As we show, the problem with hedging and no communication is equivalent to a problem with communication but without hedging. Specifically, the optimal allocations achieved in our setting, with hedging and no communication, is identical to those achieved in a mechanism design setting where the optimal contract induces the agent to truthfully reveal the firm's risk exposure.

[4] Hirshleifer and Suh (1992) characterize special cases of the agent's utility and the output distribution

absolute risk aversion implies that as output increases, the agent becomes more risk tolerant, which implies that the slope of the contract is steeper at higher output levels. This convex contract, combined with the agent's utility, results in convex indirect utility if the agent's utility function is not too concave.

If the compensation contract, with contractible hedging, generates a convex indirect utility function, then this contract will not be optimal when the hedging choice and risk exposure is not contractible. This is because an agent offered such a contract has an incentive to speculate rather than hedge. As we show, when this is the case, the agent's derivative choices may be restricted, however, if it is not restricted, the optimal compensation contract will depend on both profits and the realization of the firm's hedgeable risk. Specifically, the optimal contract penalizes the agent when both profits and hedgeable risk have extreme realizations.[5] Such a contract can induce the agent to hedge rather than speculate, even when the indirect utility function under the contract conditioned on full hedging is convex in profits.

We next consider the case where the agent selects from among several possible investments that are exposed to different risks and have different expected rates of return. In this setting, in addition to incentive issues that affect the derivative choice and the effort choice, there is a natural conflict between the investment preferred by the risk averse agent and the risk neutral principal who prefers the investment with the highest expected return. One question we ask is whether the introduction of derivatives mitigates or amplifies this conflict.

As we show, to induce the risk averse agent to select riskier projects, the compensation contract will tend to be convex in profits. This additional channel can generate a convex indirect utility function, that can induce the agent to take derivative positions that speculate rather than hedge. In contrast to the earlier case where the real investment is given, in this case it is the more risk averse agent rather than the less risk averse agent that ends up with a convex indirect utility function. In any event, we again obtain the result that the optimal contract is either altered to penalize extreme output realizations that arise when extreme hedgeable risk outcomes, or alternatively, derivative transactions are restricted, or even banned. We also show that when derivatives are used for hedging, the agent is induced to choose efforts and investments with higher expected returns more efficiently.

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function that leads to the agent's indirect utility function being convex.

[5] Intuitively, to induce hedging, the principal penalizes the agent for any covariance (positive or negative) between profits and the hedgeable element of profits.

Intuitively, hedging creates value in the settings we examine because it allows contracts to be designed that are contingent on a measure of output that is more highly correlated with the agent's effort. In this sense, our analysis is closely related to the seminal Holmström (1979) paper, which shows that the optimal contract is a function of various state variables that can provide information about the agent's effort. Our contribution is that we extend the analysis to the case where the exposure of profits to these state variables is unknown to the principal. Specifically, we consider a setting with asymmetric information about an element of risk that can be affected by the agent's choice.

We also contribute to the literature that explores how firms make risk management choices. Our paper is most closely related to papers by DeMarzo and Duffie (1991, 1995), who also point out that with hedging, the profits of a firm may provide more precise information about managerial inputs. In DeMarzo and Duffie (1991, 1995), hedging allows the owners of the firm to more precisely learn about managerial ability, which increases the value of the option to either continuing or abandoning firm projects. In contrast, in our model the owners are learning about effort, and with hedging, the contract more efficiently elicits better effort. It should be noted, however, that DeMarzo and Duffie (1991, 1995) ignore the incentive issues with the hedging choice, which is the focus of this paper.

It should also be noted that others have examined the optimal contract between a risk neutral principal and a risk averse agent that makes both risk and effort choices, e.g., Hirshleifer and Suh (1992), Sung (1995), Palomino and Prat (2003), DeMarzo et al. (2011), Barron et al. (2020).[6] The main difference between our paper and these previous papers is that we explicitly consider derivative contracts that can be distinguished from real investments considered in earlier work in two important ways. The first is that the outcome of these contracts are observable, they are not affected by the agent's effort and are contractible, and can thus be included in the optimal contract. The second is that exposure to this element of risk is a zero net present value bet, which means that to the extent possible, the agent's exposure to this element of risk should be minimized.

In summary, our model builds on the prior literature that highlights the importance of including state variables as well as output in optimal agency contracts. However, we are the first to provide a solution to this problem when, in addition to effort, the agent takes a hidden action that influences the relation between the state variable, i.e., hedgeable risk,

[6] Hébert (2018) assumes that the agent picks his effort and risk-shifting by choosing the distribution of state in a non-parametric way. Under special cost functions (e.g., Kullback-Leibler divergence), debt becomes optimal.

and the output.[7]

While we believe that we are the first to model the derivative choices of self-interested managers under moral hazard, the idea that these choices may not be made in the interests of shareholders is not new. For example, in Tufano (1996) study of the gold mining industry, he found that managerial incentives were the most important determinant of corporate derivatives choices. Policymakers are also aware of potential incentive problems. For example, during the global financial crisis, Ben Bernanke stated that "compensation practices at some banking organizations have led to misaligned incentives and excessive risk-taking, contributing to bank losses and financial instability."[8] While poorly written incentive contracts are clearly inconsistent with our model, it is possible that contract changes that should have been introduced along with the introduction and growth of derivative contracts were in fact slow to be enacted.

The paper is organized as follows: In Section 2, we provide a simple model with fixed real investments (i.e., project choices). In Section 3, we formulate our model with hidden and flexible real investment choices. Concluding remarks are provided in Section 4, and the proofs of the Lemmas and Propositions as well as omitted derivations are all given in the Appendix A. We consider cases where the free communication between the principal and the agent is possible, and discuss the optimal truth-telling mechanism in Appendix B. Finally, Appendix C provides the detailed analysis of Section 3 when the agent speculates in derivative markets under the benchmark contract.

## 2 A Model without Project Choice

This section presents a two-person single-period agency model in which a risk averse agent works for a risk neutral principal. The principal can be thought of as the firm's shareholders, and the agent can be regarded as the firm's top manager or CEO. Alternatively, we can think of the principal as the CEO and the agent as the head of one of the firm's divisions. Hereafter, we use the terms 'agent' and 'manager' interchangeably.

[7]Replacing the agent's original incentive compatibility constraint with its first-order condition, which is called the first-order approach, has been typically adopted in the literature. Deriving optimal contracts in our environment turns out to be tricky, as we cannot use the so-called first-order approach, and we make a novel methodological contribution to the literature by deriving optimal contracts based on a methodology that circumvents the first-order approach. For the first-order approach in the canonical agency model, see Grossman and Hart (1983), Rogerson (1985), Jewitt (1988), Sinclair-Desgagné (1994), Conlon (2009), Jung and Kim (2015), Jung et al. (2022) among others.

[8]Fed press release (2009): https://www.federalreserve.gov/newsevents/pressreleases/bcreg20091022a.htm

After his wage contract is finalized, the agent chooses two actions, $a_1 \in [0, \infty)$ and $a_d \in (-\infty, +\infty)$. The first action $a_1$ is a productive effort that increases expected output, that is, a high effort generates an output level that first-order stochastically dominates the output level generated by a low effort. The agent's second action, $a_d$, is his derivative choice. We can think of $a_d$ as forward contract that has zero upfront cost and pays $\eta$ per unit at the end of the period, where $\eta$ can for example, be the difference between the price of oil and its risk neutral expectation.

After the agent chooses $a_1$ and $a_d$, the firm's output, $x$, is realized and publicly observable without cost. Thus, output $x$ can be used in the manager's wage contract denoted by $w$. The output is determined not only by the agent's choice of $(a_1, a_d)$ but also by the state of nature, $(\eta, \theta)$. For simplicity, we assume that the output function exhibits the following additively separable form:

$$x = \phi(a_1) + \sigma\theta + (R - a_d)\eta. \tag{1}$$

The first element, $\phi(a_1)$, is the firm's expected output, which is a function of $a_1$ and is not affected by the agent's derivatives choice, $a_d$. The firm's risk consists of two components, $\eta$ and $\theta$, where $\eta \sim N(0, 1)$ represents one unit of the firm's hedgeable risks, and $\theta \sim N(0, 1)$ represents one unit of the firm's non-hedgeable risks. For simplicity, we assume that $\eta$ and $\theta$ are uncorrelated. As denoted by (1), the firm's total non-hedgeable risk is fixed at $\sigma$, whereas the firm's hedgeable risks are determined by market variables such as commodity prices, interest rates, and exchange rates, which become publicly observable after the agent chooses both $a_1$ and $a_d$.[9] Accordingly, we assume $\eta$ is observable at the end of the period, and thus can also be used in the manager's wage contract if necessary. In (1), $R \sim N(R_m, \sigma_R^2)$ denotes the firm's innate exposure to the hedgeable risks (e.g., the amount of oil underground for a drilling company). We assume that the manager can observe the true value of $R$ after the contract is signed but before he chooses $a_1$ and $a_d$. In contrast, the principal knows only its distribution. We assume that the management effort $a_1$ does not affect $R$, the firm's innate exposure to the hedgeable risks. However, the firm's final risk exposure is determined by the manager's transaction $a_d$ in the derivative market. The manager hedges, i.e., reduces risk, as long as $|R - a_d| < |R|$ and minimizes risk by setting $a_d = R$. On the other hand, if $|R - a_d| > |R|$, the manager speculates in the derivative

[9] In fact, if the relevant derivative market observable is denoted as $p$, then $\eta = p - \overline{p}$ where $\overline{p}$ is the expected value of $p$.

market, and $a_d = 0$ implies that the manager does not trade derivatives. Finally, we assume that the manager's only risk exposure comes from the compensation contract, i.e., he cannot hedge or speculate on his own account.

In addition, we make the following assumptions:

**Assumption 1** The agent's preferences on wealth and productive effort are additively separable:

$$U(w, a_1, a_d) = u(w) - v(a_1), \quad u' > 0, u'' < 0,$$

where $v$, the agent's disutility of exerting productive effort, has the properties $v' > 0, v'' > 0, \ \forall a_1$.

**Assumption 2** $\dfrac{\partial \phi}{\partial a_1}(a_1) \equiv \phi_1(a_1) > 0, \ \dfrac{\partial^2 \phi}{\partial a_1^2}(a_1) \equiv \phi_{11}(a_1) < 0.$

Assumption 1 implies that the agent is risk-averse and effort-averse, and the agent's derivatives choices have no direct effect on his utility.[10] Assumption 2 indicate that the effort $a_1$ affects the expected output with a usual property of decreasing marginal increase in output.

## 2.1 Benchmark Case

Throughout Section 2.1, we consider a benchmark case where both the risk exposure $R$ and the agent's derivative choice $a_d$ are observed by the principal.[11] In this case, the optimal contracts are written based on $y \equiv x - (R - a_d)\eta$, and we have the following result:

**Lemma 1** *If the principal observes the firm's innate risk exposure $R$, and the level of derivative transaction $a_d$, then the compensation of the agent will be independent of the realization of the hedgable risk.*

Lemma 1 can be understood as follows: when $R$ and $a_d$ can be observed by the principal, $(R - a_d)\eta$ provides no information about effort, and thus, following Holmström (1979), does not affect compensation in the optimal contract. It should be noted that the optimal contract will undo the effect of derivative transactions, i.e., the contract offsets the effect of

[10]For the derivative choice $a_d$, we assume that a direct hedging cost (e.g., option premium) is negligible compared with the nominal amounts of a firm's cash flows. Therefore, we assume away costs for derivative choice $a_d$.

[11]This includes cases where there is no derivative market, i.e., $a_d = 0$.

$(R - a_d)\eta$ by compensating the agent based on a new signal $y = x - (R - a_d)\eta = \phi(a_1) + \sigma\theta$, which is independent of $\eta$. In this sense, the actual derivative choice is a matter of indifference.

The optimal wage contract $w(y)$, in this case, is found by solving for the contract that maximizes the combined utilities of the principal and agent subject to the restriction that the agent's effort $a_1$ is chosen to maximize the agent's utility given the contract. The optimization is given by[12]

$$\begin{aligned} \max_{a_1, w(\cdot)} \phi(a_1) - \int w(y) f(y|a_1) dy + \lambda \left( \int u(w(y)) f(y|a_1) dy - v(a_1) \right) \quad \text{s.t.} \\ (i)\ \ a_1 \in \arg\max_{a_1'} \int u(w(y)) f(y|a_1') dy - v(a_1'),\ \ \forall a_1', \\ (ii)\ \ w(y) \geq k,\ \ \forall y, \end{aligned} \tag{2}$$

where $f(y|a_1)$ denotes a probability distribution function of $y \sim N(\phi(a_1), \sigma^2)$ given the agent's action $a_1$, and $\lambda$ is a welfare weight placed on the agent's utility in the joint optimization (2). As shown, the combined utilities of the principal and the agent are maximized subject to the agent's incentive compatibility constraint which specifies that the agent chooses his effort for his own optimization, and his limited liability constraint which specifies that the agent receives at least $k$, the subsistence level of utility.[13] Based on the first-order approach, instead of the optimization (2), we solve the following alternative:

$$\begin{aligned} \max_{a_1, w(\cdot) \geq k} \phi(a_1) - \int w(y) f(y|a_1) dy + \lambda \left( \int u(w(y)) f(y|a_1) dy - v(a_1) \right) \quad \text{s.t.} \\ (i)\ \ \int u(w(y)) f_1(y|a_1') dy - v'(a_1') = 0, \end{aligned} \tag{3}$$

where we replace the agent's incentive compatibility constraint with his first-order condition.[14]

[12]The optimization in (2) yields a mathematically equivalent solution to an agency model where a principal maximizes her utility subject to an optimizing agent receiving his reservation utility level: see e.g., Holmström (1979). Our purpose here is to analyze the overall efficiency implication of financial market innovations and thus we choose to fix $\lambda$, which is usually an endogenous Lagrange multiplier in the literature.

[13]The limited liability constraint is introduced to guarantee the existence of optimal solution for $w(y)$. This condition is needed because we assume that the signal is normally distributed. For details about this 'unpleasantness', see Mirrlees (1974) and Jewitt et al. (2008).

[14]We assume that the first-order approach is valid. Grossman and Hart (1983) and Rogerson (1985) show that MLRP and CDFC are sufficient for the validity of the first-order approach when the signal space is of one dimension. Jewitt (1988) finds less restrictive conditions for the validity of the first-order approach, based on

Following the literature (see e.g., Kim (1995)), to find the optimal solution $(a_1^*, w^*(y|a_1^*))$ for the optimization in (2), we first derive an optimal contract for an arbitrarily given action $a_1$. Let $w^*(y|a_1)$ be a contract that optimally motivates the agent to choose a particular level of $a_1$. Subject to some technical assumptions, by solving the Euler equation of the above program (2) after fixing $a_1$, we derive that $w^*(y|a_1)$ must satisfy

$$\frac{1}{u'(w^*(y|a_1))} = \lambda + \mu_1(a_1)\frac{f_1}{f}(y|a_1), \tag{4}$$

for almost every $y$ for which the solution in (4) satisfies $w^*(y|a_1) \geq k$, and otherwise $w^*(y|a_1) = k$. In (4), $\mu_1(a_1)$ denotes the optimized Lagrange multiplier for the agent's incentive compatibility constraint associated with $a_1$. Since $y \sim N(\phi(a_1), \sigma^2)$, (4) is reduced to:

$$\frac{1}{u'(w^*(y|a_1))} = \lambda + \mu_1(a_1)\frac{y - \phi(a_1)}{\sigma^2}\phi_1(a_1). \tag{5}$$

Before analyzing the optimal contract $w^*(\cdot)$, we first define social welfare $SW^*$ as a function of $a_1$ as follows:

$$SW^*(a_1) \equiv \phi(a_1) - C^*(a_1) - \lambda v(a_1), \tag{6}$$

which denotes the joint benefits when $w^*(y|a_1)$ is designed where

$$C^*(a_1) \equiv \int (w^*(y|a_1) - \lambda u(w^*(y|a_1)))\, f(y|a_1)dy \tag{7}$$

represents the efficiency loss in this case compared with the full information case, given the optimal contract $w^*(\cdot|a_1)$. In other words, $C^*(a_1)$ measures the agency cost arising from motivating the agent to take a particular action $a_1$. Finally, the optimal action $a_1^*$ can be found by

$$a_1^* \in \arg\max_{a_1'} SW^*(a_1), \tag{8}$$

and we simplify notation, thus $w^*(y) \equiv w^*(y|a_1^*)$. The optimal surplus in this case will be given by $SW^* \equiv SW^*(a_1^*)$.

the agent's risk preferences as well as the distribution function of the signal. Sinclair-Desgagné (1994) shows that more general versions of MLRP and CDFC in a multi-dimensional space are sufficient for the validity of the first-order approach when the signal space is of multiple dimensions. For more recent treatments along this line, see Conlon (2009) and Jung and Kim (2015) among others. Recently, Jung et al. (2022) justifies the first-order approach when the technology follows normal distributions, which corresponds to our problem in (2).

Given the optimal contract $w^*(\cdot)$, we define the agent's indirect utility function $V(\cdot)$ as

$$V(y) \equiv u(w^*(y)). \tag{9}$$

We know from Rothschild and Stiglitz (1970) that if $V(\cdot)$ is convex (concave), then the agent wants to raise (reduce) the level of overall risk to the output $x$ if possible. In general, the curvature of the agent's indirect utility function $V(\cdot)$ depends on the distribution of random state variables and the utility function $u(\cdot)$ itself. To see how different utility functions affect this curvature differently, we consider the case where the agent has constant relative risk aversion with degree $1-t$, where $t<1$ ($u(w)=\frac{1}{t}w^t, t<1$). We obtain from equation (5) that

$$w^*(y) = \left(\lambda + \mu_1(a_1^*)\left(\frac{y-\phi(a_1^*)}{\sigma^2}\right)\phi_1(a_1^*)\right)^{\frac{1}{1-t}}, \tag{10}$$

and the agent's indirect utility under this wage contract is

$$V(y) \equiv u(w^*(y)) = \frac{1}{t}\left(\lambda + \mu_1(a_1^*)\left(\frac{y-\phi(a_1^*)}{\sigma^2}\right)\phi_1(a_1^*)\right)^{\frac{t}{1-t}}. \tag{11}$$

The above equation shows that the agent's indirect utility $V(\cdot)$ becomes strictly concave in $y$[15] if $t<\frac{1}{2}$, linear if $t=\frac{1}{2}$, and convex if $t>\frac{1}{2}$ for $y$ satisfying $w^*(y)\geq k$. If we assume $w^*(y)=k$ for sufficiently low $y$, as far as the agent's induced risk preferences are concerned, the agent acts as if he is risk-loving if and only if $t\geq\frac{1}{2}$, i.e., the agent's risk aversion is lower than $\frac{1}{2}$.

## 2.2 The Model with Derivative Markets

Now, we go back to the model's original specification, where the hedgeable risk, $\eta \sim N(0,1)$, is contractible, but the firm's inherent risk-exposure, $R$, is observed only by the agent. With access to the derivative market, the agent can choose any level of $a_d$. However, because of the asymmetry of information between the two parties (i.e., principal and agent) about the value of risk-exposure $R$, the efficiency of the agency relationship can be hurt, even if $\eta$ can be used in contracts. In this section, we study this issue in depth.

[15] It is widely known in the literature that $\mu_1(a_1^*)>0$ at the optimum. For the proof, see Holmström (1979), Jewitt (1988), Jung and Kim (2015), Jung et al. (2022) among others.

Since the principal does not observe $R$ or $a_d$, contracts can depend only on the output $x$ and the hedgeable risk $\eta$. We start our analysis by assuming the same contract $w^*(x)$,[16] given in (5), which is optimal in the benchmark case, is given to the agent, and analyze cases where the agent's indirect utility function, $V(x) = u(w^*(x))$, given this contract $w^*(x)$, is either convex or concave in output $x$.

#### 2.2.1 When the agent's indirect utility $V(x)$ is concave in output $x$

As we discussed in the last subsection, With a concave indirect utility function $V(x)$, the agent has an incentive to minimize the risk of $x$, which implies that his optimal strategy is to eliminate the risk-exposure $R$ by choosing $a_d = R$.[17] In this case there is no efficiency loss from the information asymmetry between shareholders and the manager. Specifically, the hedging choice of the agent eliminates the risk-exposure, and thus no longer affects the output $x$. In this case, the optimal contract as well as the optimal action remains the same at $w^*(x)$ and $a_1^*$ in (5) and (8) and social welfare is the same as in equation (6) of Section 2.1.

This result is summarized by the following Proposition 1.

**Proposition 1** *When the agent's indirect utility $V(x) = u(w^*(x))$, under the benchmark optimal contract $w^*(x)$ defined in (9), is concave in output $x$, the agent always chooses $a_d = R$ (i.e., complete hedging) in derivative markets, eliminating the welfare loss arising from asymmetric information about the risk-exposure $R$. The surplus remains at $SW^*$.*

For the case of constant relative risk aversion with degree $1 - t$ (i.e., $u(w) = \frac{1}{t}w^t$, $t < 1$), for example, the manager whose preference shows a higher risk aversion than $t = \frac{1}{2}$ case (i.e., $t < \frac{1}{2}$) features a concave $V(\cdot)$, thereby belonging to the case of the above Proposition 1.

#### 2.2.2 When the agent's indirect utility $V(x)$ is convex in output $x$

If the agent's indirect utility $V(x)$ is convex in output $x$ when the risk exposure is observable, then there is a cost associated with the risk exposure being unobserved. To show this, we first consider the case where the principal offers the agent the same contract $w^*(\cdot)$

[16] Note that the contract $w^*(\cdot)$ is now a function of the output $x$, not the constructed signal $y$ of Section 2.1.

[17] Since $x = \phi(a_1) + \sigma\theta + (R - a_d)\eta$, the agent can minimize the risk by choosing $a_d = R$.

described in Section 2.1. As we show in the following Lemma 2, when this is the case, the agent has an incentive to increase the risk of output $x$ as much as possible, choosing $a_d = \infty$.

**Lemma 2** *If the agent's indirect utility $V(x) = u(w^*(x))$, under the benchmark contract $w^*(x)$ defined in (9), is convex in output $x$, the agent chooses $a_d = \infty$ (i.e., infinite speculation) given $w^*(x)$.*

Note that Lemma 2 implies that the agent's indirect utility function $V(\cdot)$ cannot be convex if the agent is allowed to trade, without constraints, in the derivative market. In other words, the contract $w^*(\cdot)$ in (5) is not optimal in this setting.

To examine how the contract can be designed to motivate the agent to choose 'finite' risk we consider two cases: In the first case, we consider a contract that motivates the agent to engage in complete hedging (i.e., $a_d = R$). Alternatively, shareholders can restrict access to derivative markets, so that the manager can only choose $a_d = 0$. Both cases result in welfare losses relative to the efficient contract in Section 2.1 where there is no information asymmetry.

Our analysis starts with the case where the use of derivatives is banned.

**Case 1: Banning the use of derivatives** In this case, $a_d = 0$, regardless of the level of $R$ that the agent observes before taking effort $a_1$. Due to the asymmetric information about $R$ between shareholders and manager, the optimized joint welfare in this case, which we denote by $SW^N$, is lower than $SW^*$ in Section 2.1. Since shareholders do not observe $R$, the compensation contract must be based on $(x, \eta)$, i.e., $w = w(x, \eta)$.

The principal's maximization program is thus:

$$SW^N \equiv \max_{\substack{a_1(\cdot) \\ w(\cdot)\geq k}} \int_R \left( \int_{x,\eta} (x - w(x,\eta) + \lambda u(w(x,\eta))\, g(x,\eta|a_1(R),R)h(R)dxd\eta - \lambda v(a_1(R)) \right) dR$$

$$\text{s.t. } (i) \;\; a_1(R) \in \arg\max_{a_1} \int_{x,\eta} u(w(x,\eta))g(x,\eta|a_1,R)dxd\eta - v(a_1), \forall R, \tag{12}$$

where

$$g(x,\eta|a_1,R) = \frac{1}{2\pi\sigma} \exp\left(-\frac{1}{2}\left(\frac{(x-\phi(a_1)-R\eta)^2}{\sigma^2} + \eta^2\right)\right) \tag{13}$$

denotes the probability density function of $(x, \eta)$ given $(a_1, a_2, R)$ when $a_d = 0$, and $h(R)$ denotes the probability density function of $R$. $SW^N$ is defined as the optimized surplus in this case. Note that as the agent is the only one seeing the realized value of $R$, his action $a_1$ would depend on observed $R$, given the contract $w(x,\eta)$.

For each $R$, let $\left\{a_1^N(R), w^N(x,\eta)\right\}$ be the solution of the above optimization program in (12). If we let $\mu_1(R)$ be the optimized Lagrange multiplier attached to the incentive constraint in $a_1(R)$, the optimal contract $w^N(x,\eta)$ can be written as[18]

$$\frac{1}{u'(w^N(x,\eta))} = \lambda + \int_R \mu_1(R) \left[ \frac{g_1(x,\eta|a_1^N(R),R)}{\int_{R'} g(x,\eta|a_1^N(R'),R')h(R')dR'} \right] h(R)dR \tag{14}$$

when $w(x,\eta) \geq k$ and otherwise $w(x,\eta) = k$.

$SW^N$ in this case is lower than $SW^*$ of Section 2.1, since the principal can no longer use $R$, which is an informative signal about the agent's effort, in the design of the compensation contract. This is summarized in the following Proposition 2.

**Proposition 2** *When the principal bans derivative contracts and any communication between the principal and the agent is not possible, the principal's inability to observe the firm's risk exposure reduces welfare, i.e.,*

$$SW^N < SW^*.$$

Intuitively, when the principal observes the firm's risk exposure, $R$, this information can be used to design a compensation contract that eliminates the influence of the hedgeable risks, i.e., $w = w^*(y \equiv x - R\eta)$.[19] However, if $R$ is not observable and cannot be communicated this is impossible. We can clearly see the benefit of hedging that reduces $R$ here: hedging mitigates this concern where you cannot use an informative signal $R$ in a contract, and achieves the better welfare.

**Case 2: Designing an optimal contract that allows derivative trading** As we discussed above, the contract $w^*(\cdot)$ in (5) induces infinite speculation, i.e., $a_d = \infty$. In this subsection, we consider compensation contracts that induce the manager to make finite derivative choices. We will show that there is a menu of optimal contracts that achieve identical effort

[18] We provide the derivation for equation (17) in Appendix.

[19] As we explained in Section 2.1, this is related to the 'informativeness' principle in Holmström (1979), who argues a signal has a positive value (i.e., should be used in contracts) if it affects the local likelihood ratio.

and utilities for both the agent and the principal. The indeterminacy of the optimal contract arises because any change in risk exposure specified in a contract can be costlessly offset by the derivative positions chosen by the agent.

To understand this indeterminacy, note that since the agent observes $R$ before choosing the actions $a_1$ and $a_d$, his choice of $a_d$ can be characterized as his choice of $b \equiv R - a_d$. Suppose that the principal wants to induce the action $a_1^o$ and $\hat{b} = R - a_d^o$ with some contract $w^o(x, \eta)$, i.e., given contract $w^o(x, \eta)$, the agent will choose $a_1^o$ and $a_d^o$ such that $b = R - a_d^o = \hat{b}$, satisfying

$$\hat{b} \in \arg\max_{b} \mathbb{E}\left[u\left(w^o\left(\underbrace{\phi\left(a_1^o\right) + \sigma\theta + b\eta}_{\equiv x}, \eta\right)\right)\right] - v(a_1^o), \tag{15}$$

which can be translated into

$$0 \in \arg\max_{b} \mathbb{E}\left[u\left(w^o\left(\underbrace{\phi\left(a_1^o\right) + \sigma\theta + \hat{b}\eta + b\eta}_{\equiv x + \hat{b}\eta}, \eta\right)\right)\right] - v(a_1^o), \tag{16}$$

which implies, if the principal induces $b = \hat{b}$ with contract $w^o(x, \eta)$, then she can always motivate $b = 0$ with another contract $w^{oo}(x, \eta) \equiv w^o(x + \hat{b}\eta, \eta)$. As the principal is risk neutral, in this case without loss of generality, we can assume the principal induces $b = 0$ (i.e., perfect hedging).

Given that the agent choosing $a_d$ given his private information $R$ is equivalent to his choosing $b = R - a_d$, a new optimal contract, $w^o(x, \eta)$, inducing the agent to take $(a_1^o, b = 0)$ must solve the following optimization problem:[20]

$$\begin{aligned} \max_{w(\cdot)\geq k} \int (x - w(x,\eta)) g(x, \eta | a_1^o, b=0) dx d\eta + \lambda \left( \int u(w(x,\eta)) g(x, \eta | a_1^o, b=0) dx d\eta - v(a_1^o) \right) \\ \text{s.t. } (i) \int u(w(x,\eta)) g_1(x, \eta | a_1^o, b=0) dx d\eta - v'(a_1^o) = 0, \\ (ii)\ b = 0 \in \arg\max_{b'} \int u(w(x,\eta)) g(x, \eta | a_1^o, b') dx d\eta,\ \forall b. \end{aligned} \tag{17}$$

[20]Here the distribution $g(x, \eta | a_1, b)$ is of the joint normal distribution of $(x, \eta)$ implied by equation (1).

**First-order approach with $b = 0$.** Note that the optimization problem (17) takes the *optimal* $a_1^o$ as given, and relies on the first-order approach for the incentive constraint associated with the effort $a_1$, as we do in optimization (3).[21] However, we do not use the same first-order approach for the incentive compatibility constraint associated with the hedging choice $b$. The following Lemma 3 demonstrates the reason we cannot use the first-order approach for the incentive compatibility around $b$.

**Lemma 3** *If $w^*(x)$ in (5) is designed, the agent will be indifferent between taking $b$ and taking $-b$, $\forall b$.*

Lemma 3 shows that if $w^*(x)$, the optimal contract in the benchmark case, is offered, the manager's expected utility is symmetric around $b = 0$ (i.e., $a_d = R$) in the space of $b$ (i.e., in the space of $a_d$). As we know:

$$\int u(w^*(x))g(x, \eta|a_1^o, b)dzd\eta \tag{18}$$

is continuous and differentiable in $b$, that Lemma 3 implies:

$$\int u(w^*(x))g_b(x, \eta|a_1^o, b = 0)dzd\eta = 0. \tag{19}$$

Since $(w^*(x), a_1^*)$ is the solution to the optimization in (2), where there is no incentive constraint of $b$,[22] if we use the first-order approach for the incentive constraint associated with $b$ in the above program (17), we always end up with $w^*(x)$ in (5) as the optimal contract. However, we can see from Lemma 2 that this contract incentivizes the agent to take $b = \pm\infty$ instead of taking the stipulated $b = 0$. Therefore, the first-order approach cannot be used in this setting. In particular, we cannot rely only on the first-order condition at $b = 0$, but must instead explicitly include the incentive constraint that must hold for all $b$.

Without relying on the first-order approach, we follow Grossman and Hart (1983), re-

[21] $g_1(x, \eta|a_1, b)$ is defined as a partial derivative of $g(x, \eta|a_1, b)$ with respect to $a_1$. Likewise, we define $g_b(x, \eta|a_1, b)$ as $g(x, \eta|a_1, b)$'s partial derivative with respect to $b$.

[22] Since for $b = 0$, the likelihood ratios can be represented as

$$\frac{g_1}{g}(x, \eta|a_1, b = 0) = \frac{x - \phi(a_1)}{\sigma^2}\phi_1(a_1), \quad \frac{g_b}{g}(x, \eta|a_1, b = 0) = \frac{(x - \phi(a_1))\eta}{\sigma^2}, \tag{20}$$

we see that $w^*(x)$ becomes the solution of the (2) without the incentive constraint of $b$.

placing the incentive constraint for $b$ (i.e., (ii) in (17)) with:

$$\int u(w(x,\eta))\left(g(x,\eta|a_1^o,b=0)-g(x,\eta|a_1^o,b)\right)dxd\eta \geq 0,\ \forall b, \tag{21}$$

which implies that the manager's indirect utility is maximized when he takes $b=0$ (i.e., $a_d=R$).

Now we state formally the optimization problem of choosing the optimal contract $w^o(\cdot)$ given $a_1^o$ as:

$$\begin{aligned} SW^o \equiv \max_{w(\cdot)\geq k} & \int (x-w(x,\eta))g(x,\eta|a_1^o,b=0)dxd\eta + \lambda\left(\int u(w(x,\eta))g(x,\eta|a_1^o,b=0)dxd\eta - v(a_1^o)\right) \\ \text{s.t. } (i) & \int u(w(x,\eta))g_1(x,\eta|a_1^o,b=0)dxd\eta - v'(a_1^o)=0, \\ (ii) & \int u(w(x,\eta))\left(g(x,\eta|a_1^o,b=0)-g(x,\eta|a_1^o,b)\right)dxd\eta \geq 0,\ \forall b, \end{aligned} \tag{22}$$

where we define $SW^o$ as the optimized surplus in this case. The first-order condition of the above program (22) yields the optimal contract, $w^o(x,\eta)$, that satisfies

$$\frac{1}{u'(w^o(x,\eta))} = \lambda + \mu_1^o \frac{x-\phi(a_1^o)}{\sigma^2}\phi_1(a_1^o) + \underbrace{\int \mu_b^o(b)\left(1-\frac{g(x,\eta|a_1^o,b)}{g(x,\eta|a_1^o,b=0)}\right)db}_{\text{Additional term to (5)}}, \tag{23}$$

for $(x,\eta)$ satisfying $w^o(x,\eta)\geq k$ and $w^o(x,\eta)=k$ otherwise. In (23), $\mu_1^o$ and $\mu_b^o(b)$ are the optimized Lagrange multipliers associated with the first constraint (i.e., (i)) and the second constraint for a particular $b$ (i.e., (ii)) in the above optimization program (22), respectively.[23]

As we formally prove in the Appendix A, equation (23) implies the following proposition:

**Proposition 3** *Suppose that the agent's indirect utility $V(x)$ in (9) is convex in output $x$. The agent can be motivated to hedge completely with a new contract, $w^o(x,\eta)$ in (23), which (i) satisfies $w^o(x,\eta)=w^o(x,-\eta)$ for all $x,\eta$; (ii) penalizes the manager for having a higher realized $(x-\phi(a_1^o))^2\eta^2$. Specifically, given the realized $(x,\eta)$, in the optimal contract, a higher $(x-\phi(a_1^o))^2\eta^2$ reduces $w^o(x,\eta)$, and for any given output $x$ and $(x-\phi(a_1^o))^2\eta^2$, the wage $w^o(x,\eta)$ increases in $\eta$.*

[23] For general reference about the variational approach to the optimization (22), see e.g., Luenberger (1969).

Proposition 3 can be understood as follows: with the output $x = \phi(a_1^o) + \sigma\theta + b\eta$, $b$ can be expressed as $Cov(x, \eta) \equiv \mathbb{E}((x - \phi(a_1^o))\eta)$. If the agent fully hedges (i.e., $b = 0$), the covariance between output $x$ and hedgeable risk $\eta$ becomes zero, whereas any other $b \neq 0$ generates non-zero expected covariance. Since the manager voluntarily chooses $b = 0$ given $w^o(x, \eta)$, $x = \phi(a_1^o) + \sigma\theta$ is generated, which is independent of $\eta$, $w^o(x, \eta) = w^o(x, -\eta)$ for all $x$. At the optimum $\eta$ is thus insured to minimize the amount of risk imposed on the agent, as $\eta$ is now irrelevant in inducing $a_1^o$ (as $x$ does not depend on $\eta$ under $b = 0$) and has a symmetric distribution around $0$.

Ideally, by penalizing the covariance between $x$ and $\eta$, shareholders can effectively induce full hedging (i.e., $b = 0$) from the manager. The problem is that the principal does not observe the population covariance between $x$ and $\eta$. Therefore, as our model is static, any positive or negative realized sample covariance $\widehat{Cov} = (x - \phi(a_1^o))\eta = b\eta^2 + \sigma\theta\eta$, instead of a population covariance, is punished by the principal through a lower compensation $w^o(x, \eta)$. Since the optimization (22) is symmetric around $b = 0$, the optimal contract $w^o(x, \eta)$ punishes positive and negative sample covariance $(x - \phi(a_1^o))\eta$ in a symmetric way, i.e., penalizes higher $((x - \phi(a_1^o)\eta)^2$. If the realized covariance $\widehat{Cov} = (x - \phi(a_1^o))\eta = b\eta^2 + \sigma\theta\eta$ is large, not because of the manager's speculation (i.e., $b \neq 0$) but from a high level of realized market observable, $|\eta|$, then the principal takes it into account and raises $w^o(x, \eta)$. In contrast, given realized output and market observables $(x, \eta)$, a bigger realization of $\widehat{Cov}$ is likely to be generated by $b \neq 0$ with a bigger $|b|$, thus the agent is penalized and her compensation $w^o(x, \eta)$ falls.

Note that social welfare in this case $SW^o$ is lower than the benchmark level $SW^*$, as we impose an additional incentive compatibility constraint in (17) compared with (2). This is summarized by the following Proposition.

**Proposition 4** *When the agent's indirect utility $V(x)$ given in (9) is convex in output $x$, the introduction of a derivative market will reduce the firm's welfare to $SW^o$ from $SW^*$ in Section 2.1. Therefore,*

$$SW^o < SW^*.$$

**Comparing $SW^N$ and $SW^o$** From Propositions 2 and 4, we observe that in cases where the agent infinitely speculates given the optimal $w^*(x)$ from Section 2.1, i.e., when $V(x)$ is convex in output $x$, the social welfare (i.e., $SW^N$ or $SW^o$) is reduced from $SW^*$ as either (i) the principal does not allow the manager to transact in derivative markets; or (ii) alter the optimal contract in a way that the manager voluntarily hedges, which distorts from the original risk-sharing in (5) and hurts the efficiency.

The following Proposition 5 compares social welfare levels $SW^N$ and $SW^o$ in some cases.

**Proposition 5** *When the agent's indirect utility $V(x)$ given in (9) is convex in output $x$, then $SW^o$, the level of welfare when the agent's derivative choices are unrestricted, can be lower than $SW^N$ in (22), the level of welfare when derivative use is banned. This will be the case when uncertainty about the firm's risk exposure, $\sigma_R^2$, is small.*[24]

As we illustrated in Proposition 3, if the agent can transact in derivative markets and choose $a_d$, the optimal contract must be altered from $w^N(x,\eta)$ in equation (14) to $w^o(x,\eta)$ in equation (23). In the cases where the manager chooses to hedge when the derivative market is introduced, given his original optimal contract $w^*(\cdot)$ (i.e., $V(x)$ is concave in $x$), the compensation contract remains unchanged from $w^*(x)$, and welfare unambiguously increases because of the informational gain $SW^* - SW^N$ generated by using derivatives to eliminate $R$.[25] However, when a convex $V(x)$ given $w^*(x)$ induces the manager to speculate in the derivative market, shareholders must revise the manager's contract from $w^*(x)$ to $w^o(x,\eta)$ to provide an incentive to hedge. This is costly because it imposes additional risk on the risk-averse manager, so welfare declines.

Note that we do not require expectation with respect to the firm's initial risk exposure $R$ to calculate joint benefits $SW^*$ and $SW^o$, since they are both independent of $R$. When there is no hedgeable risk, i.e., $\eta \equiv 0$, then joint benefits, $SW^*$, become obviously independent of the $R$'s realization because the optimal action $a_1^*$ is independent of $R$. Similarly, when $w^o(x,\eta)$ is designed in the presence of derivative markets, the joint benefits $SW^o$ are independent of $R$ as agent is always induced to take $b = R - a_d = 0$ no matter what $R$ is realized. However, in calculating joint benefits $SW^N$, the expectation with respect to $R$ is taken, implying that the distribution of $R$ affects the level of $SW^N$. Generally, as $\sigma_R^2$ decreases, the degree of asymmetric information between two parties falls, reducing $SW^N$. When $\sigma_R^2 \to 0$ especially, it would be $SW^N \to SW^*$.

The above discussion implies that informational gains from the manager's hedging decline as the amount of uncertainty (i.e., $\sigma_R^2$) in the firm's risk exposure $R$ falls. On the other

[24]Or the principal can put some restriction on derivative transactions, which might weakly dominate the complete banning.

[25]In cases where $V(x)$ is concave, the social welfare is improved to $SW^*$ from $SW^N$ by Proposition 2.

hand, the cost of controlling the additional incentive problem associated with $a_d$ (or equivalently $b = R - a_d$) is independent of the firm's risk exposure $R$, and thus $\sigma_R^2$. For instance, even if $R$ is known to the principal (i.e., $\sigma_R^2 = 0$), the moral hazard problem associated with inducing $b = 0$ still remains to the same degree. Therefore, $\sigma_R^2$ is indeed a matter of indifference in incentivizing the agent's choice of $b$, and we can conclude that as $\sigma_R^2 \to 0$, we definitely would have $SW^o < SW^N$, as $SW^o$ is unaffected while $SW^N \to SW^*$ which is greater than $SW^o$.

**Remark** We conclude Section 2 by noting that sometimes, it is better for shareholders to shut down the manager's access to derivative markets, due to the agency problem around his hedging choices. In Appendix B, we consider possible communication between shareholders and manager about the value of $R$ that the manager observes. As we show, the conditions under which welfare is improved by inducing the agent to truthfully reveal the firm's risk exposure are identical to the conditions under which welfare is improved by inducing the agent to fully hedge. This result illustrates our claim that hedging effectively improves efficacy of the agency relationship through its informational provision.

# 3 A Model with Discretionary Project Choice

This section extends the model to include the agent's real investment choices. Specifically, after his wage contract is finalized, the agent takes three kinds of actions, $a_1 \in [0, \infty)$, $a_2 \in [\underline{a}_2, \overline{a}_2]$, and $a_3 \in (-\infty, +\infty)$. The agent's first action, $a_1$ is the productive effort choice, which increases expected output as before, that is, high effort generates an output level that first-order stochastically dominates the output level generated by low effort. The agent's second action $a_2$ is his (real) project choice. We assume there exists projects with different risks with more risky projects having higher expected output. The agent's preference is still the same as in Assumption 1. The third action $a_3$ is his choice in the derivatives market.[26] We assume that although the set of projects available to the agent is bounded, i.e., $a_2 \in [\underline{a}_2, \overline{a}_2]$, the agent can choose any position in the derivatives market, i.e., $a_3 \in (-\infty, +\infty)$ as in Section 2.

After the agent chooses $a_1$, $a_2$, and $a_3$, the firm's output, $x$, is realized and publicly observable without cost. Thereby, an output $x$ can be used in the manager's wage contract that is denoted by $w$. The output is determined not only by the agent's choice of $(a_1, a_2, a_3)$

[26]We use the notation $a_3$ instead of $a_d$ of Section 2.

but also by the state of nature, $(\eta, \theta)$. For simplicity, we assume that the output function exhibits the following additively separable form:

$$x = \phi(a_1, a_2) + a_2\theta + (R - a_3)\eta. \tag{24}$$

Equation (24) looks like equation (1), except that (i) the agent's project choice $a_2$ affects the expected output level $\phi(a_1, a_2)$; and (ii) the firm's level of non-hedgeable risk is not fixed *a priori*, but determined by the agent's project choice $a_2$. Now, an expected output, $\phi(a_1, a_2)$, is a function of both $a_1$ and $a_2$, whereas the agent's derivatives choice, $a_3$, does not directly affect it. As in (1), we assume that (i) $\eta \sim N(0, 1)$ and $\theta \sim N(0, 1)$ are uncorrelated; and (ii) $\eta$ is observable at the end of the contracting period, and thereby can be used in the manager's wage contract if necessary. As in Section 2, the manager observes $R$ after the contract is signed but before choosing $a_1$, $a_2$, and $a_3$. Again, shareholders do not observe $R$, but know its distribution $R \sim N(R_m, \sigma_R^2)$. Management effort $a_1$ and project choice $a_2$ do not affect $R$, the firm's innate exposure to the hedgeable risks.[27] However, the firm's final risk exposure is determined by the manager's transaction $a_3$ in the derivative market. If $a_3 = 0$, the manager does not trade derivatives. The manager hedges, i.e., reduces risk, as long as $|R - a_3| < |R|$ and minimizes risk by setting $a_3 = R$. On the other hand, if $|R - a_3| > |R|$, the manager speculates in the derivative market.

In addition to assumptions in Section 2, we make the following additional assumptions:

**Assumption 3** $\dfrac{\partial \phi}{\partial a_1}(a_1, a_2) \equiv \phi_1(a_1, a_2) > 0,\ \dfrac{\partial^2 \phi}{\partial a_1^2}(a_1, a_2) \equiv \phi_{11}(a_1, a_2) < 0.$

**Assumption 4** $\dfrac{\partial \phi}{\partial a_2}(a_1, a_2) \equiv \phi_2(a_1, a_2) > 0,\ \phi_{22}(a_1, a_2) < 0,\ \phi_2(a_1, \underline{a}_2) = \infty,$ *and* $\phi_2(a_1, \overline{a}_2) = 0.$

**Assumption 5** $0 < \underline{a}_2 < \overline{a}_2 < \infty.$

**Assumption 6** $\phi_{12}(a_1, a_2) \cdot a_2 < \phi_1(a_1, a_2)$ *for* $\forall (a_1, a_2).$

Assumptions 3 and 4 specify that $a_1$ affects expected output with a usual property of decreasing marginal increase in output, while a higher $a_2$ increases expected output as well

[27] In general, a firm's risk exposure can depend on the real investment undertaken and if we allow the firm's risk exposure to be affected by the manager's project choice $a_2$, most results in this paper do not change qualitatively.

as output variability, i.e., there is a trade-off between return and risk.[28]. Assumption 5 states that there is neither a completely safe project nor a project with unbounded risk.

If $\phi_{12}(a_1, a_2)$ is positive and decreasing in $a_2$, and $\phi_1(a_1, \underline{a}_2) \simeq 0$, $\underline{a}_2$ is close to $0$, then Assumption 6 holds as we see in Figure 1. As the manager raises a project risk level $a_2$, an increase in effort $a_1$ results in a *higher* increase in expected output $\phi(a_1, a_2)$, i.e., $\phi_{12}(a_1, a_2)$ is positive.[29] We assume this complementarity between $a_1$ and $a_2$ become weaker as the project becomes riskier, i.e., $a_2$ increases.

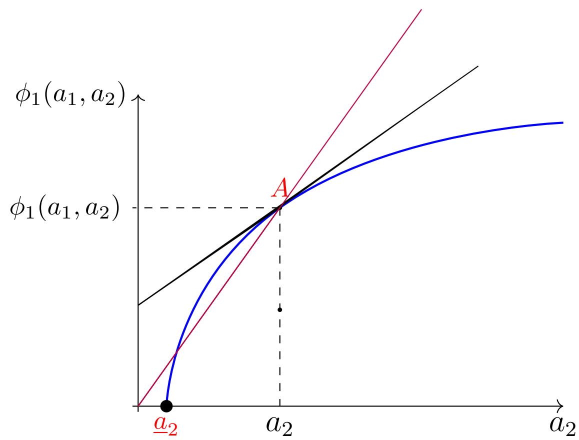


Figure 1: Illustration of the Assumption 6

## 3.1 When There Is No Derivative Market

### 3.1.1 The Principal Knows the Firm's Exposure to the Hedgeable Risks

In this section, we consider a benchmark case where there is no derivatives market and the principal knows the firm's innate risk exposure, $R$. We thus specify $a_3 = 0$ so that the production function in equation (24) reduces to

$$x = \phi(a_1, a_2) + R\eta + a_2\theta. \tag{25}$$

Since there is no derivative market, the manager's incentive problem arises only in in-

[28] As noted from equation (24), reducing the firm's non-hedgeable risks requires the firm to sacrifice a part of an expected output. This trade-off guarantees the existence of an optimal project choice $a_2$ in our agency setting.

[29] For example, if we regard the action $a_1$ as managing the project on a day-to-day basis, it is natural to assume that when the manager takes additional project risk $a_2$, the role of action $a_1$ in generating output becomes more important, i.e., $\phi_1(a_1, a_2)$ rises.

ducing $(a_1, a_2)$. As $R$ and $\eta$ are observable and thus contractible, $y \equiv x - R\eta$ is a sufficient statistic for $(x, \eta)$ in assessing $(a_1, a_2)$. Therefore, the principal uses $y$ as a contractual variable to induce $(a_1, a_2)$, and the above equation can be expressed as

$$y = \phi(a_1, a_2) + a_2\theta. \tag{26}$$

**Benchmark: without incentive problem in $a_2$** In general, designing a contract to optimally induce project choice $a_2$ as well as effort choice $a_1$ is different than designing a contract that only induces the agent's effort choice $(a_1)$. To illustrate this distinction, we first consider the case in which the agent's project choice, $a_2$, is observable, or equivalently, selected by the principal. The optimal compensation contract $w(\cdot)$, in this case, maximizes the combined utilities of the principal and agent subject to the restriction that the agent's effort $a_1$ is chosen to maximize his utility given the contract.

$$\begin{aligned} \max_{a_1,a_2,w(\cdot)} \phi(a_1, a_2) - &\int w(y) f(y|a_1, a_2) dy + \lambda \left( \int u(w(y)) f(y|a_1, a_2) dy - v(a_1) \right) \quad \text{s.t.} \\ &(i)\ \ a_1 \in \arg\max_{a_1'} \int u(w(y)) f(y|a_1', a_2) dy - v(a_1'),\ \ \forall a_1', \\ &(ii)\ \ w(y) \geq k,\ \ \forall y, \end{aligned} \tag{27}$$

where $f(y|a_1, a_2)$ denotes a probability density function of $y$ given the agent's three actions, and $\lambda$ denotes the weight placed on the agent's utility in the joint optimization. As shown, the combined utilities of the principal and the agent are maximized subject to the agent's incentive compatibility constraint, which specifies that the agent optimally chooses his effort, and his limited liability constraint, which specifies that the agent receives at least $k$, the subsistence level of utility.

Based on the first-order approach as in Section 2, the above maximization problem (27) reduces to:

$$\begin{aligned} \max_{\substack{a_1,a_2 \\ w(\cdot)\geq k}} \phi(a_1, a_2) - &\int w(y) f(y|a_1, a_2) dy + \lambda \left( \int u(w(y)) f(y|a_1, a_2) dy - v(a_1) \right) \quad \text{s.t.} \\ &(i)\ \ \int u(w(y)) f_1(y|a_1, a_2) dy - v'(a_1) = 0, \end{aligned} \tag{28}$$

where $f_1$ denotes the first derivative of $f$ taken with respect to $a_1$.

To find the solution $(a_1^P, a_2^P, w^P(y|a_1^P, a_2^P))$ for the above program, we first derive an optimal contract for an arbitrarily given $(a_1, a_2)$. Let $w^P(y|a_1, a_2)$ be a contract which optimally motivates the agent to take a particular level of $a_1$ when an arbitrary level of $a_2$ is chosen by the principal. By solving the Euler equation of the above program after fixing $(a_1, a_2)$, we derive that $w^P(y|a_1, a_2)$ must satisfy

$$\frac{1}{u'(w^P(y|a_1, a_2))} = \lambda + \mu_1(a_1, a_2)\frac{f_1}{f}(y|a_1, a_2), \tag{29}$$

for almost every $y$ for which (29) has a solution $w^P(y|a_1, a_2) \geq k$, and otherwise $w^P(y|a_1, a_2) = k$. In (29), $\mu_1(a_1, a_2)$ denotes the optimized Lagrange multiplier for the agent's incentive constraint associated with effort $a_1$ when the project choice is pinned down at $a_2$. Since $f(y|a_1, a_2)$ is a normal density function with mean $\phi(a_1, a_2)$ and variance $a_2^2$, (29) is reduced to:

$$\frac{1}{u'(w^P(y|a_1, a_2))} = \lambda + \mu_1(a_1, a_2)\frac{y - \phi(a_1, a_2)}{a_2^2}\phi_1(a_1, a_2). \tag{30}$$

Before analyzing the optimal contract, we first define given $(a_1, a_2)$:

$$SW^P(a_1, a_2) \equiv \phi(a_1, a_2) - C^P(a_1, a_2) - \lambda v(a_1), \tag{31}$$

which denotes the joint benefits when $w^P(y|a_1, a_2)$ is designed and $a_2$ is instructed by the principal where

$$C^P(a_1, a_2) \equiv \int \left(w^P(y|a_1, a_2) - \lambda u(w^P(y|a_1, a_2))\right) f(y|a_1, a_2)dy \tag{32}$$

represents the efficiency loss of this case compared with the full information case. In other words, $C(a_1, a_2)$ measures the agency cost arising from inducing the agent to take that particular $a_1$ when $a_2$ is chosen by the principal.

We start our analysis with the following Lemma 4, which is analogous to Kim (1995).

**Lemma 4** *$C^P(a_1, a_2^0) < C^P(a_1, a_2^1)$ for any given $a_1$ if $a_2^0 < a_2^1$.*

Since the principal dictates the agent's project choice $a_2$ here, an agency problem arises only in inducing $a_1$. Lemma 4 implies that under Assumption 6, when the project choice $a_2$ is selected by the principal, the agency cost associated with motivating the agent to take any given action $a_1$, i.e., $C^P(a_1, a_2)$, is reduced if the principal chooses a less risky project. A lowered risk $a_2$ improves the efficiency of the agency relationship by providing a more precise signal $y$ about the agent's effort, $a_1$, which enables the principal to design a contract inducing a particular $a_1$ in a less costly way. If $\phi_{12}(a_1, a_2)$ is large enough to break Assumption 6, then lower $a_2$ might lower $\phi_1(a_1, a_2)$ a lot, which in turn makes harder for the principal to give the proper incentive for the action $a_1$ and raise the incentive cost $C^P(a_1, a_2)$. Assumption 6 guarantees that this incentive drawback is lower than the informational rent from lower $a_2$, so that a lower level of $a_2$ reduces the agency cost $C^P(a_1, a_2)$.

**Value of hedging** Lemma 4 indicates that firms should take all zero net present value projects that reduce output risk, if possible. For example, when the agent can be induced to hedge in the derivative market, the principal can generically induce the agent to choose higher efforts and investments with higher expected returns, given any initial risk exposure level $R$.

**Risk-return trade-off in project choice** However, given the trade-off between return and risk, i.e., $\phi_2 > 0$, the exact level of $a_2$ that the principal prefers will be determined by the loss in expected return as well as the benefit from achieving a more precise signal of effort. Let $a_2^P$ be the project that is most preferred by the principal, and $a_1^P$ the agent's optimal effort choice for the above program when $a_2^P$ is chosen by the principal. Then, as we prove in Appendix A, from the above optimization we obtain that $(a_1^P, a_2^P, w^P(\cdot))$ should satisfy

$$\int \left(y - w^P(y) + \lambda u(w^P(y))\right) f_2(y|a_1^P, a_2^P)dy + \mu_1(a_1^P, a_2^P) \int u(w^P(y)) f_{12}(y|a_1^P, a_2^P)dy = 0, \tag{33}$$

where $w^P(\cdot) = w^P(\cdot|a_1^P, a_2^P)$, $f_2$ denotes the first derivative of $f$ with respect to $a_2$ and $f_{12}$ is the second derivative with respect to $a_1$ and $a_2$. The optimal contract $w^P(y|a_1^P, a_2^P)$ satisfies,

$$\frac{1}{u'(w^P(y|a_1^P, a_2^P))} = \lambda + \mu_1(a_1^P, a_2^P)\frac{y - \phi(a_1^P, a_2^P)}{(a_2^P)^2}\phi_1(a_1^P, a_2^P), \tag{34}$$

for $y$ satisfying $w^P(y|a_1^P, a_2^P) \geq k$ and $w^P(y|a_1^P, a_2^P) = k$ otherwise.

**The manager's incentive to select $a_2$ under contract $w^P(\cdot)$** The above analysis assumes that shareholders essentially select the projects. In this subsection we ask whether the manager will voluntarily choose the project that would be chosen by informed shareholders, i.e., $a_2^P$. If the answer to this question is no, then the moral-hazard problem arises not only in motivating $a_1$ but also in incentivizing $a_2$, which implies that the optimal wage contract must be modified from the contract, $w^P(y|a_1^P, a_2^P)$, in (34).

To formally analyze this issue, we denote $a_2^A(a_2^P)$ as a solution to

$$a_2^A(a_2^P) \in \arg\max_{a_2} \int u(w^P(y|a_1^P, a_2^P))f(y|a_1^P, a_2)dy. \tag{35}$$

Thus, $a_2^A(a_2^P)$ represents the project choice that the agent would take under $w^P(y|a_1^P, a_2^P)$ described in (34) when $a_2$ is not enforceable. Thus, our previous question, "Will the agent voluntarily choose $a_2^P$ when $w^P(y|a_1^P, a_2^P)$ is designed?", is equivalent to the question, "Will $a_2^A(a_2^P)$ be equal to $a_2^P$?"

As previously shown, the principal balances two considerations when he directs the agent to take a certain project: the informational benefits from risk reduction and the lower mean return associated with lower risk. However, the risk level chosen by the agent depends on his indirect risk preferences induced by contract $w^P(y|a_1^P, a_2^P)$, i.e., the curvature of $u(w^P(y|a_1^P, a_2^P))$ with respect to $y$, and the effect that a trade-off between return and risk would have on his utility *via* $w^P(y|a_1^P, a_2^P)$.

In general, the curvature of the agent's indirect utility function depends on the distribution of the random state variable and his utility function. To see how different utility functions affect this curvature differently, we again consider the case where the agent has constant relative risk aversion with degree $1-t$ as we did in Section 2.1, where $t < 1$ (i.e., $u(w) = \frac{1}{t}w^t, t < 1$). We obtain from equation (34) that

$$w^P(y|a_1^P, a_2^P) = \left(\lambda + \mu_1(a_1^P, a_2^P)\left(\frac{y - \phi(a_1^P, a_2^P)}{(a_2^P)^2}\right)\phi_1(a_1^P, a_2^P)\right)^{\frac{1}{1-t}}, \tag{36}$$

and the agent's indirect utility under this wage contract is

$$u(w^P(y|a_1^P, a_2^P)) = \frac{1}{t}\left(\lambda + \mu_1(a_1^P, a_2^P)\left(\frac{y - \phi(a_1^P, a_2^P)}{(a_2^P)^2}\right)\phi_1(a_1^P, a_2^P)\right)^{\frac{t}{1-t}}. \tag{37}$$

The above equation shows that the agent's indirect utility becomes strictly concave in $y$ if $t < \frac{1}{2}$, linear if $t = \frac{1}{2}$, and convex if $t > \frac{1}{2}$ for $y$ satisfying $w^P(y|a_1^P, a_2^P) \geq k$. If we assume $w^P(y|a_1^P, a_2^P) = k$ for sufficiently low $y$, as far as the agent's induced risk preferences are concerned, $u(w^P(y|a_1^P, a_2^P))$ makes the agent risk-loving if $t \geq \frac{1}{2}$. Furthermore, since the compensation contract $w^P(y|a_1^P, a_2^P)$ is positively related to the absolute output level (i.e., $\mu_1(a_1^P, a_2^P) > 0$),[30] if $t \geq \frac{1}{2}$, the agent is induced to take the most risky project, i.e., $a_2^A(a_2^P) = \overline{a}_2$ when $w^P(y|a_1^P, a_2^P)$ is designed even if $\phi_2(a_1, \overline{a}_2) = 0$ by Assumption 4. However, in this case, principal prefers to have a firm's risk level $a_2$ lower than $\overline{a}_2$. This is because, from his standpoint, the informational benefits from risk reduction are still substantial, while the costs of risk reduction are zero at $\overline{a}_2$ (i.e., $\phi_2(\overline{a}_2) = 0$). Thus, $a_2^P < a_2^A(a_2^P)$ in this case. In other words, the principal prefers less risk than the agent under $w^P(y|a_1^P, a_2^P)$.

On the other hand, if $t$ is close to $-\infty$ (i.e., the agent is extremely risk-averse), the agent's indirect utility function induces him to choose a lower level of risk than what the principal prefers (i.e., $a_2^A(a_2^P) < a_2^P$) even if a lower $a_2$ yields on average lower output.

Incentive problems associated with project choice $a_2$, in general, exist in all cases except for those where both of the following conditions are satisfied: (i) the agent's indirect utility is sufficiently concave and (ii) there is no trade-off between return and risk, i.e., $\phi_2 = 0,\ \forall a_2$. Under these conditions, both the principal and the agent agree that the firm should choose the least risky project, i.e., $a_2 = \underline{a}_2$, and there is no efficiency loss due to the existence of the manager's unobservable project choice. However, when either the agent's induced risk preferences are convex, or the trade-off between return and risk exists as assumed in Assumption 4, the principal and the agent will not generally agree on the firm's optimal project choice, and the compensation contract, $w^P(y|a_1^P, a_2^P)$, described in equation (34) will no longer be optimal.

**Optimal contracts with moral hazard in $a_2$** In this situation, the principal must determine the optimal compensation contract by solving the following optimization problem:

$$\max_{\substack{a_1,a_2 \\ w(\cdot)\geq k}} \phi(a_1,a_2) - \int w(y) f(y|a_1,a_2)dy + \lambda\left(\int u(w(y)) f(y|a_1,a_2)dy - v(a_1)\right) \quad \text{s.t.}$$
$$(i)\ \ (a_1,a_2) \in \arg\max_{a_1',a_2'} \int u(w(y)) f(y|a_1',a_2')dy - v(a_1'),\ \ \forall a_1', a_2'. \tag{38}$$

The optimization problem (38) accounts for the fact that the agent selects $a_2$ to maximize his own expected utility. If an interior solution for $(a_1, a_2)$ exists and the first-order ap-

[30] For the proof of $\mu_1(a_1^P, a_2^P) > 0$, see e.g., Holmström (1979), Jewitt (1988), Jung and Kim (2015).

proach is valid, the above maximization problem can be expressed as:

$$\max_{\substack{a_1,a_2\\ w(\cdot)\geq k}} \phi(a_1,a_2) - \int w(y) f(y|a_1,a_2)dy + \lambda\left(\int u(w(y)) f(y|a_1,a_2)dy - v(a_1)\right) \quad \text{s.t.}$$
$$(i) \quad \int u(w(y)) f_1(y|a_1,a_2)dy - v'(a_1) = 0,$$
$$(ii) \quad \int u(w(y)) f_2(y|a_1,a_2)dy = 0, \tag{39}$$

Let $(a_1^*, a_2^*)$ be the optimal action combination for the above program. Then, by solving the Euler equation, we obtain that the optimal wage contract, $w^*(y)$, which satisfies,

$$\frac{1}{u'(w^*(y))} = \lambda + \mu_1^* \frac{f_1}{f}(y|a_1^*, a_2^*) + \mu_2^* \frac{f_2}{f}(y|a_1^*, a_2^*), \tag{40}$$

for almost every $y$ for which equation (40) has a solution $w^*(y) \geq k$, and otherwise $w^*(y) = k$. $\mu_1^*$ and $\mu_2^*$ are the optimized Lagrange multipliers for both incentive constraints, respectively.

Since $f(y|a_1^*, a_2^*)$ is a normal distribution with mean $\phi(a_1^*, a_2^*)$ and variance $(a_2^*)^2$, from (40),

$$\frac{1}{u'(w^*(y))} = \lambda + \mu_1^* \underbrace{\frac{y - \phi(a_1^*, a_2^*)}{(a_2^*)^2}\phi_1^*}_{\equiv SS_1} + \mu_2^* \left[ \underbrace{\frac{y - \phi(a_1^*, a_2^*)}{(a_2^*)^2}\phi_2^*}_{\equiv SS_2^1} + \underbrace{\frac{1}{a_2^*}\left(\frac{(y - \phi(a_1^*, a_2^*))^2}{(a_2^*)^2} - 1\right)}_{\equiv SS_2^2} \right], \tag{41}$$

where we define $SS_1$, $SS_2 \equiv SS_2^1 + SS_2^2$ as sufficient statistics for unobservbable action $a_1$ and project choice $a_2$, respectively. Compared with (34), (41) shows that when both $a_1$ and $a_2$ are not observable, the optimal wage contract is based not only on the absolute output $y$, but also on its (standardized) deviation from the expected level, $\frac{\left(y - \phi(a_1^*, a_2^*)\right)^2}{(a_2^*)^2}$. Since $\left(y - \phi(a_1^*, a_2^*)\right)^2$ is a sample (i.e., realized) variance of a single observation with mean zero and variance $(a_2^*)^2$, the term $\frac{\left(y - \phi(a_1^*, a_2^*)\right)^2}{(a_2^*)^2}$ in (41) can be regarded a standardized output deviation. Note that $SS_2$, the sufficient statistic for the project choice $a_2$, can be now decomposed into two parts: $SS_2^1$ and $SS_2^2$. $SS_2^1$ takes account of the effects that an increase in $a_2$ has on the mean cash flow $\phi(a_1, a_2)$,[31] while $SS_2^2$ is about how an increase

[31] This term is present since we assume the risk-return trade-off in $a_2$, i.e., $\phi(a_1, a_2)$ is increasing in $a_2$.

in $a_2$ affects the signal $y$'s volatility. By including the sample variance as a contractual parameter, the principal effectively motivates the agent to take the appropriate level of $a_2$, i.e., $a_2^*$. (41) can be written in a simpler way as

$$\frac{1}{u'(w^*(y))} = \lambda + (\mu_1^*\phi_1^* + \mu_2^*\phi_2^*)\,\frac{y-\phi(a_1^*, a_2^*)}{(a_2^*)^2} + \mu_2^*\frac{1}{a_2^*}\left(\frac{(y-\phi(a_1^*, a_2^*))^2}{(a_2^*)^2} - 1\right), \quad (42)$$

for $y$ satisfying $w^*(y) \geq k$ and $w^*(y) = k$ otherwise. Here, $\phi_i^* \equiv \phi_i(a_1^*, a_2^*), i = 1, 2$. We call $w^*(y)$ as an *optimal dual-agency contract* à la Hirshleifer and Suh (1992).

The optimal dual agency contract is characterized in the following propositions.

**Proposition 6** $\mu_1^*\phi_1^* + \mu_2^*\phi_2^* > 0.$

Proposition 6 implies that holding the cash flow variance constant, the manager's payout increases when the firm's output increases, which implies that the manager is rewarded for a higher effort. However, this does not necessarily mean that the contracted payout is monotonically increasing in output. For example, if $\mu_2^* < 0$ in equation (42), the agent can be paid less when the output is very high.

Thus, a more interesting question has to do with the relation between the agent's rewards and the output deviation, i.e., the sign of $\mu_2^*$.

**Proposition 7** *If the principal prefers a less risky project than the agent under $w^P(y|a_1^P, a_2^P)$ in equation (34), i.e., $a_2^P < a_2^A(a_2^P)$, then the optimal dual agency contract will penalize the agent if output differs substantially from the expected level, i.e., $\mu_2^* < 0$ for $w^*(y)$ in equation (42). If the principal prefers a more risky project than the agent under $w^P(y|a_1^P, a_2^P)$, i.e., $a_2^P > a_2^A(a_2^P)$, then the optimal dual agency contract will reward the agent for having unusual output deviation, i.e., $\mu_2^* > 0$ for $w^*(y)$ in equation (42).*

If the principal prefers a lower level of project risk than the agent under the contract $w^P(y|a_1^P, a_2^P)$, the contract will be revised in a way that motivates the agent to reduce risk. This can be done by setting $\underline{\mu_2^* < 0}$ in equation (42) which penalizes the agent for the unusual output deviation and makes the agent act as if he is more risk-averse. On the other hand, if the principal prefers a higher risk than the agent when $w^P(y|a_1^P, a_2^P)$ is designed, the contract is revised to motivate the agent to increase risk. This can be done by setting $\underline{\mu_2^* > 0}$ in equation (42) which rewards the agent for unusual output deviation and makes the agent act as though he is less risk-averse. As discussed earlier, the later case is more likely to occur when the manager is more risk averse and when the firm's investment opportunities offer a non-trivial trade-off between return and risk.[32]

We denote the optimized joint benefits in this case as

$$SW^*(a_1^*, a_2^*) \equiv \phi(a_1^*, a_2^*) - C^*(a_1^*, a_2^*) - \lambda v(a_1^*), \tag{43}$$

where

$$C^*(a_1^*, a_2^*) \equiv \int (w^*(y) - \lambda u(w^*(y)))\, f(y|a_1^*, a_2^*)dy \tag{44}$$

denotes the agency cost arising from inducing $(a_1^*, a_2^*)$ when $a_3$ is fixed at 0 and $R$ is observable.

### 3.1.2 The Principal Does Not Know the Firm's Risk Exposure

We now consider the case of asymmetric information, where the firm's innate exposure to hedgeable risks, $R$, is observed only by the agent. In this case, the wage contract cannot explicitly include $y \equiv x - R\eta$ as a contractual variable. Furthermore, we rule out the possibility of any communication between principal and the agent that allows the agent to reveal $R$.[33]

If principal does not observe $R$, the compensation contract must be based on $(x, \eta)$, i.e., $w = w(x, \eta)$. The principal's maximization program in this case is thus:[34]

$$\begin{aligned}
&\max_{\substack{a_1(\cdot),a_2(\cdot)\\ w(\cdot)\geq k}} \int_R \int_{x,\eta} (x - w(x,\eta))\, g(x,\eta|a_1(R), a_2(R), R)h(R)dxd\eta dR \\
&\qquad + \lambda \int_R \left( \int_{x,\eta} u(w(x,\eta))g(x,\eta|a_1(R), a_2(R), R)dxd\eta - v(a_1(R)) \right) h(R)dR \quad \text{s.t.} \\
&\qquad (i)\ \ (a_1(R), a_2(R)) \in \arg\max_{a_1,a_2} \int_{x,\eta} u(w(x,\eta))g(x,\eta|a_1, a_2, R)dxd\eta - v(a_1), \forall R,
\end{aligned} \tag{45}$$

[32] For example, in cases of constant relative risk aversion with degree $1 - t$, it is more likely that $\mu_2^* > 0$ when $1 - t$ is higher (i.e., $t$ is lower).

[33] In general, communication between principals and agents are likely to be very costly, especially when actually the principal stands for multiple shareholders. For a more detailed discussion of communication costs, see Laffont and Martimort (1997). We study issues of potential communication in Appendix B.

[34] In this case, since the agent is the only one that observes $R$, his actions $a_1, a_2$ both depend on $R$, given the contract $w(x, \eta)$.

where

$$g(x,\eta|a_1,a_2,R) = \frac{1}{2\pi a_2}\exp\left(-\frac{1}{2}\left(\frac{(x-\phi(a_1,a_2)-R\eta)^2}{a_2^2}+\eta^2\right)\right) \tag{46}$$

denotes a joint probability density function of $(x,\eta)$ given $(a_1,a_2,R)$ and $h(R)$ denotes the probability density function of $R$.

For each $R$, let $(a_1^N(R), a_2^N(R), w^N(x,\eta))$ be the solution for the above optimization program in (45). If we let $\mu_1(R)$, $\mu_2(R)$ be Lagrange multipliers attached to incentive constraints in $a_1(R)$ and $a_2(R)$, respectively, the optimal contract $w^N(x,\eta)$ can be written as[35]

$$\begin{aligned}\frac{1}{u'(w^N(x,\eta))} =&\lambda + \int_R \mu_1(R)\left[\frac{g_1(x,\eta|a_1^N(R),a_2^N(R),R)}{\int_{R'} g(x,\eta|a_1^N(R'),a_2^N(R'),R')h(R')dR'}\right]h(R)dR \\ &+ \int_R \mu_2(R)\left[\frac{g_2(x,\eta|a_1^N(R),a_2^N(R),R)}{\int_{R'} g(x,\eta|a_1^N(R'),a_2^N(R'),R')h(R')dR'}\right]h(R)dR,\end{aligned} \tag{47}$$

when $w(x,\eta)\geq k$ and otherwise $w(x,\eta)=k$. The optimized joint benefit in this case is denoted as:

$$SW^N \equiv \int_R \left(\phi(a_1^N(R),a_2^N(R)) - C^N(a_1^N(R),a_2^N(R)) - \lambda v(a_1^N(R))\right)h(R)dR, \tag{48}$$

where

$$C^N(a_1^N(R),a_2^N(R)) \equiv \int_{x,\eta}\left(w^N(x,\eta)-\lambda u(w^N(x,\eta))\right)g(x,\eta|a_1^N(R),a_2^N(R),R)dxd\eta \tag{49}$$

denotes the agency cost arising from inducing $(a_1^N(R),a_2^N(R))$ given a realized value of $R$. In this case, we obtain the following comparison between two welfare measures: $SW^N$ and $SW^*(a_1^*,a_2^*)$ as in Proposition 2.

**Proposition 8** *When there is no derivative market and no communication between the principal and the agent, the principal's inability to observe the firm's risk exposure reduces*

[35] We provide the derivation for equation (47) in Appendix A.

*welfare, i.e.,*

$$SW^N < SW^*(a_1^*, a_2^*).$$

Intuitively, when the principal observes the firm's risk exposure, $R$, this information can be used to design a wage contract that eliminates the influence of hedgeable risk, i.e., $w = w^*(y \equiv x - R\eta)$. However, if $R$ is not observable and cannot be communicated, this is impossible.

## 3.2 When Managers Can Trade Derivatives

In this subsection we consider how the introduction of an opportunity to trade derivatives (i.e., when $a_3$ is not fixed at $0$) affects the optimal contract and the firm's efficiency. Continuing from Section 3.1.2, we assume that a manager's project choice, $a_2$, is not observable, and in addition, we assume that the derivatives choice, $a_3$ and the firm's risk exposure, $R$, cannot be observed by or communicated to the principal.

Following the logic of Section 2.2.2: since the firm's exposure to hedgeable risks, $R$, is observed by the agent before he takes actions $(a_1, a_2, a_3)$, the agent's choice of $a_3$ can be characterized as his choice of $b \equiv R - a_3$. Then given a compensation contract, the principal can rationally anticipate the agent's choice of $b = R - a_3$. We denote the principal's anticipation of the agent's choice of $R - a_3$ by $\hat{b}$, and define $z(\hat{b}) \equiv x - \hat{b}\eta$ as a variable that can potentially be in the wage contract, i.e., $w(z(\hat{b}))$ is a potential contract. If the principal's beliefs are consistent,[36] it must be the case that the agent chooses $a_3$ satisfying $b \equiv R - a_3 = \hat{b}$ given this contract.

Thus, since

$$z(\hat{b}) \equiv x - \hat{b}\eta = \phi(a_1, a_2) + (b - \hat{b})\eta + a_2\theta, \tag{50}$$

if the principal offers the contract $w(z(\hat{b}))$ and the agent chooses $a_3$ satisfying $b = R - a_3 = \hat{b}$, then

$$z(\hat{b}) = \phi(a_1, a_2) + a_2\theta = y. \tag{51}$$

Note that the maximum level of joint benefits that can be obtained in this case is $SW(a_1^*, a_2^*, a_3 = 0)$ in equation (43).[37] Therefore, we first consider the case in which the principal designs

[36] As the principal predicts the agent with risk-exposure $R$ to choose $\hat{b} = R - a_3$, a contract that relies on $\hat{b}$ induces the agent to take $b = \hat{b}$.

[37] Given the contract $w(z(\hat{b}))$, if there is no incentive problem associated with $b = R - a_3$, i.e., the agent voluntarily chooses $a_3$ such that $R - a_3 = \hat{b}$, then we obtain the maximal joint benefit $SW(a_1^*, a_2^*, a_3 = 0)$. The issue is whether the agent would voluntarily choose $a_3$ such that $R - a_3 = \hat{b}$ given $w(z(\hat{b}))$.

the contract the same as $w^*(y)$ in the benchmark case (i.e., equation (42)) but based on $z(\hat{b})$ instead of $y \equiv x - R\eta$, and examine whether the agent chooses $b \equiv R - a_3 = \hat{b}$ under $w^*(z(\hat{b}))$. If this is indeed the case, there is no welfare loss associated with $R$ (and $a_3$) being unobservable when the agent is able to transact in the derivatives market.

**The optimal contract in the benchmark case (i.e., (42)) as a potential contract** Suppose that the principal designs a contract $w^*(z(\hat{b}) \equiv x - \hat{b}\eta)$ satisfying

$$\frac{1}{u'(w^*(z(\hat{b})))} = \lambda + (\mu_1^*\phi_1^* + \mu_2^*\phi_2^*)\frac{z(\hat{b}) - \phi(a_1^*, a_2^*)}{(a_2^*)^2} + \mu_2^*\frac{1}{a_2^*}\left(\frac{\left(z(\hat{b}) - \phi(a_1^*, a_2^*)\right)^2}{(a_2^*)^2} - 1\right), \tag{52}$$

for $z(\hat{b})$ satisfying $w^*(z(\hat{b})) \geq k$ and $w^*(z(\hat{b})) = k$ otherwise. Because $w^*(z(\hat{b}))$ in (52) is of the same functional form as $w^*(y)$ in (42),[38], we easily see the agent will take $(a_1^*, a_2^*)$ under $w^*(z(\hat{b}))$ *if* he chooses $a_3$ satisfying $b \equiv R - a_3 = \hat{b}$. But, the real question is: "Will the agent always choose $a_3$ satisfying $b = \hat{b}$ when $w^*(z(\hat{b}))$ is designed and offered?".

The following Lemma 5 provides an answer to the above question.

**Lemma 5** [Speculation and Hedging with $w^*(z(\hat{b}))$]
(1) *If $\mu_2^* < 0$ for the contract, $w^*(z(\hat{b}))$, described in equation (52) for any given $\hat{b}$,[39] then the manager will choose $a_3$ such that $b = \hat{b}$ when the contract $w^*(z(\hat{b}))$ is offered.*
(2) *If $\mu_2^* > 0$ for $w^*(z(\hat{b}))$ in equation (52) for any given $\hat{b}$, then the manager will take $a_3$ such that $|R - a_3| = \infty$ when $w^*(z(\hat{b}))$ is offered.*

From Lemma 5, we directly obtain the following proposition:

**Proposition 9** *If $\mu_2^* < 0$ for $w^*(z(\hat{b}))$ described in (52) for any given $\hat{b}$, then the level of $b \equiv R - a_3$ that is induced is a matter of indifference as long as it is bounded, i.e., $|b| < \infty$. For example, If $\mu_2^* < 0$ for $w^*(z(0))$ in (52), then the manager will choose $a_1^*, a_2^*, a_3 = R$ (i.e., $b = 0$) when $w^*(z(0))$ is offered. Therefore, the optimized joint benefits in this case are the same as $SW^*(a_1^*, a_2^*)$ in (43), implying that the firm's welfare with a derivative market will be the same as it is in the case where the risk exposure is observed by the principal.*[40]

[38]Note that $\mu_1^*, \mu_1^*, a_1^*, a_2^*$ in (42) and (52) are endogenous variables characterized by solving the optimization in (38).

[39]One can easily see that if $\mu_2^* < 0$ in $w^*(z(\hat{b}))$ for any given $\hat{b}$, then $\mu_2^* < 0$ in $w^*(z(\hat{b}))$ for all $\hat{b}$. This is because the principal's anticipating different $\hat{b}$ does not change the functional form of $w^*(\cdot)$.

[40]Therefore, the introduction of derivative markets in this case improves the welfare compared with the

Proposition 9 is quite intuitive. If $\underline{\mu_2^* < 0}$ for $w^*(z(\hat{b}))$ in (52), the agent is induced to engage in perfect hedging to minimize the variance of $z(\hat{b})$. Intuitively, the contract $w^*(z(\hat{b}))$ with $\mu_2^* < 0$ induces the agent to sacrifice expected payoffs to lower risk.[41] If the risk can be reduced through a channel that does not decrease the expected payoff (e.g., here $a_3$ does not have risk-return trade-off.), then agent will clearly do so. In addition, $\mu_1^*\phi_1^* + \mu_2^*\phi_2^* > 0$ means a higher $z(\hat{b})$ yields the higher compensation $w^*(z(\hat{b}))$ given its squared deviation from the average of $z(\hat{b})$.

In this case, the optimal contract can be designed *as if* $R - a_3$ is observable to the principal, and it allows the principal and the agent to achieve the welfare $SW^*(a_1^*, a_2^*)$ that can be achieved when the risk-exposure $R$ is observable. We will discuss more thoroughly about this *informational gain* from the manager's derivative transaction later.[42]

However, this is not possible if $\underline{\mu_2^* > 0}$ for $w^*(z(\hat{b}))$ in (52), since the agent speculates infinitely, i.e., choose $a_3$ such that $|R - a_3| = \infty$. This is because, as shown from equation (52), the manager will be paid an infinite amount when $z(\hat{b}) = x - \hat{b}\eta$ is either positive or negative infinity if $\mu_2^* > 0$ for $w^*(z(\hat{b}))$. Given that it is impossible to design a wage contract $w^*(z(\hat{b}))$ based on the belief $\hat{b} = \infty$, the principal has to either alter the wage contract to ensure $|R - a_3| < \infty$ or retain the optimal contract without a derivative market, $w^N(x, \eta)$ and prohibit the manager from engaging in derivative transactions.

**Comparison with Section 2: different implications?** It is useful to compare the results in this section to the Section 2 analysis that takes the real investment choice as given (i.e., we only consider action $a_1$ and derivative transactions $a_d$). Recall, we start from the benchmark case without hedgeable risk $\eta$, which reduces the problem to the canonical principal-agent model (e.g., Holmström (1979)). The optimal contract $w^*(x)$ in this benchmark scenario generates the agent's indirect utility function $V(x)$. As we show, (i) if $V(x)$ is concave (convex) in $x$, then the manager will choose to perfectly hedge (infinitely specu-

case where the principal does not observe the firm's risk-exposure $R$ and the communication between the principal and the agent is prohibitively costly (i.e., $SW^*(a_1^*, a_2^*) > SW^N$ in Proposition 8). In practice, benefits derivative markets provide to firms are multi-dimensional (e.g., firms can prevent themselves from going bankrupt through proper hedging. In this paper, we focus on the new channel, in which derivative markets can eliminate the informational asymmetry between shareholders and the manager about firms' innate risk-exposure, only if the manager properly hedges in the derivative market.

[41] Since the optimal contract $w^*(z(\hat{b}))$ features $\mu_2^* < 0$ when $a_2^P < a_2^A(a_2^P)$ holds.

[42] Derivative markets exists for many reasons (e.g., allowing market participants to hedge against various risks, thereby preventing bankruptcies). We focus on the new channel through which the derivative market affects the efficiency: it eliminates informational asymmetry about the firm's risk-exposure, thereby making shareholders less vulnerable to adverse selection issues around the risk exposure.

late) when there is a derivatives market and (ii) $V(x)$ is more likely to be concave (convex) when the agent's utility function features higher (lower) risk-aversion. Therefore, a less risk-averse manager is more likely to speculate in derivative markets given the contract $w^*(\cdot)$.

In cases with a flexible project choice $a_2$, we obtain the opposite result: (i) the agent with $\mu_2^* > 0$ speculates infinitely when derivative markets open; (ii) under the benchmark case, i.e., neither asymmetric information nor derivative market, the principal initially offers a contract with $\mu_2^* > 0$ since she prefers a higher level of project risk $a_2$ than the agent, implying generically that the agent's risk aversion is very high. To be specific, when the manager's risk aversion is sufficiently high, shareholders will design a contract to induce the manager to choose a higher project risk level $a_2$, to benefit from the positive risk-return tradeoff. Such a contract will reward a higher level of output variance (i.e., $\mu_2^* > 0$), which can in turn induce the manager to speculate infinitely, choosing $a_3 = \pm\infty$ due to the additional incentive effect from $\mu_2^* > 0$.

It can be understood as a side effect of inducing the project risk-taking which is productive (i.e., $\phi_2(a_1, a_2) > 0$) through incentive contracts. A contract that induces risk-taking in the real investment choice makes the manager speculate infinitely when derivative transaction is possible, as he acts as effectively risk-loving under the contract (52) with $\mu_2^* > 0$.

**Optimal contracts when $\mu_2^* > 0$** When the agent takes infinite speculation in derivative markets under the contract $w^*(z(\hat{b}))$ in (52) with $\mu_2^* > 0$, our analysis becomes close to Section 2.2.2. First, the principal might design a new optimal contract, $w^o(x, \eta)$ to induce the agent's perfect hedging. This new optimal contract satisfies conditions in Proposition 3, and thus penalizes the agent for having both positive and negative realization of $(x - \phi(a_1^o, a_2^o))\eta$, which we regard as sample covariance between output and hedgeable risks. As the new optimal contract $w^o(x, \eta)$ imposes additional risks on the agent, it incurs efficiency costs, thereby lowering the social welfare from $SW^*$ to $SW^o$, i.e., $SW^o < SW^*$.

Instead, the principal might just ban the derivative trading, in which case we go back to Section 3.1.2 and achieve $SW^N$ as welfare. When the degree of asymmetric information is small enough, i.e., the principal's prior distribution $h(R)$ on risk exposure $R$ is tight with $\sigma_R \to 0$, then hedging benefits shrink, and therefore, the principal is better off banning the derivative trading, as summarized in Proposition 5.

We provide a detailed analysis of this case in Appendix C.

# 4 Conclusion

As we stated in the introduction, large public firms take the management of risk quite seriously. However, to a large extent, the academic literature on this topic ignores the issues that are most relevant to large public firms. In particular, a large part of the literature focuses on financially constrained firms, and ignore the fact that risk management choices are made by self-interested managers rather than by value-maximizing equity holders.

An important result in the paper is that in some situations asymmetric information about the firm's risk exposure does not result in a loss in welfare. In these situations, the manager chooses the same hedging choice as would be chosen if the derivative choice was directed by shareholders that are fully informed about the firm's risk exposure. If that is the case, derivatives markets contribute to welfare because they effectively allow the manager to communicate the firm's risk exposure to the shareholders at no cost.

However, this is not always the case. In some situations, the manager's compensation contract must be altered to motivate him to hedge appropriately. In these situations, derivatives markets can still contribute to welfare, but if the required alteration in the compensation contract is too costly, the firm is better off banning the use of derivatives.

While our model most closely resembles the relationship between the top executives of a corporation and its shareholders, one can also apply the model to describe the relationship between the top executives of a firm and the individuals managing the firm's divisions. In such a setting, the division heads can be interpreted as agents, who report to the firm's CEO, who may not observe the risk exposure of the individual divisions. The CEO thus has an incentive to design a contract with its division heads that elicits information about the divisions' risk exposure and simultaneously induces effort.

There are three reasons why information about risk exposure can be useful for the CEO. The first reason, which we emphasize in our model, is that by taking out the effect of hedgeable risk exposure, the contract can be designed to more efficiently induce effort. The second reason, which is similar to the motivation in DeMarzo and Duffie (1991, 1995), is that a better-informed CEO may be able to better allocate resources to the different divisions. The third, is that by aggregating information from the divisions, the CEO can reveal a more accurate estimate of the firm's total risk exposure to the firm's board of directors, who can use this information to better evaluate and compensate the CEO.

This description of incentives and risk choices of multi-divisional firms can potentially be applied to the financial sector, and can, perhaps, provide insights about what may have gone wrong in the financial crisis. As our model illustrates, designing optimal compensation contracts that optimally elicit both effort and risk choices can potentially be quite complicated, and in some situations, the firm is better off banning the use of derivatives. Perhaps, it is not surprising that in the 1990s and 2000s, when the use of derivatives was somewhat novel, that individuals in this industry were inadvertently compensated in ways that induced speculation rather than hedging. If such speculation generates spillover costs, as observed in the 2008/2009 global financial crisis, then there are likely policy implications associated with these incentive contracts.[43]

Although the model is already quite complex, there are a number of possible extensions that may be considered in future work. The first is to consider this problem in a dynamic setting. We have shown that the optimal compensation contract sometimes penalizes the agent for realizing unusually high or low output when the payoff from the derivative contract is either unusually high or low, respectively. We interpret this as penalizing covariance between hedgeable risk and output. Since our model is static, this interpretation is a bit loose. In a dynamic model, we can consider explicitly penalizing estimates of the covariance, which can be more or less precise depending on the number of observations and the other sources of noise effecting the firm's output. We conjecture that as the estimate of the covariance becomes more precise, contracting becomes more efficient and the gains from hedging increases.

A second potential extension has to do with uncertainty about the risk aversion of the agent. In our model, the agent's risk aversion plays an important role, because it affects the convexity of the indirect utility function. When the agent's risk aversion is unknown to the principal, it might be difficult to induce all the agents with different levels of risk aversion to take the appropriate project and hedge, and therefore optimal for the principal to restrict the use of derivatives. This extension will be relevant to the financial industry as it attracts individuals who may not be risk averse. Given this, we expect the optimal contracts will include limits on the use of derivatives.

[43] https://www.nytimes.com/2009/10/23/business/23pay.html says that "The Federal Reserve is working to ensure that compensation packages appropriately tie rewards to longer-term performance and do not create undue risk for the firm or the financial system."

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# Appendix A. Derivations and Proofs

**Derivation of equation (14)**: With Lagrange multiplier $\mu_1(R)$ attached to the incentive constraint for action $a_1(R)$ for any given $R$, we set up the Lagrangian for the optimization in (12) as follows:

$$\mathcal{L} = \int_R \left[ \int_{x,\eta} (x - w(x,\eta) + \lambda u(w(x,\eta)))\, g(x,\eta|a_1(R),R)dxd\eta - v(a_1(R)) \right] h(R)dR \\ + \int_R \mu_1(R) \left[ \int_{x,\eta} u(w(x,\eta)) g_1(x,\eta|a_1(R),R)dxd\eta - v'(a_1(R)) \right] h(R)dR \tag{A1}$$

from which we obtain the following first-order condition about $w(\cdot,\cdot)$:

$$(-1 + \lambda u'(w(x,\eta))) \int_R g(x,\eta|a_1(R),R)h(R)dR + u'(w(x,\eta)) \left[ \int_R \mu_1(R) g_1(x,\eta|a_1(R),R)h(R)dR \right] = 0,$$

which derives the optimal contract $w^N(x,\eta)$ in (14). ■

**Proof of Proposition 2**: Consider the principal's following *alternative* maximization program:

$$\max_{a_1(\cdot),w(\cdot)\geq k} \int_R \int_{x,\eta} (x - w(x,R,\eta))\, g(x,\eta|a_1(R),R)h(R)dxd\eta dR \\ + \lambda \int_R \left( \int_{x,\eta} u(w(x,R,\eta)) g(x,\eta|a_1(R),R)dxd\eta - v(a_1(R)) \right) h(R)dR \quad \text{s.t.} \\ (i) \quad \int_{x,\eta} u(w(x,R,\eta)) g_1(x,\eta|a_1(R),R)dxd\eta - v'(a_1(R)) = 0, \forall R. \tag{A2}$$

Note that the program in (A2) is different from the original program (12) in that here contract can be written on the realized value of $R$. If we let the Lagrange multipliers to the constraint be $\mu_1(R)h(R)$, we get the following optimal contractual form:

$$\frac{1}{u'(w(x,R,\eta))} = \lambda + \mu_1(R) \frac{\overbrace{x - R\eta}^{\equiv y} - \phi(a_1(R))}{\sigma^2} \phi_1(a_1(R)), \tag{A3}$$

when $w(x,R,\eta) \geq k$. The above (A3) implies that the optimal contract only depends on $y \equiv x - R\eta$ and the solution $\{a_1(R), w(x,R,\eta)\}$ becomes $\{a_1^*, w^*(y) \equiv w^*(x - R\eta)\}$ in equation (5) in Section 2.1. By comparing (A3) with the program in (12) where the principal does not observe $R$, one can easily see that the set of wage contracts, $\{w(x, R, \eta)\}$, satisfying the incentive constraints for a given action $a_1(R)$ in the above program (A2) always contains the set of wage contracts available when the principal does not know $R$, $\{w(x, \eta)\}$, satisfying the incentive constraints for the same action. Therefore, we have

$$SW^N \leq SW^*. \tag{A4}$$

However, we easily observe that $w^*(y) = w^*(x - R\eta)$, which is a unique solution for the above program (A2), is not in the set of $\{w(x, \eta)\}$. As a result, we finally derive

$$SW^N < SW^*. \tag{A5}$$

**<u>Proof of Lemma 3</u>**: Given $w^*(x)$ described in equation (5) is designed,[1] if the agent takes $(a_1^o, b)$ under $w^*(x)$, then his expected utility becomes:

$$\int u(w^*(x))g(x, \eta|a_1^o, b)dzd\eta - v(a_1^o) = \int u(w^*(x))q(x|a_1^o, b, \eta)l(\eta)dzd\eta - v(a_1^o), \tag{A6}$$

where $q(\cdot)$ denotes the conditional density of $x$ given $(a_1^o, b, \eta)$ and $l(\cdot)$ denotes the density function of $\eta \sim N(0, 1)$. Now, suppose the agent takes $(a_1^o, -b)$ under $w^*(x)$. Then,

$$\int u(w^*(x))g(x, \eta|a_1^o, -b)dzd\eta - v(a_1^o) = \int u(w^*(x))q(x|a_1^o, -b, \eta)l(\eta)dzd\eta - v(a_1^o). \tag{A7}$$

Since

$$q(x|a_1^o, b, \eta) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x - \phi(a_1^o) - b\eta)^2}{2\sigma^2}\right), \tag{A8}$$

we have

$$q(x|a_1^o, b, \eta) = q(x|a_1, -b, -\eta). \tag{A9}$$

Since $\eta \sim N(0, 1)$ is symmetrically distributed around 0 and $l(\eta) = l(-\eta)$, $\forall \eta$, we finally have

$$\int u(w^*(x))g(x, \eta|a_1^o, b)dzd\eta - v(a_1^o) = \int u(w^*(x))g(x, \eta|a_1^o, -b)dzd\eta - v(a_1^o). \tag{A10}$$

[1] The output $x$ is given by $x = \phi(a_1^o) + \sigma\theta + b\eta$ given $a_1^o$ and $b = R - a_d$.

■

<u>**Proof of Proposition 3**</u>: To prove this proposition, we start with the following Lemma 6.

**Lemma 6** *When the agent's indirect utility $V(x)$ in (9) in the absence of the hedgeable risk $\eta$ is convex in output $x$, then the optimal contract $w^o(x,\eta)$ guaranteeing that the agent takes $a_1^o, a_d^o = R$ (i.e., $b = 0$), i.e., $w^o(x,\eta)$ in equation (23), must satisfy*

*(1) $\mu_b^o(b) \neq 0$ $(> 0)$ for a positive Borel-measure of $b$.*[2]

*(2) $w^o(x,\eta) = w^o(x,-\eta)$ for all $x, \eta$ and $\mu_b^o(b) = \mu_b^o(-b)$ for all $b$.*

**Proof.** (1) <u>$\mu_b^o(b) \neq 0$ for a positive Borel-measure of $b$</u>: Assume $\mu_b^o(b) = 0$, $a.s.$ Then the optimal contract $w^o(x,\eta)$ in (23) can be written as

$$\frac{1}{u'(w^o(x,\eta))} = \lambda + \mu_1^o \frac{x - \phi(a_1^o)}{\sigma^2}\phi_1(a_1^o), \tag{A11}$$

for $(x,\eta)$ satisfying $w^o(x,\eta) \geq k$ and $w^o(x,\eta) = k$.

Because we already know $(w^o(x,\eta), \mu_1^o, a_1^o)$ becomes $(w^*(x), \mu_1^*, a_1^*)$ in this case and $V(x)$ (i.e., the agent's indirect utility given $w^*(x)$) is convex in $x$ by assumption, $(w^o(x,\eta), \mu_1^o)$ will induce $b = \pm\infty$ instead of $b = 0$ from the agent, a contradiction to the constraint (ii) in the optimization (17).

(3) <u>$w^o(x,\eta) = w^o(x,-\eta)$ for all $x, \eta$ and $\mu_4^o(b) = \mu_4^o(-b)$ for all $b$</u>: We first see:[3]

$$g(x,\eta|b) = \frac{1}{2\pi\sigma}\exp\left(-\frac{1}{2}\frac{(x - \phi(a_1^o) - b\eta)^2}{\sigma^2} - \frac{1}{2}\eta^2\right), \tag{A13}$$

where

$$\frac{g(x,\eta|b)}{g(x,\eta|b=0)} = \exp\left(\frac{b\eta(x - \phi(a_1^o))}{\sigma^2}\right)\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right). \tag{A14}$$

[2]We already know $\mu_4^o(b) \geq 0$ for every $b$ (almost surely), since it is derived from the inequality constraint at each $b$.

[3]We suppress $a_1^o$ in $g(x,\eta|a_1^o,b)$ in (17). Note that $g(x,\eta|a_1,b)$ yields the following likelihood ratios:

$$\frac{g_1}{g}(x,\eta|a_1,b) = \frac{x - b\eta - \phi(a_1)}{\sigma^2}\phi_1(a_1), \quad \frac{g_b}{g}(x,\eta|a_1,b) = \frac{(x - b\eta - \phi(a_1))\eta}{\sigma^2}. \tag{A12}$$

From (A12), (A13), and (A14), we observe that $g(x, \eta|b = 0)$ and $g_1(x, \eta|b = 0)$ are both even with $\eta$ where $g_1$ is a partial derivative of $g$ with respect to $a_1$: i.e., (i) $g(x, -\eta|b = 0) = g(x, \eta|b = 0)$; (ii) $g_1(x, -\eta|b = 0) = g_1(x, \eta|b = 0)$. Also from (A13), we acknowledge:

$$g(x, -\eta|b) = g(x, \eta| - b), \;\; \forall (x, \eta, b). \tag{A15}$$

Our strategy is to prove that: (i) if $w^o(x, \eta)$ is an optimal contract, then $w^o(x, -\eta)$ satisfies all the constraints in (17); (ii) Related to (i), if $w^o(x, \eta)$ is an optimal contract, then $w^o(x, -\eta)$ also becomes an optimal contract; and (iii) $\mu_b^o(-b) = \mu_b^o(b)$ for $\forall b$ at the optimum.

**Step 1. If $w^o(x, \eta)$ is an optimal contract, then $w^o(x, -\eta)$ satisfies all the constraints in (17).**

(i) As $w^o(x, \eta)$ is optimal, it satisfies constraints in (17). We start from the incentive compatibility in action $a_1$: based on that $g_1(x, \eta|b = 0)$ is even in $\eta$,

$$\begin{aligned}\int u(w^o(x, -\eta))g_1(x, \eta|b = 0)dxd\eta - v'(a_1^o) &= \int u(w^o(x, -\eta))g_1(x, -\eta|b = 0)dxd\eta - v'(a_1^o) \\ &= \int u(w^o(x, \eta))g_1(x, \eta|b = 0)dxd\eta - v'(a_1^o) = 0,\end{aligned}$$

where we use the change of variable (i.e., $-\eta$ to $\eta$) in the second equality.

(ii) Incentive compatibility in *after-hedging* risk exposure $b$: as $w^o(x, \eta)$ is optimal,

$$\int u(w^o(x, \eta)) \left(g(x, \eta|b = 0) - g(x, \eta|b)\right) dxd\eta \geq 0, \; \underline{\forall b}. \tag{A16}$$

From (A15) and that $g(x, \eta|b = 0)$ is even in $\eta$, we obtain for $\forall b$,

$$\begin{aligned}\int u(w^o(x, -\eta)) \left(g(x, \eta|b = 0) - g(x, \eta|b)\right) dxd\eta &= \int u(w^o(x, -\eta)) \left(g(x, -\eta|b = 0) - g(x, -\eta| - b)\right) dxd\eta \\ &= \int u(w^o(x, \eta)) \left(g(x, \eta|b = 0) - g(x, \eta| - b)\right) dxd\eta \geq 0,\end{aligned}$$

where the first equality is from (A15) and the second equality is from the change of variable (i.e., $-\eta$ to $\eta$). Thus, we proved that if $w^o(x, \eta)$ is an optimal contract, then $w^o(x, -\eta)$ satisfies all the constraints in (17).

**Step 2. If $w^o(x, \eta)$ is an optimal contract, then $w^o(x, -\eta)$ also becomes an optimal contract.**

From the above Step 1, $w^o(x, -\eta)$ satisfies all the constraints in (17). It is sufficient to show that $w^o(x, -\eta)$ achieves the same efficiency as $w^o(x, \eta)$. It follows from:

$$\begin{aligned}
&\int (x - w^o(x, -\eta))g(x, \eta|b = 0)dxd\eta + \lambda \left( \int u(w^o(x, -\eta))g(x, \eta|b = 0)dxd\eta - v(a_1^o) \right) \\
&\quad = \int (x - w^o(x, -\eta))g(x, -\eta|b = 0)dxd\eta + \lambda \left( \int u(w^o(x, -\eta))g(x, -\eta|b = 0)dxd\eta - v(a_1^o) \right) \\
&\quad = \int (x - w^o(x, \eta))g(x, \eta|b = 0)dxd\eta + \lambda \left( \int u(w^o(x, \eta))g(x, \eta|b = 0)dxd\eta - v(a_1^o) \right), \qquad \text{(A17)}
\end{aligned}$$

where the first equality is from that $g(x, \eta|b = 0)$ is symmetric in $\eta$, and the second equality is from the change of variable (i.e., $-\eta$ to $\eta$). Therefore, if $w^o(x, \eta)$ is an optimal contract, then $w^o(x, -\eta)$ becomes an optimal contract and we obtain $w^o(x, -\eta) = w^o(x, \eta)$.[4]

**Step 3. $\mu_b^o(-b) = \mu_b^o(b)$ for $\forall b$.**

Note from the Lagrange duality theorem (see e.g., Luenberger (1969)) that the optimal solution $(\mu_1^o, \{\mu_b^o(b)\}, w^o(\cdot))$ is the one that solves $\min_{\mu_1, \{\mu_b(\cdot)\}} \max_{w(\cdot)} \mathcal{L}$ where $\mathcal{L}$ is given by

$$\begin{aligned}
\mathcal{L} \equiv &\int (x - w(x, \eta))g(x, \eta|b = 0)dxd\eta + \lambda \left( \int u(w(x, \eta))g(x, \eta|b = 0)dxd\eta - v(a_1^o) \right) \\
&+ \mu_1 \left( \int u(w(x, \eta))g_1(x, \eta|b = 0)dxd\eta - v'(a_1^o) \right) \\
&+ \int_b \mu_b(b) \left( \int u(w(x, \eta)) \left( g(x, \eta|b = 0) - g(x, \eta|b) \right) dxd\eta \right) db,
\end{aligned}$$

while satisfying $\mu_b^o(b) \geq 0$ for $\forall b$ and the following complementary slackness at the optimum:

$$\mu_b^o(b) \left( \int u(w^o(x, \eta)) \left( g(x, \eta|b = 0) - g(x, \eta|b) \right) dxd\eta \right) = 0, \ \ \forall b. \qquad \text{(A18)}$$

The last term in the above Lagrangian $\mathcal{L}$ given the optimal contract $w^o(x, \eta)$ can be written

[4] We implicitly assume that the optimal contract is unique in this environment, following the literature (e.g., Jewitt et al. (2008)).

as

$$\int_b \mu_4(b)\left(\int u(w^o(x,\eta))\left(g(x,\eta|b=0)-g(x,\eta|b)\right)dxd\eta\right)db$$
$$=\int_b \mu_4(-b)\left(\int u(w^o(x,-\eta))\left(g(x,\eta|b=0)-g(x,\eta|-b)\right)dxd\eta\right)db, \tag{A19}$$

where we use a change of variable (i.e., $b$ to $-b$) and $w^o(x,-\eta)=w^o(x,\eta)$. Now with (A15) and that $g(x,\eta|b=0)$ is even in $\eta$, we know:

$$\int u(w^o(x,-\eta))\left(g(x,\eta|b=0)-g(x,\eta|-b)\right)dxd\eta=\int u(w^o(x,-\eta))\left(g(x,-\eta|b=0)-g(x,-\eta|b)\right)dxd\eta$$
$$=\int u(w^o(x,\eta))\left(g(x,\eta|b=0)-g(x,\eta|b)\right)dxd\eta, \tag{A20}$$

where we use the change of variable (i.e., $-\eta$ to $\eta$) for the second equality. With (A19) and (A20), the last term in Lagrangian $\mathcal{L}$ can be therefore written as

$$\int_b \mu_4(b)\left(\int u(w^o(x,\eta))\left(g(x,\eta|b=0)-g(x,\eta|b)\right)dxd\eta\right)db$$
$$=\int_b \mu_4(-b)\left(\int u(w^o(x,\eta))\left(g(x,\eta|b=0)-g(x,\eta|b)\right)dxd\eta\right)db. \tag{A21}$$

Plugging in (A21) to the original Lagrangian $\mathcal{L}$ yields $\mu_4^o(-b)=\mu_4^o(b)$.

**Step 4**. We have:

$$\int u(w^o(x,\eta))g(x,\eta|b)dxd\eta=\int u(w^o(x,\eta))g(x,\eta|-b)dxd\eta, \tag{A22}$$

which implies that the agent's indirect utility is symmetric in $b$ around $b=0$.

It follows from:

$$\int u(w^o(x,\eta))g(x,\eta|-b)dxd\eta=\int u(w^o(x,\eta))g(x,-\eta|b)dxd\eta=\int u(w^o(x,-\eta))g(x,-\eta|b)dxd\eta$$
$$=\int u(w^o(x,\eta))g(x,\eta|b)dxd\eta,$$

where we use (A15) in the first equality, $w^o(x,-\eta)=w^o(x,\eta)$ in the second, and and the change of variable (i.e., $-\eta$ to $\eta$) in the third equality.

■

**Proof of Proposition 3**: Given the optimal action $a_1^o$, we define $\widehat{Cov} \equiv (x - \phi(a_1^o))\eta$.[5] Since

$$\exp\left(\frac{b\eta(x-\phi(a_1^o))}{\sigma^2}\right) = \exp\left(\frac{b}{\sigma^2}\widehat{Cov}\right) = \sum_{k=0}^{\infty}\frac{1}{k!}\frac{b^k}{\sigma^{2k}}\widehat{Cov}^k, \tag{A23}$$

From equation (A14), we obtain

$$\frac{g(x,\eta|b)}{g(x,\eta|b=0)} = \left(\sum_{k=0}^{\infty}\frac{1}{k!}\frac{b^k}{\sigma^{2k}}\widehat{Cov}^k\right)\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right), \tag{A24}$$

and therefore, we attain

$$\begin{aligned}\int \mu_4^o(b)\left(1-\frac{g(x,\eta|b)}{g(x,\eta|b=0)}\right)db &= \int \mu_4^o(b)db - \int \mu_4^o(b)\left(\sum_{k=0}^{\infty}\frac{1}{k!}\frac{b^k}{\sigma^{2k}}\widehat{Cov}^k\right)\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right)db \\ &= \int \mu_4^o(b)db - \sum_{k=0}^{\infty}\left(\frac{1}{k!}\frac{1}{\sigma^{2k}}\underbrace{\left(\int \mu_4^o(b)b^k\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right)db\right)}_{\equiv C_k(\eta)}\right)\widehat{Cov}^k.\end{aligned} \tag{A25}$$

When $k$ is odd, the coefficient $C_k(\eta)$ becomes 0 for $\forall\eta$, since $\mu_4^o(b) = \mu_4^o(-b)$ for all $b$ from Lemma 6 implies

$$C_{k:odd}(\eta) = \int \mu_4^o(b)b^k\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right)db = \int_{b\geq 0}\left(\underbrace{\mu_4^o(b)-\mu_4^o(-b)}_{=0}\right)b^k\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right)db = 0. \tag{A26}$$

When $k$ is even, the coefficient $C_k(\eta)$ becomes strictly positive for $\forall\eta$, since $\mu_4^o(b) \neq 0$ for the non-zero measure of $b$ from Lemma 6 implies

$$\begin{aligned}C_{k:even}(\eta) = \int \mu_4^o(b)b^k\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right)db &= \int_{b\geq 0}(\mu_4^o(b)+\mu_4^o(-b))b^k\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right)db \\ &= 2\int_{b\geq 0}\mu_4^o(b)b^k\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right)db > 0.\end{aligned} \tag{A27}$$

[5] This is a realized value of sample covariance between $x$ and $\eta$, as our framework is in single-period setting.

Therefore, (A25) can be written as

$$\int \mu_4^o(b)\left(1-\frac{g(x,\eta|b)}{g(x,\eta|b=0)}\right)db = \int \mu_4^o(b)db - 2\sum_{k:even}^{\infty}\left(\frac{1}{k!}\frac{1}{\sigma^{2k}}\left(\int_{b\geq 0}\mu_4^o(b)b^k \exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right)db\right)\right)\widehat{Cov}^k. \tag{A28}$$

Finally, we can plug the expression (A28) into our optimal contact $w^o(x,\eta)$ in (23) when $w^o(x,\eta) \geq k$ and obtain

$$\begin{aligned}\frac{1}{u'(w^o(x,\eta))} =& \lambda + \mu_1^o\frac{x-\phi(a_1^o)}{\sigma^2}\phi_1(a_1^o) + \underbrace{\int \mu_4^o(b)db}_{>0} \\ & - 2\sum_{k:\text{even}}^{\infty}\underbrace{\frac{1}{k!}\frac{1}{\sigma^{2k}}\underbrace{\left(\int_{b\geq 0}\mu_4^o(b)b^k\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right)db\right)}_{\equiv C_{k:\text{even}}(\eta)>0}}_{\equiv D_{k:\text{even}}(\eta)>0}\widehat{Cov}^k.\end{aligned} \tag{A29}$$

Since $D_{k:\text{even}}(\eta) > 0$ for all even numbers $k$, given $(x,\eta)$ a higher $\widehat{Cov}$ results in a lower compensation $w^o(x,\eta)$. Also as $D_{k:\text{even}}(\eta) > 0$ decreases in $\eta^2$, given $(x,\widehat{Cov})$, a higher $\eta^2$ results in a higher $w^o(x,\eta)$. In sum the principal punishes a sample covariance $|\widehat{Cov}|$ but becomes lenient when a high $|\widehat{Cov}|$ comes from the high $\eta$ realization, not from the agent's speculation activity ($b \neq 0$).

■

**Proof of Lemma 4**: We know from $y \sim N(\phi(a_1,a_2),a_2^2)$ that

$$\frac{y-\phi(a_1,a_2)}{a_2} \sim N(0,1),\ \ \frac{f_1}{f}(y|a_1,a_2) = \frac{y-\phi(a_1,a_2)}{a_2^2}\phi_1(a_1,a_2) \sim N\left(0,\frac{\phi_1(a_1,a_2)^2}{a_2^2}\right). \tag{A30}$$

Therefore, we observe that if $\frac{\phi_1(a_1,a_2)}{a_2}$ is decreasing in $a_2$, for any pair $a_2^0 < a_2^1$, $\frac{f_1}{f}(y|a_1,a_2^0)$'s distribution is mean-preserving spread (MPS) of that of $\frac{f_1}{f}(y|a_1,a_2^1)$. Assumption 6 guarantees that this condition holds, and the following Lemma 7, a slightly changed form of Kim (1995), proves $C(a_1,a_2^0) < C(a_1,a_2^1)$ for $\forall a_1$.

**Lemma 7** *For given action $a_1$ and technology $h(\cdot|a_1)$, let the solution of the following* *optimization problem be $w_h(\cdot)$:*

$$\max_{w(\cdot)} \int (y - w(y)) h(y|a_1) dy + \lambda \left( \int u(w(y)) h(y|a_1) dy - v(a_1) \right) \quad \textit{s.t.}$$
$$(i) \int u(w(y)) h_1(y|a_1) dy - v'(a_1) = 0, \tag{A31}$$
$$(ii)\ w(y) \geq k, \forall y.$$

For two different technologies $h = f,\ g$ such that $\frac{f_1}{f}(y|a_1)$ is a mean-preserving spread of $\frac{g_1}{g}(y|a_1)$, we have:

$$C_f(a_1) \equiv \int \left(w_f(y) - \lambda u(w_f(y))\right) f(y|a_1) dy < \int \left(w_g(y) - \lambda u(w_g(y))\right) g(y|a_1) dy \equiv C_g(a_1). \tag{A32}$$

**Proof.** We know that the solution of (A31) would be given as

$$\frac{1}{u'(w_h(y))} = \max\left\{\lambda + \mu_h \frac{h_1}{h}(y|a_1), \frac{1}{u'(k)}\right\}, \tag{A33}$$

where $\mu_h$ is the Lagrange multiplier attached to the incentive constraint for the given $a_1$. If we define $q_h \equiv \lambda + \mu_h \frac{h_1}{h}(y|a_1)$, we can rewrite the optimal contract $w_h(\cdot)$ as a function of $q_h$ so that $w_h(y) = r(q_h)$ where $r(\cdot) = (\frac{1}{u'})^{-1}(\cdot)$ is increasing and does not rely on the technology $h$. Therefore, (A33) can be written as

$$u'(r(q_h)) q_h = 1, \tag{A34}$$

if $q_h \geq u(k)^{-1}$ and otherwise $r(q_h) = k$. Now, we obtain

$$\begin{aligned}\mathbb{E}_h\left(u(r(q_h))q_h\right) &= \int u(r(q_h)) \cdot q_h \cdot h(y|a_1) dy = \int u(r(q_h)) \left[\lambda + \mu_h \frac{h_1}{h}(y|a_1)\right] h(y|a_1) dy \\ &= \lambda \underbrace{\int u(r(q_h)) h(y|a_1) dy}_{\equiv B_h} + \mu_h \underbrace{\int u(r(q_h)) h_1(y|a_1) dy}_{= v'(a_1)} = \lambda B_h + \mu_h v'(a_1),\end{aligned} \tag{A35}$$

where we used the fact that $r(q_h)$ satisfies the agent's incentive constraint in $a_1$. Following Kim (1995), we define

$$\psi(q) \equiv r(q) - u(r(q))q, \tag{A36}$$

which is <u>concave</u> in $\forall q$, since: (i) with $q \geq u(k)^{-1}$, we obtain $\psi'(q) = \cancel{r'(q)} - \cancel{u'(r(q))r'(q)q} -$

$u(r(q)) = -u(r(q))$ as $u'(r(q))q = 1$ and $\psi''(q) = -u'(r(q))r'(q) < 0$; (ii) with $q < u(k)^{-1}$, we have $r(q) = k$ so $\psi(q)$ becomes linear.[6] Now we can introduce two different technologies $f(\cdot|a_1)$ and $g(\cdot|a_1)$ such that $\frac{f_1}{f}(y|a_1)$ is a mean-preserving spread of $\frac{g_1}{g}(y|a_1)$, and define

$$\bar{q} \equiv \lambda + \mu_f \frac{g_1}{g}(y|a_1), \tag{A37}$$

which is possibly different from $q_g$ as $\mu_f$ is possibly different from $\mu_g$. As $\psi(q)$ is globally concave, we obtain

$$\begin{aligned}\mathbb{E}_g\left(\psi(\bar{q})\right) - \mathbb{E}_g\left(\psi(q_g)\right) \le \mathbb{E}_g\left(\psi'(q_g)(\bar{q} - q_g)\right) &= \mathbb{E}_g\left(-u(r(q_g))(\mu_f - \mu_g)\frac{g_1}{g}\right) \\ &= (\mu_g - \mu_f)\int u(r(q_g))g_1(y|a_1)dy = (\mu_g - \mu_f)v'(a_1) \\ &= \left(\mathbb{E}_g\left(u(r(q_g))q_g\right) - \lambda B_g\right) - \left(\mathbb{E}_f\left(u(r(q_f))q_f\right) - \lambda B_f\right),\end{aligned} \tag{A38}$$

where we used (A35). Finally, it leads to the following agency cost comparison:

$$\begin{aligned}C_g(a_1) - C_f(a_1) &= \mathbb{E}_g\left(r(q_g) - \lambda B_g\right) - \mathbb{E}_f\left(r(q_f) - \lambda B_f\right) = \mathbb{E}_g(r(q_g)) - \mathbb{E}_f(r(q_f)) - (\lambda B_g - \lambda B_f) \\ &= \mathbb{E}_g(\psi(q_g)) + \mathbb{E}_g(u(r(q_g))q_g) - \mathbb{E}_f(\psi(q_f)) - \mathbb{E}_f(u(r(q_f))q_f) - (\lambda B_g - \lambda B_f) \\ &\ge \cancel{\mathbb{E}_g(\psi(q_g))} - \mathbb{E}_f(\psi(q_f)) + \mathbb{E}_g(\psi(\bar{q})) - \cancel{\mathbb{E}_g(\psi(q_g))} = \mathbb{E}_g(\psi(\bar{q})) - \mathbb{E}_f(\psi(q_f)) \\ &= \int \psi\left(\lambda + \mu_f \frac{g_1}{g}(y|a_1)\right) g(y|a_1)dy - \int \psi\left(\lambda + \mu_f \frac{f_1}{f}(y|a_1)\right) f(y|a_1)dy\end{aligned} \tag{A39}$$

where we used (A38) in the above (A39)'s inequality part. Finally, if $\frac{f_1}{f}(y|a_1)$ is a mean-preserving spread of $\frac{g_1}{g}(y|a_1)$, then (A39) with Rothschild and Stiglitz (1970) implies $C_g(a_1) \ge C_f(a_1)$, as $\mu_f > 0$ and $\psi(\cdot)$ is globally concave.

■

Finally, with $f(y|a_1) \equiv f(y|a_1, a_2^0)$ and $g(y|a_1) \equiv f(y|a_1, a_2^1)$ in our specification, Lemma 7 proves Lemma 4.

■

**Derivation of equation (33):**

Given the fixed $a_1 = a_1^P$, $\phi_2(a_1^P, a_2^P) = C_2(a_1^P, a_2^P)$ holds at optimum. We write $C_2(a_1^P, a_2^P)$

[6] We see that $\psi(q)$ is continuously differentiable at all points including $q = u(k)^{-1}$.

as follows:

$$\phi_2(a_1^P, a_2^P) = C_2(a_1^P, a_2^P) = \int \left(w^P(y|a_1^P, a_2^P) - \lambda u(w^P(y|a_1^P, a_2^P))\right) f_2(y|a_1^P, a_2^P)dy$$
$$+ \int \left(\frac{\partial w^P}{\partial a_2}(y|a_1^P, a_2^P) - \lambda u'(w^P(y|a_1^P, a_2^P))\frac{\partial w^P}{\partial a_2}(y|a_1^P, a_2^P)\right) f(y|a_1^P, a_2^P)dy, \tag{A40}$$

where we know the following equation is satisfied:[7]

$$\frac{\partial w^P}{\partial a_2}(y|a_1^P, a_2^P) - \lambda u'(w^P(y|a_1^P, a_2^P))\frac{\partial w^P}{\partial a_2}(y|a_1^P, a_2^P) = \frac{\partial w^P}{\partial a_2}(y|a_1^P, a_2^P)\left(1 - \lambda u'(w^P(y|a_1^P, a_2^P))\right), \tag{A41}$$

which leads to

$$\begin{aligned} &\frac{\partial w^P}{\partial a_2}(y|a_1^P, a_2^P) - \lambda u'(w^P(y|a_1^P, a_2^P))\frac{\partial w^P}{\partial a_2}(y|a_1^P, a_2^P) \\ &\quad = \frac{\partial w^P}{\partial a_2}(y|a_1^P, a_2^P) \cdot \mu_1(a_1^P, a_2^P)\frac{f_1}{f}(y|a_1^P, a_2^P)u'(w^P(y|a_1^P, a_2^P)). \end{aligned} \tag{A42}$$

Thus by plugging equation (A41) into equation (A40), we obtain

$$\begin{aligned} &\int \left(\frac{\partial w^P}{\partial a_2}(y|a_1^P, a_2^P) - \lambda u'(w^P(y|a_1^P, a_2^P))\frac{\partial w^P}{\partial a_2}(y|a_1^P, a_2^P)\right) f(y|a_1^P, a_2^P)dy \\ &\quad = \mu_1(a_1^P, a_2^P)\int \frac{\partial w^P}{\partial a_2}(y|a_1^P, a_2^P)f_1(y|a_1^P, a_2^P)u'(w^P(y|a_1^P, a_2^P))dy. \end{aligned} \tag{A43}$$

When $w^P(y|a_1^P, a_2)$ is designed for $\forall a_2$, it should satisfy the such incentive constraint (where $w^P(y|a_1^P, a_2 = a_2^P) \equiv w^P(y|a_1^P, a_2^P)$) as

$$\int u(w^P(y|a_1^P, a_2))f_1(y|a_1^P, a_2)dy = v'(a_1^P). \tag{A44}$$

We get the following by differentiating both side of equation (A44) by $a_2$ at $a_2 = a_2^P$:

$$\int u'(w^P(y|a_1^P, a_2^P))\frac{\partial w^P}{\partial a_2}(y|a_1^P, a_2^P)f_1(y|a_1^P, a_2^P)dy = -\int u(w^P(y|a_1^P, a_2^P)f_{12}(y|a_1^P, a_2^P)dy. \tag{A45}$$

[7]The second equality below holds even in the region where the limited liability constraint binds and $w^P(y|a_1^P, a_2^P) = k$ as its derivative with respect to $a_2$ is 0, except on measure 0. A small change in $a_2$ leads to only a small change in the region of a binding limited liability.

Plugging equation (A45) into equation (A40), we get the following equation (33).[8]

$$\phi_2(a_1^P, a_2^P) = \int y f_2(y|a_1^P, a_2^P)dy = \int \left(w^P(y|a_1^P, a_2^P) - \lambda u(w^P(y|a_1^P, a_2^P))\right) f_2(y|a_1^P, a_2^P)dy$$
$$- \mu_1(a_1^P, a_2^P) \int u(w^P(y|a_1^P, a_2^P) f_{12}(y|a_1^P, a_2^P)dy. \tag{A48}$$

■

**Proof of Proposition 6**: Assume to the contrary that $\mu_1^* \phi_1^* + \mu_2^* \phi_2^* \leq 0$. Then, pick up any two levels of $y$: $y_1$ and $y_2$, such that

$$y_1 < y_2, \quad \text{and} \quad \frac{y_1 + y_2}{2} = \phi(a_1^*, a_2^*). \tag{A49}$$

That is, $y_1$ and $y_2$ are located at the same distance from the mean value $\phi(a_1^*, a_2^*)$. If $\mu_1^* \phi_1^* + \mu_2^* \phi_2^* \leq 0$, we have from equation (42) that

$$w^*(y_1) \geq w^*(y_2), \quad \text{and} \quad u(w^*(y_1)) \geq u(w^*(y_2)). \tag{A50}$$

Since $f_1(y_1|a_1^*, a_2^*) = -f_1(y_2|a_1^*, a_2^*) < 0$ for any $y_1$ and $y_2$ satisfying equation (A49), we have:

$$\int u(w^*(y)) f_1(y|a_1^*, a_2^*)dy \leq 0, \quad \text{and} \quad \int u(w^*(y)) f_1(y|a_1^*, a_2^*)dy - v'(a_1^*) < 0. \tag{A51}$$

Therefore, there is a contradiction.

[8] We can derive (33) based on the envelope theorem. If we regard the principal's optimization as the one in which, given a fixed $a_2$, we find optimal $\{a_1, w(\cdot)\}$ that maximizes joint utility of both principal and agent under the incentive constraint for $a_1$ and the limited liability constraint, the principal solves:

$$SW(a_2) = \min_{\mu_1} \max_{w(\cdot), a_1} L(a_2) \equiv \phi(a_1, a_2) - \int w(y) f(y|a_1, a_2)dy + \lambda \left( \int u(w(y)) f(y|a_1, a_2)dy - v(a_1) \right)$$
$$+ \mu_1 \left( \int u(w(y)) f_1(y|a_1, a_2)dy - v'(a_1) \right) \tag{A46}$$

As $(a_1^P, w^P(\cdot|a_1^P, a_2^P))$ are the solution of (A46) given $a_2^P$, $SW'(a_2^P) = 0$ must be satisfied at the optimum. Therefore, the envelope theorem applied to (A46) yields

$$SW'(a_2^P) = \int \left(y - w^P(y) + \lambda u(w^P(y)\right) f_2(y|a_1^P, a_2^P)dy + \mu_1(a_1^P, a_2^P) \int u(w^P(y)) f_{12}(y|a_1^P, a_2^P)dy = 0, \tag{A47}$$

which is (33), where $\mu_1(a_1^P, a_2^P)$ are the endogenous Lagrange multiplier for incentive constraint for $a_1$ at $a_1^P$ given $a_2^P$.

■

**Proof of Proposition 7**:

**Case 1: $\mu_2^* > 0$ if $a_2^A(a_2^P) < a_2^P$**. Let us compare the following two optimizations:[9]

$$\max_{a_1,a_2,w(\cdot)} \int (y - w(y))f(y|a_1,a_2)dy + \lambda\left(\int u(w(y))f(y|a_1,a_2)dy - v(a_1)\right) \quad \text{s.t.}$$
$$(i) \quad \int u(w(y))f_1(y|a_1,a_2)dy - v'(a_1) = 0,$$
$$(ii) \quad \int u(w(y))f_2(y|a_1,a_2)dy = 0,$$
$$(iii) \;\; w(y) \geq k, \;\; \forall y, \tag{A52}$$

and

$$\max_{a_1,a_2,w(\cdot)} \int (y - w(y))f(y|a_1,a_2)dy + \lambda\left(\int u(w(y))f(y|a_1,a_2)dy - v(a_1)\right) \quad \text{s.t.}$$
$$(i) \quad \int u(w(y))f_1(y|a_1,a_2)dy - v'(a_1) = 0,$$
$$(ii) \quad \int u(w(y))f_2(y|a_1,a_2)dy \geq 0,$$
$$(iii) \;\; w(y) \geq k, \;\; \forall y, \tag{A53}$$

where the incentive constraint associated with the non-hedgeable risk choice $a_2$ takes the form of inequality in the latter program, instead of equality in the original optimization program.

We know that $(w^*(y), a_1^*, a_2^*, \mu_1^*, \mu_2^*)$ are the optimal solution for the first program. Let $(\hat{w}(y), \hat{a}_1, \hat{a}_2, \hat{\mu}_1, \hat{\mu}_2)$ be the optimal solution for the second program. We will show that the above two programs are equivalent in that two solutions align perfectly with each other when $a_2^A(a_2^P) < a_2^P$. Then, we can directly derive $\mu_2^* \geq 0$ when $a_2^A(a_2^P) < a_2^P$, since $\hat{\mu}_2 \geq 0$ by Kuhn-Tucker theorem.

Assume that the second constraint in the second program is not binding. Then, $\hat{\mu}_2 = 0$, and $\hat{w}(y)$ should satisfy:

$$\frac{1}{u'(\hat{w}(y))} = \lambda + \hat{\mu}_1 \frac{y - \phi(\hat{a}_1, \hat{a}_2)}{(\hat{a}_2)^2}\phi_1(\hat{a}_1, \hat{a}_2), \tag{A54}$$

[9]Following Rogerson (1985), we replace the incentive constraint with the corresponding inequality constraint, and exploit the fact that a multiplier to the inequality constraint must be non-negative.

for $y$ satisfying $\hat{w}(y) \geq k$ and $\hat{w}(y) = k$ otherwise. As the second constraint is not binding, $\hat{a}_2$ becomes the best (from the principal's perspective) $a_2$, i.e., $\hat{a}_2 = a_2^P$. Then we must have $\hat{a}_1 = a_1^P$ and $\hat{w}(y) = w^P(y|a_1^P, a_2^P)$. Therefore, the fact that the second constraint in the second program is not binding implies

$$\int u(w^P(y|a_1^P, a_2^P)) f_2(y|a_1^P, a_2^P, a_3 = 0) dy > 0. \tag{A55}$$

However, equation (A55) implies $a_2^A(a_2^P) > a_2^P$, a contradiction.[10] Thus, the second constraint in the second program must be binding, and the above two programs are equivalent so $\mu_2^* = \hat{\mu}_2 \geq 0$. And also, $\mu_2^* \neq 0$, because $\mu_2^* = 0$ implies $a_2^A(a_2^P) = a_2^P$.

**<u>Case 2</u>: $\mu_2^* < 0$ if $a_2^A(a_2^P) > a_2^P$.** By applying the same method as in Case 1, we can easily prove it. We compare following two optimizations similar to (A52) and (A53) in Case 1.

$$\begin{aligned} \max_{a_1, a_2, w(\cdot)} & \int (y - w(y)) f(y|a_1, a_2) dy + \lambda \left( \int u(w(y)) f(y|a_1, a_2) dy - v(a_1) \right) \quad \text{s.t.} \\ & (i) \quad \int u(w(y)) f_1(y|a_1, a_2) dy - v'(a_1) = 0, \\ & (ii) \quad \int u(w(y)) f_2(y|a_1, a_2) dy = 0, \\ & (iii) \quad w(y) \geq k, \quad \forall y, \end{aligned} \tag{A56}$$

and

$$\begin{aligned} \max_{a_1, a_2, w(\cdot)} & \int (y - w(y)) f(y|a_1, a_2) dy + \lambda \left( \int u(w(y)) f(y|a_1, a_2) dy - v(a_1) \right) \quad \text{s.t.} \\ & (i) \quad \int u(w(y)) f_1(y|a_1, a_2) dy - v'(a_1) = 0, \\ & (ii) \quad \int u(w(y)) f_2(y|a_1, a_2) dy \leq 0, \\ & (iii) \quad w(y) \geq k, \quad \forall y, \end{aligned} \tag{A57}$$

Solutions of the above two optimization programs must be the same, and due to the property that the multiplier attached to the incentive constraint associated with $a_2$ in the second program must be non-positive, we conclude $\mu_2^* < 0$ when $a_2^A(a_2^P) > a_2^P$.

[10] We assume that $\int u(w(y|a_2^P)) f(y|a_1^P, a_2, a_3 = 0) dy$ is concave in $a_2$, which is based on the first-order approach associated with $a_2$.

■

**Derivation of equation (47)**: With the Lagrange multipliers $\mu_1(R)$, $\mu_2(R)$ attached to the incentive constraints for action $a_1(R)$ and the project choice $a_2(R)$ given $R$, respectively, we can set up the Lagrangian for the optimization in (45) as follows:

$$\begin{aligned}\mathcal{L} = &\int_R \left[\int_{x,\eta} (x - w(x,\eta) + \lambda u(w(x,\eta)))\, g(x,\eta|a_1(R), a_2(R), R) dx d\eta - v(a_1(R))\right] h(R)dR \\ &+ \int_R \mu_1(R) \left[\int_{x,\eta} u(w(x,\eta)) g_1(x,\eta|a_1(R), a_2(R), R) dx d\eta - v'(a_1(R))\right] h(R)dR \\ &+ \int_R \mu_2(R) \left[\int_{x,\eta} u(w(x,\eta)) g_2(x,\eta|a_1(R), a_2(R), R) dx d\eta\right] h(R)dR,\end{aligned} \tag{A58}$$

from which we get the following first-order Euler equation about $w(x,\eta)$:

$$\begin{aligned}&(-1 + \lambda u'(w(x,\eta))) \int_R g(x,\eta|a_1(R), a_2(R), R) h(R) dR \\ &+u'(w(x,\eta)) \left[\int_R \{\mu_1(R) g_1(x,\eta|a_1(R), a_2(R), R) + \mu_2(R) g_2(x,\eta|a_1(R), a_2(R), R)\} \cdot h(R)dR\right] = 0,\end{aligned}$$

which derives (47).

■

**Proof of Proposition 8**: Proof will be similar to Proposition 2, except that now we have the project choice $a_2(R)$ that depends on the observed $R$ by the manager. Consider the principal's following alternative maximization program:

$$\begin{aligned}\max_{a_1(\cdot), a_2(\cdot), w(\cdot)} &\int_R \int_{x,\eta} (x - w(x,R,\eta))\, g(x,\eta|a_1(R), a_2(R), R) h(R) dx d\eta dR \\ &+ \lambda \int_R \left(\int_{x,\eta} u(w(x,R,\eta)) g(x,\eta|a_1(R), a_2(R), R) dx d\eta - v(a_1(R))\right) h(R) dR \quad \text{s.t.} \\ &(i) \int_{x,\eta} u(w(x,R,\eta)) g_1(x,\eta|a_1(R), a_2(R), R) dx d\eta - v'(a_1(R)) = 0, \forall R, \\ &(ii) \int_{x,\eta} u(w(x,R,\eta)) g_2(x,\eta|a_1(R), a_2(R), R) dx d\eta = 0, \forall R, \\ &(iii)\ w(x,R,\eta) \geq k, \ \ \forall (x,\eta).\end{aligned} \tag{A59}$$

Note that the above program is different from the original program (45) in that here contract can be written on the realized value of $R$. If we let the Lagrange multipliers to the constraints $(i)$ and $(ii)$ be $\mu_1(R)h(R)$ and $\mu_2(R)h(R)$ respectively, we get the following optimal contractual form:[11]

$$\begin{aligned}\frac{1}{u'(w(x,R,\eta))} &= \lambda + \mu_1(R)\frac{x-R\eta-\phi_R}{a_2(R)^2}\phi_{1,R} + \mu_2(R)\Big[-\frac{1}{a_2(R)} + \frac{x-R\eta-\phi_R}{a_2(R)^2}\phi_{2,R} + \frac{(x-R\eta-\phi_R)^2}{a_2(R)^3}\Big] \\ &= \lambda + (\mu_1(R)\phi_{1,R} + \mu_2(R)\phi_{2,R})\frac{\overbrace{x-R\eta}^{\equiv y}-\phi_R}{a_2(R)^2} + \frac{\mu_2(R)}{a_2(R)}\Big[\frac{(\overbrace{x-R\eta}^{\equiv y}-\phi_R)^2}{a_2(R)^2} - 1\Big],\end{aligned} \tag{A60}$$

when $w(x,R,\eta) \geq k$. The above equation (A60) implies that optimal contract only depends on $y \equiv x - R\eta$ and the solution $(w(x,R,\eta), a_1(R), a_2(R))$ becomes $(a_1^*, a_2^*, w^*(y) \equiv w^*(x-R\eta))$. By comparing the above (A59) with the program in (45) where the principal does not know $R$, one can easily see that the set of wage contracts, $\{w(x,R,\eta)\}$, satisfying the incentive constraints for a given action combination $(a_1(R), a_2(R))$ in the above program always contains the set of wage contracts that would be available when the principal does not know $R$, $\{w(x,\eta)\}$, satisfying the incentive constraints for the same action combination. Therefore, we have

$$SW^N \leq SW^*(a_1^*, a_2^*). \tag{A61}$$

However, one can easily see that $w^*(y) = w^*(x - R\eta)$ which is a unique solution for the wage contract of the above program is not in the set of $\{w(x,\eta)\}$. As a result, we finally derive

$$SW^N < SW^*(a_1^*, a_2^*). \tag{A62}$$

■

**Proof of Lemma 5**:

(1) Suppose $\underline{\mu_2^* < 0}$ in equation (52) for any given $\hat{b}$. Proposition 7 implies that if the shareholders want their manager to reduce the risk through the project choice (i.e., if $a_2^P < a_2^A(a_2^P)$), the optimal contract in equation (42) features $\mu_2^* < 0$. Note that risk reduction through the real project choice (i.e., lowering $a_2$) is costly to the manager in the sense that a less risky project generates the lower expected return, and thereby reduces the agent's expected payoff (i.e., $\mu_1^*\phi_1^* + \mu_2^*\phi_2^* > 0$). Thus, the fact that even costly risk reduction

[11] We define $\phi_R \equiv \phi(a_1(R), a_2(R))$, $\phi_{i,R} \equiv \phi_i(a_1(R), a_2(R))$ for $\forall i = 1, 2$, where $\{a_1(R), a_2(R)\}$ are optimal actions for each $R$.

is encouraged by $w^*(z(\hat{b}))$ implies that any risk reduction (i.e., reducing the variance of $z(\hat{(b)}))$ in the absence of expected return reduction will be taken by the manager under $w^*(z(\hat{b}))$. Risk reduction through derivative transaction is costless to the agent because there is no risk-return trade-off for derivative transaction (i.e., manipulating $a_3$). Whenever taking further risk reduction is encouraged, therefore, the manager would like to do it through the derivative choices first.

Thus, the manager will choose $a_3$ so that $b \equiv R - a_3 = \hat{b}$ which minimizes the variance of $z(\hat{b})$, when $w^*(z(\hat{b}))$ with $\mu_2^* < 0$ is designed.

■

(2) Suppose $\underline{\mu_2^* > 0}$ for $w^*(z(\hat{b}))$ in equation (52). Given $(a_1^*, a_2^*)$, $z(\hat{b}) = x - \hat{b}\eta = \phi(a_1^*, a_2^*) + (b - \hat{b})\eta + a_2^*\theta$ holds. Let $w(\eta, \theta, b|w^*)$ be the wage that the manager will receive under $w^*(z(\hat{b}))$ when he takes $(a_1^*, a_2^*, b)$ and $(\eta, \theta)$ are realized. Then, by substituting equation (50) into equation (52), we have

$$\frac{1}{u'(w(\eta,\theta,b|w^*))} = \lambda + (\mu_1^*\phi_1^* + \mu_2^*\phi_2^*)\frac{(b-\hat{b})\eta + a_2^*\theta}{(a_2^*)^2} + \mu_2^*\frac{1}{a_2^*}\left(\frac{((b-\hat{b})\eta + a_2^*\theta)^2}{(a_2^*)^2} - 1\right), \tag{A63}$$

when $w(\eta, \theta, b|w*) \geq k$ and otherwise $w(\eta, \theta, b|w*) = k$. Therefore, for two different $b$, say $b^0$ and $b^1$, given some realized $(\eta, \theta)$, we have

$$\begin{aligned}\frac{1}{u'(w(\eta,\theta,b^1|w^*))} - \frac{1}{u'(w(\eta,\theta,b^0|w^*))} =& (\mu_1^*\phi_1^* + \mu_2^*\phi_2^*)\frac{(b^1-b^0)\eta}{(a_2^*)^2} \\ &+ \mu_2^*\frac{1}{a_2^*}\left(\frac{\left((b^1-\hat{b})^2 - (b^0-\hat{b})^2\right)\eta^2 + 2a_2^*(b^1-b^0)\eta\theta]}{(a_2^*)^2}\right).\end{aligned} \tag{A64}$$

Assume that $b^1 = +\infty$ or $-\infty$, and $-\infty < b^0 < +\infty$. Since $\mu_2^* > 0$, we have

$$\frac{1}{u'(w(\eta,\theta,b^1|w^*))} - \frac{1}{u'(w(\eta,\theta,b^0|w^*))} > 0, \quad \forall(\eta,\theta). \tag{A65}$$

Therefore, we have

$$w(\eta,\theta,b^1|w^*) > w(\eta,\theta,b^0|w^*), \quad \forall(\eta,\theta). \tag{A66}$$

which implies that the agent takes $a_3$ satisfying $b = +\infty$ or $-\infty$ with $w^*(z(\hat{b}))$ with $\mu_2^* > 0$.

■

# Appendix B. The Truth-Telling Mechanism

In Sections 2 and 3, we assume there is no communication between the principal and the agent after the contract is written, or even if they can communicate, the contract cannot be contingent on communication between the principal and the agent. We now relax this assumption and consider the case where the agent can costlessly report the firm's risk exposure $R$ to the principal, and receive a payoff that is contingent on the communicated risk exposure as well as on the output and hedgeable risks. Note that the agent's message about his observed $R$ might or might not be truthful.

We assume the flexible project choice setting as in Section 3. As we will show below, for the case where $\mu_2^* < 0$ for $w^*(z(0))$ in (52), a contract that is similar to $w^*(z(0))$ can be designed to induce the agent to truthfully reveal the firm's risk exposure $R$. In other words, there is no loss associated with the risk exposure being unobservable and thus no gain from the introduction of derivative market. The intuition is the same as the one for why the manager would voluntarily hedge under $w^*(z(0))$ with $\mu_2^* < 0$. Essentially, the truth-telling contract will allow the agent to make a *side bet* with the principal. If the agent hedges in the derivative market with the contract $w^*(z(0))$, he will truthfully reveal what he observes (i.e., true $R$) to minimize the additional risk associated with this side bet.

However, when $\mu_2^* > 0$ for $w^*(z(0))$ in (52), any contract similar to $w^*(z(0))$ does not induce truth-telling since the manager wants to add more risks, as he does by engaging in speculation in derivative markets. Again, a new contract similar to the sample covariance punishing contract in (C.8) must be designed to induce him to reveal the truth.

**Equivalence between derivative market games and communication games** Suppose the principal does not know the firm's innate risk exposure $R$ and there is no derivative market (i.e., $a_3$ is again fixed at 0). Since the agent observes $R$ before he takes $(a_1, a_2)$ and communication regarding $R$ is freely allowed, the principal can design a truth-telling mechanism, $w(x, r, \eta)$, without incurring cost where $r$ represents the value of $R$ reported by the agent. Let $(a_1^T(R), a_2^T(R))$ be agent's optimal action combination after observing $R$ and $w^T(x, r, \eta)$ be the wage contract that optimally induces $(a_1^T(R), a_2^T(R))$ with the agent telling the truth. Knowing that $r = R, \forall R$, under $w^T(x, r, \eta)$, we denote optimized joint benefits in this case as

$$SW^T \equiv \int \left(\phi(a_1^T(R), a_2^T(R)) - C^T(a_1^T(R), a_2^T(R)) - \lambda v(a_1^T(R))\right) h(R)dR, \tag{B1}$$

where

$$C^T(a_1^T(R), a_2^T(R)) \equiv \int \left(w^T(x, R, \eta) - \lambda u(w^T(x, R, \eta))\right) g(x, \eta | a_1^T(R), a_2^T(R)) dx d\eta \tag{B2}$$

denotes the agency cost arising from inducing $(a_1^T(R), a_2^T(R))$ through $w^T(x, r, \eta)$ when $R$ is realized. In the above equation, $g(x, \eta | a_1^T(R), a_2^T(R))$ denotes the joint density function of $(x, \eta)$ given that $(a_1^T(R), a_2^T(R))$ is chosen by the manager when $a_3$ is fixed at $0$.

Since $SW^*(a_1^*, a_2^*)$ in (43) is the maximum level of joint benefits that $SW^T$ can attain, we first consider the case in which principal designs a wage contract, $w^*(y_r)$, that is the same as $w^*(y)$ in (40) except that it is based on $y_r \equiv x - r\eta$ instead of $y \equiv x - R\eta$. That is, $w^*(y_r)$ satisfies

$$\frac{1}{u'(w^*(y_r))} = \lambda + (\mu_1^*\phi_1^* + \mu_2^*\phi_2^*)\frac{y_r - \phi(a_1^*, a_2^*)}{(a_2^*)^2} + \mu_2^*\frac{1}{a_2^*}\left(\frac{(y_r - \phi(a_1^*, a_2^*))^2}{(a_2^*)^2} - 1\right), \tag{B3}$$

for $y_r$ satisfying $w^*(y_r) \geq k$ and $w^*(y_r) = k$ otherwise. We call $w^*(y_r)$ the *full-trust contract* as $w^*(y_r)$ simply is based on the agent's report instead of the realized $R$ in (B3). Note that

$$y_r \equiv x - r\eta = \phi(a_1, a_2) + (R - r)\eta + a_2\theta. \tag{B4}$$

Since, from (50),

$$z(0) = x = \phi(a_1, a_2) + (R - a_3)\eta + a_2\theta, \tag{B5}$$

the principal's problem of designing a truth-telling mechanism based on $y_r$ when there is no derivative market is equivalent to his problem of designing an incentive scheme based on $z(0)$ to induce $b = 0$ (i.e., $a_3 = R$) when derivative transactions are allowed. As a result, as is the case for $w^*(z(0))$ in Lemma 5, we directly obtain following results for $w^*(y_r)$.

**Lemma 8** [Speculation and Hedging with $w^*(y_r)$]
(1) *If $\underline{\mu_2^* < 0}$ for the wage contract, $w^*(y_r)$, described in (B3), then the manager will always report truly, i.e., $r = R, \forall R$, when $w^*(y_r)$ is offered.*
(2) *If $\underline{\mu_2^* > 0}$ for $w^*(y_r)$, then the manager will report $r$ such that $|R - r| = \infty$ when $w^*(y_r)$ is offered.*

From Lemma 8, we obtain the following propositions.

**Proposition 10** *When there is no derivative market and communication between the principal and the agent is costless, then $w^*(y_r)$ described in (B3) is the optimal truth-telling contract for the firm's hidden risk exposure, $R$, if $\underline{\mu_2^* < 0}$ for $w^*(y_r)$. In this case:*
*(1) The principal's inability to observe $R$ does not reduce the firm's welfare (i.e., no adverse selection).*
*(2) An introduction of a derivative market does not improve the firm's welfare compared with $SW^T$ in (B1).*

Proposition 10 along with Propositions 8 and 9 reaffirms that the benefits that derivative markets bring are actually informational gains, as the agent engages in perfect hedging in derivative markets. If the principal and the agent cannot communicate with each other due to huge communication costs, these benefits are actually associated with saving communication costs[1] that would realistically incur when principal has to design a truth-telling contract that induces the agent to reveal his exact information about the firm's risk exposure $R$. When the communication between the principal and the agent becomes free, the principal, by designing $w^*(y_r)$, can easily reproduce the same results as when she observes the firm's innate risk exposure, if $\mu_2^* < 0$ for $w^*(y_r)$. However, in reality, the costs associated with communicating this information and updating the compensation contract based on the reavealed $R$ may be greater than the hedging cost. As shown in (B4), allowing the manager to choose $a_3$ in derivative transactions is observationally equivalent to allowing him to freely report the firm's realized risk exposure $R$.

On the other hand, if $\mu_2^* > 0$ for $w^*(y_r)$, the manager does not report the true $R$ under $w^*(y_r)$, and shareholders have to redesign a truth-telling mechanism, $w^T(x, r, \eta)$ different from $w^*(y_r)$. It can be similar to (C.8), except that we design $w^o(y_r, \eta)$ instead of $w^o(x, \eta)$. Therefore, two games (i.e., with derivative markets and with free communication) are of the isomorphic structure.

**Proposition 11** *When $\underline{\mu_2^* > 0}$ for $w^*(y_r)$ described in (B3), the introduction of a derivative market does not improve on the firm's efficiency when communication between the principal and the agent is freely allowed, and it actually lowers the firm's efficiency if $\sigma_R^2$ is very small, when the restriction on derivative transaction is not feasible.*

[1]In the presence of derivative markets, the principal and the agent do not need to communicate about the realized $R$, since the agent can eliminate this innate risk $R$ through derivative transactions (i.e., $a_3 = R$).

As explained in Proposition 13, when communication between shareholders and the manager is not available and $\mu_2^* > 0$ for $w^*(y_r)$ in (52), the manager's opportunity to transact derivatives may or may not improve the firm's welfare compared to the case without the derivative market depending on the size of uncertainty $\sigma_R^2$ on the exposure $R$.

If the communication becomes free between the principal and the agent however, the access to the derivative market reduces the firm's welfare when $\sigma_R^2$ is small enough. It is because both $w^o(x, \eta)$ in (C.8) and $w^N(x, \eta)$ in (49)[2] are actually truth-telling contracts. Therefore, when there is no derivative market, the principal designs either $w^o(y_r, \eta)$ or $w^N(x, \eta)$ under the free communication depending on which of two gives the better welfare. As shown in Proposition 13, the principal prefers designing $w^N(x, \eta)$ to $w^o(y_r, \eta)$ if $\sigma_R^2$ is very small. The optimal truth-telling mechanism $w^N(x, \eta)$, which actually does not elicit any information from the agent, thus performs weakly better than $w^o(x, \eta)$. However, after the derivative market is introduced, the principal has to shift from $w^N(x, \eta)$ to $w^o(x, \eta)$ because there now exists an incentive problem associated with $a_3$.

In summary, when the communication between shareholders and the manager becomes free, the manager's access to derivative market transactions does not change the firm's welfare if $\mu_2^* < 0$, and might lower it if $\mu_2^* > 0$ and no restriction on the derivative trading can be imposed by the principal.

## B.1. Proof of Appendix B

**Proof of Proposition 10**: From Lemma 8, we see that $w^*(y_r)$ is a truth-telling mechanism for the firm's hidden risk exposure, $R$, if $\mu_2^* < 0$ for $w^*(y_r)$. Since $r = R, \forall R$, under $w^*(y_r)$, we have

$$y \equiv x - R\eta = \phi(a_1, a_2) + a_2\theta = y_r. \tag{B6}$$

Furthemore, we have that $w^*(y_r)$ in equation (B3) has the same contractual form as $w^*(y)$ in (42). Thus, the optimal action combination to be chosen by the agent under $w^*(y_r)$ is $(a_1^*, a_2^*)$, i.e., $(a_1^T(R), a_2^T(R)) = (a_1^*, a_2^*), \forall R$. Therefore, we derive

$$SW^T = SW^*(a_1^*, a_2^*), \tag{B7}$$

[2]Note that both $w^o(x, \eta)$ and $w^N(x, \eta)$ do not depend on the reported value of $R$, so we regard both two contracts as truth-telling mechanism.

and from Proposition 9, we derive that $SW^T$ is the same as the joint benefits that will be obtained under $w^*(z(0))$ when there is a derivative market.

■

**Proof of Proposition 11** Note that both non-communication contracts $w^N(x,\eta)$ and $w^o(x,\eta)$ in (C.8) are truth-telling mechanisms.[3] Therefore, if $\mu_2^* > 0$ for $w^*(y_r)$ in equation (B3), we have

$$SW^T \geq max\{SW^N, SW^o(a_1^o, a_2^o, R)\}. \tag{B8}$$

Furthermore, from Proposition 13, we have $SW^N > SW^o(a_1^o, a_2^o, R)$ when $\sigma_R^2$ is very small. Thus, we obtain that $SW^T > SW^o(a_1^o, a_2^o, R)$ when $\sigma_R^2$ is very small.

■

[3]The principal can design $w^N(x,\eta)$ without using $r$. Also, by designing $w^o(y_r,\eta)$ as a truth-telling mechanism, he can obtain the same result as $w^o(x,\eta)$ would provide.

# Appendix C. Optimal Contracts when $\mu_2^* > 0$ in Section 3

**New optimal contract when $\mu_2^* > 0$ in (52) in the presence of derivative markets** In deriving the optimal contract in the presence of derivative markets when $\mu_2^* > 0$ for $w(z(\hat{b}))$ in (52), we first consider the case in which the principal designs a contract that ensures a finite $a_3$ when $\mu_2^* > 0$ for $w^*(z(\hat{b}))$ in (52). Let $(a_1^o, a_2^o, a_3^o)$, where $|b^o \equiv R - a_3^o| < \infty$, be the optimal action combination and $\underline{w^o(z(\hat{b}), \eta)}$ be the wage contract which optimally induces that action combination $(a_1^o, a_2^o, a_3^o)$ where $\hat{b} = b^o \equiv R - a_3^o$. We denote the optimized joint benefits in this case as

$$SW^o(a_1^o, a_2^o, a_3^o) \equiv \phi(a_1^o, a_2^o) - C^o(a_1^o, a_2^o, b^o) - \lambda v(a_1^o), \tag{C.1}$$

where

$$C^o(a_1^o, a_2^o, b^o) \equiv \int \left[w^o(z(\hat{b}), \eta) - \lambda u(w^o(z(\hat{b}), \eta))\right] g(z(\hat{b}), \eta | a_1^o, a_2^o, b^o) dz d\eta \tag{C.2}$$

denotes the agency cost arising from inducing $(a_1^o, a_2^o, a_3^o)$ when there is a derivative market and $\mu_2^* > 0$ for $w^*(z(\hat{b}))$ in equation (52).

**Lemma 9** *If $w^o(z(\hat{b}), \eta)$ is an optimal contract that induces $(a_1^o, a_2^o, a_3^o)$ where $\hat{b} = R - a_3^o \equiv b^o \neq 0$, then $w^o(z(0), \eta) \equiv w^o(x, \eta)$*[1] *is also an optimal contract which induces $(a_1^o, a_2^o, a_3 = R)$. Therefore,*

$$SW^o(a_1^o, a_2^o, a_3^o) = SW^o(a_1^o, a_2^o, a_3 = R).$$

*Therefore, without loss of generality, the principal chooses to induce complete hedging from the agent (i.e., $a_3 = R$ or $b = 0$).*

Lemma 9 indicates that when the principal has to design a compensation contract to guarantee the agent's choice of $a_3$ satisfying $|R - a_3| < \infty$ due to the fact that $\mu_2^* > 0$ for $w^*(z(\hat{b}))$, the level of $a_3$ to be induced by $w^o(z(\hat{b}), \eta)$ is a matter of indifference as long as it is finite and the efficiency is concerned. This is because, as shown in equation (50), the agent's derivative choice, $a_3$, is additively separable from his other two productive action choices, $(a_1, a_2)$, in determining the output level, $x$, and not only the output level but also the derivative market variables, $\eta$, are observable (thus contractible). This feature allows the

[1]Note that $z(0) = x$ from equation (50).

principal to always eliminate or add hedgeable risks to the agent's compensation, making a level of remaining hedgeable risks irrelevant. Without loss of generality, therefore, from now on we focus on the case where $a_3 = R$ (i.e., complete hedging) is induced.[2] The optimal contract $w^o(x, \eta)$ that induces the agent's complete hedging (i.e., $b = 0$ or $a_3 = R$) is obtained by solving a maximization problem that is similar to (39) with an added requirement that a contract induces the manager to take $a_3 = R$.

Given that the agent's choosing $a_3$ given his private information $R$ is equivalent to his choosing $b = R - a_3$, a new optimal contract, $w^o(x, \eta)$, inducing the agent to take $(a_1^o, a_2^o, b = 0)$ when $\mu_2^* > 0$ for $w^*(z(\hat{b}))$ in (52), must solve the following optimization problem:[3,4]

$$\max_{w(\cdot)\geq k} \int (x - w(x,\eta))g(x,\eta|a_1^o, a_2^o, b=0)dxd\eta + \lambda \left( \int u(w(x,\eta))g(x,\eta|a_1^o, a_2^o, b=0)dxd\eta - v(a_1^o) \right)$$

$$\text{s.t.} \quad (i) \int u(w(x,\eta))g_1(x,\eta|a_1^o, a_2^o, b=0)dxd\eta - v'(a_1^o) = 0,$$

$$(ii) \int u(w(x,\eta))g_2(x,\eta|a_1^o, a_2^o, b=0)dxd\eta = 0,$$

$$(iii) \;\; b = 0 \in \arg\max_{b'} \int u(w(x,\eta))g(x,\eta|a_1^o, a_2^o, b')dxd\eta, \; \forall b. \tag{C.3}$$

Note that we take the optimal $(a_1^o, a_2^o)$ as given, and rely on the first-order approach for incentive constraints associated with the action $a_1$ and the project choice $a_2$. However, we do not use the same approach for the incentive compatibility constraint associated with the hedging choice $b$. The following Lemma 10 demonstrates the reason we cannot rely on the first-order approach for the incentive compatibility around $b$.

**Lemma 10** *If $w^*(z(0))$ in (52) is designed, the agent will be indifferent between taking $b$ and taking $-b$, $\forall b$.*

Lemma 10 shows that, if $w^*(z(0))$ is designed and offered, the manager's expected utility becomes symmetric around $b = 0$ (i.e., $a_3 = R$) in the space of $b$ (i.e., in the space of $a_3$).

---

[2] Therefore, we have the same indeterminacy issue as in Section 2. It comes from the same linear technology with the fact that the principal is risk-neutral.

[3] Here the distribution $g(x, \eta|a_1, a_2, b)$ is of the same form as (46) with $b$ in the position of $R$ in (46).

[4] Our analysis of the new optimal contract $w^o(x, \eta)$ in this section follows closely to our treatment without project choice $a_2$ in Section 2.2. There is no specific role of $a_2^o$ in deriving a new optimal contract $w^o(x, \eta)$ that induces perfect hedging from the agent.

As we know:

$$\int u(w^*(z(0)))g(z(0),\eta|a_1^o,a_2^o,b)dzd\eta \tag{C.4}$$

is continuous and differentiable in $b$, Lemma 10 implies:

$$\int u(w^*(z(0))g_3(z(0),\eta|a_1^o,a_2^o,b=0)dzd\eta = 0, \tag{C.5}$$

where $g_3(\cdot|\cdot)$ denotes the first derivative of $g(\cdot|\cdot)$ taken with respect to $b$. Since $(w^*(z(0)),a_1^*,a_2^*)$ is the solution of the optimization in (C.3) without the incentive constraint of $b$, i.e., (iii) in (C.3), if we use the first-order approach for the incentive constraint associated with $b$ in program (C.3), we always end up obtaining $w^*(z(0))$ in (52) as an optimal contract. However, $\mu_2^* > 0$ for $w^*(z(0))$ implies from Lemma 5 that this contract incentivizes an agent to take $b = \pm\infty$ instead of taking a stipulated $b = 0$.[5] Therefore, we have to explicitly include the incentive constraint for $b$ which does not rely only on the first-order condition at $b = 0$.

Without relying on the first-order approach, we follow Grossman and Hart (1983), replacing the incentive constraint for $b$ (i.e., (iii) in (C.3)) with:

$$\int u(w(x,\eta))\left(g(x,\eta|a_1^o,a_2^o,b=0) - g(x,\eta|a_1^o,a_2^o,b)\right)dxd\eta \geq 0,\ \forall b, \tag{C.6}$$

which implies that the manager's indirect utility is maximized when he takes $b = 0$ (i.e., $a_3 = R$).

Now we state formally the optimization problem of choosing the optimal contract $w^o(\cdot)$ given $(a_1^o,a_2^o)$ as:

$$\begin{aligned}
\max_{w(\cdot)\geq k} &\int (x - w(x,\eta))g(x,\eta|a_1^o,a_2^o,b=0)dxd\eta + \lambda\left(\int u(w(x,\eta))g(x,\eta|a_1^o,a_2^o,b=0)dxd\eta - v(a_1^o)\right)\\
\text{s.t.}\ (i) &\int u(w(x,\eta))g_1(x,\eta|a_1^o,a_2^o,b=0)dxd\eta - v'(a_1^o) = 0,\\
(ii) &\int u(w(x,\eta))g_2(x,\eta|a_1^o,a_2^o,b=0)dxd\eta = 0,\\
(iii) &\int u(w(x,\eta))\left(g(x,\eta|a_1^o,a_2^o,b=0) - g(x,\eta|a_1^o,a_2^o,b)\right)dxd\eta \geq 0,\ \forall b,
\end{aligned} \tag{C.7}$$

[5]Note that a technical problem about the first-order approach does not arise in the incentive constraint about the project choice $a_2$, which also determines the firm's risks. It is because lowering risk through project choice $a_2$ is costly in terms of return, while doing it through $a_3$ is not. Thus, the manager's expected utility is not symmetric in the space of $a_2$.

Note that the set of incentive constraints for all $b$ (i.e., (C.6)) are taken into account to make sure the agent's expected indirect utility is at maximum when he chooses $b = 0$ instead of other $b > 0$ or $b < 0$.

The first-order condition of the above program (C.7) yields the optimal contract, $w^o(x, \eta)$, that satisfies[6]

$$\frac{1}{u'(w^o(x,\eta))} = \lambda + (\mu_1^o \phi_1^o + \mu_2^o \phi_2^o) \frac{x - \phi(a_1^o, a_2^o)}{(a_2^o)^2} + \mu_2^o \frac{1}{a_2^o} \left( \frac{(x - \phi(a_1^o, a_2^o))^2}{(a_2^o)^2} - 1 \right) + \underbrace{\int \mu_4^o(b) \left( 1 - \frac{g(x, \eta|b)}{g(x, \eta|b = 0)} \right) db}_{\text{Additional term to (52)}}, \tag{C.8}$$

for $(x, \eta)$ satisfying $w^o(x, \eta) \geq k$ and $w^o(x, \eta) = k$ otherwise. In equation (C.8), $\phi_i^o \equiv \phi_i(a_1^o, a_2^o), i = 1, 2$, and $\mu_1^o$, $\mu_2^o$, and $\mu_4^o(b)$ are the optimized Lagrange multipliers associated with the first, second, and third constraints (for specific $b$) in the above optimization program (C.7), respectively.

As shown in the Appendix, we obtain the following proposition from (C.8).

**Proposition 12** [Hedging through Punishment]
*If $\mu_2^* > 0$ for $w^*(z(0))$ described in (52), then the principal can motivate the manager to hedge completely by designing a new compensation contract, $w^o(x, \eta)$ in (C.8), which (i) satisfies $w^o(x, \eta) = w^o(x, -\eta)$ for all $x, \eta$, and (ii) penalizes the manager for having higher realized $(x - \phi)^2\eta^2$). To be specific, given realized $(x, \eta)$, a higher $(x - \phi)^2\eta^2$ yields a lower wage $w^o(x, \eta)$, while given the output $x$ and sample covariance $(x - \phi)^2\eta^2$, a higher $\eta$ raises the wage $w^o(x, \eta)$.*

Proposition 12 can be understood in the following way: the production function $x = \phi(a_1^o, a_2^o) + a_2\theta + b\eta$ gives us the relation $b = Cov(x, \eta) = \mathbb{E}((x - \phi(a_1^o, a_2^o))\eta)$. It implies that if the agent takes $b = 0$, a statistical covariance between output $x$ and hedgeable risk $\eta$ disappears, whereas any other $b \neq 0$ generates non-zero population covariances. Since $b = 0$ generates $x = \phi(a_1^o, a_2^o) + a_2\theta$, which is independent of $\eta$, $w^o(x, \eta) = w^o(x, -\eta)$ for all $x, \eta$ is ensured to minimize the amount of risk imposed on the agent, as $\eta$ becomes irrelevant in inducing $(a_1^o, a_2^o)$ and has a symmetric distribution around 0.

At optimum, by punishing the covariance between $x$ and $\eta$,[7] shareholders effectively incentivize the manager to engage in full hedging and take $b = 0$. As our framework is

[6] We suppress the dependence of distribution $g$ and likelihood ratios on $(a_1^o, a_2^o)$.
[7] It is possible since $\eta$ is observable at the end of the period and thus contracts can be written upon it.

in one-period setting, any positive or negative realized sample covariance $\widehat{Cov} = (x - \phi(a_1^o, a_2^o))\eta = b\eta^2 + a_2\theta\eta$, instead of a population covariance, is punished by the principal through a lower compensation $w^o(x, \eta)$. If the realized sample covariance $\widehat{Cov} = (x - \phi(a_1^o, a_2^o))\eta = b\eta^2 + a_2\theta\eta$ is large, not because of the manager's speculation ($b \neq 0$) but from a high realized market observable, $|\eta|$, then the principal takes it into account and raises $w^o(x, \eta)$. In contrast, given realized output and market observables $(x, \eta)$, a bigger realization of $\widehat{Cov}$ is likely to be generated by $b \neq 0$ with a bigger $|b|$, thus the agent is punished and her wage income $w^o(x, \eta)$ falls.

Designing the new optimal contract including this covariance punishment is not, however, costless compared with $w^N(x, \eta)$ in (49), the optimal contract in cases where there is no derivative market, since it exposes the agent to additional risks. As we show below, if this cost is relatively high compared to the informational gain that principal gets through the agent's derivative transaction, an introduction of derivative markets can actually reduce the welfare.

**Proposition 13** *If $\mu_2^* > 0$ for $w^*(z(0))$ in (52), the introduction of a derivative market will reduce welfare compared with $SW^N$ in equation* (48) *when the amount of uncertainty about the firm's risk exposure, $\sigma_R^2$, is small.*

The logic is similar to Proposition 5, except that whether the manager would speculate depend on the sign of $\mu_2^*$ in (52) here. As we show in Proposition 9, the optimal contract must be altered from $w^N(x, \eta)$ when derivative markets open. In cases where the manager voluntarily chooses to hedge after the derivative market is introduced given his original optimal dual-agency contract (i.e., $\mu_2^* < 0$ for $w^*(y)$ in (42) or equivalently $\mu_2^* < 0$ for $w^*(z(0))$ in (52)), the compensation contract remains mainly unchanged from $w^*(y)$ while being based on $z(0) = x$ rather than $y$, and the welfare unambiguously increases by the informational gain generated by an opportunity of the manager to hedge in the derivative market and eliminate the firm's risk exposure $R$. However, when $w^*(z(0))$ with $\mu_2^* > 0$ induces the manager to speculate in the derivative market, shareholders must revise the manager's contract to $w^o(x, \eta)$ to provide the manager with an incentive to hedge, which imposes additional risks on the risk-averse manager's side and incurs the cost out of it. Thus, there are costs and benefits associated with derivative trading that the principal must consider.

Altering contracts to ensure the agent hedges rather than speculates is costly since it needs to consider the agent's additional incentive problem associated with $a_3$ by exposing him to the additional risk: market observables $\eta$, whereas the principal gets informational benefits as now she does not have to know about the firm's risk exposure $R$ as the agent eliminates any hedgeable risk (i.e., $b = 0$ or $a_3 = R$) under $w^o(x, \eta)$. On the other hand, when there is no derivative market, principal's inability to observe $R$ causes welfare loss as she now should offer $w^N(x, \eta)$ instead of $w^*(y)$.[8] To illustrate these costs and benefits more precisely, we use equation (43), equation (48), and equation (C.1), and decompose the welfare change in the following way.

$$SW^o(a_1^o, a_2^o, a_3 = R) - SW^N = \left(SW^*(a_1^*, a_2^*) - SW^N\right) - \left(SW^*(a_1^*, a_2^*) - SW^o(a_1^o, a_2^o, a_3 = R)\right). \quad \text{(C.9)}$$

The first part, $SW^*(a_1^*, a_2^*) - SW^N$, represents the welfare loss due to the principal's inability to observe the firm's risk exposure when there is no derivative market (or equivalently informational gains from the introduction of a derivative market). The second part represents the welfare loss due to the additional incentive problem associated with the manager's derivative choices when the derivative market is introduced and the manager speculates under $w^*(z(0))$ in (52).

Note that no expectation with respect to $R$ is taken for joint benefits $SW^*(a_1^*, a_2^*)$ and $SW^o(a_1^o, a_2^o, a_3 = R)$, as both are independent of $R$. When there is no derivative market and the firm's risk exposure, $R$, is observed by the principal as well as the manager, joint benefits, $SW^*(a_1^*, a_2^*)$, are obviously independent of the $R$'s realization because $(a_1^*, a_2^*)$ are independent of $R$. Likewise, when $w^o(x, \eta)$ is designed in the presence of derivative markets, the joint benefits $SW^o(a_1^o, a_2^o, a_3 = R)$ are independent of $R$ as agent is always induced to take $b = R - a_3 = 0$ no matter what $R$ is realized. However, in calculating joint benefits $SW^N$, an expectation with respect to $R$ is taken, implying that the distribution of $R$ affects the level of $SW^N$.

The above discussion implies that informational gains from the manager's derivative transaction declines as the amount of uncertainty around the firm's risk exposure $R$ falls. On the other hand, the cost of controlling the additional incentive problem associated with $a_3$ (or equivalently $b = R - a_3$) is independent of the firm's risk exposure $R$ and thus $\sigma_R^2$. For instance, even if $R$ is known to the principal (i.e., $\sigma_R^2 = 0$), the moral hazard problem associated with inducing $b = 0$ still remains to the same degree. Therefore, the amount of uncertainty about $R$ is indeed a matter of indifference in incentivizing the agent's choice of

[8] Of course, principal can always design $w^o(x, \eta)$ instead of $w^N(x, \eta)$ when there is no derivative market. However, $w^o(x, \eta)$ will perform poorly without the derivative market.

$b$.

As a result, if the uncertainty around $R$, e.g., $\sigma_R^2$, is small enough, the contractual cost dominates informational gains when derivative markets are introduced, and shareholders would be better off prohibiting the manager from trading derivatives. Therefore, recent financial innovations can potentially hurt the efficiency of firms through its effects on agency relationships.

## C.1. Proof of Appendix C

**Proof of Lemma 9**: From equation (50), we have $z(\hat{b}) = \phi(a_1, a_2) + (b - \hat{b})\eta + a_2\theta$ for any given $(a_1, a_2, b)$, and $z(0) = \phi(a_1, a_2) + b'\eta + a_2\theta$ for any given $(a_1, a_2, b')$. Therefore, we obtain

$$z(\hat{b}|a_1, a_2, b) = z(0|a_1, a_2, b'), \quad \text{whenever } \; b' = b - \hat{b}. \tag{C.10}$$

Furthermore, if $b' = b - \hat{b}$, two joint density functions of $(z(\hat{b}), \eta)$ and $(z(0), \eta)$ are the same, i.e.,

$$\begin{aligned} g(z(\hat{b}), \eta|a_1, a_2, b) &= \frac{1}{2\pi a_2} \exp\left(-\frac{1}{2}\left(\frac{\left(z(\hat{b}) - \phi(a_1, a_2) - (b - \hat{b})\eta\right)^2}{a_2^2} + \eta^2\right)\right) \\ &= g(z(0), \eta|a_1, a_2, b'), \quad \forall b' = b - \hat{b}. \end{aligned} \tag{C.11}$$

Thus, we derive that for $\forall(a_1, a_2, b)$, we have

$$\int u(w^o(z(\hat{b}), \eta))g(z(\hat{b}), \eta|a_1, a_2, b)dzd\eta = \int u(w^o(z(0), \eta))g(z(0), \eta|a_1, a_2, b' = b-\hat{b}))dzd\eta. \tag{C.12}$$

Note that the manager is induced to take $(a_1^o, a_2^o, b^o \equiv R - a_3^o = \hat{b})$ under the contract $w^o(z(\hat{b}), \eta)$. Thus, the manager will be induced to take $(a_1^o, a_2^o, b' = 0$ (i.e., $a_3 = R$)) under wage contract $w^o(z(0), \eta)$. Moreover, since

$$\int w^o(z(\hat{b}), \eta)g(z(\hat{b}), \eta|a_1^o, a_2^o, b^o)dzd\eta = \int w^o(z(0), \eta)g(z(0), \eta|a_1^o, a_2^o, 0))dzd\eta, \tag{C.13}$$

using equation (C.1) and equation (C.2), we finally derive:

$$SW^o(a_1^o, a_2^o, a_3^o) = SW^o(a_1^o, a_2^o, R).$$

■

**Proof of Lemma 10**: Proof is almost the same as in Lemma 3. When $w^*(z(0))$ described in equation (52) is designed, we have

$$z(0|a_1, a_2, b) = x = \phi(a_1, a_2) + b\eta + a_2\theta. \tag{C.14}$$

If the agent takes $(a_1, a_2, b)$ under $w^*(z(0))$, then his expected utility is:

$$\int u(w^*(z(0)))g(z(0), \eta|a_1, a_2, b)dzd\eta - v(a_1) = \int u(w^*(z(0)))q(z(0)|a_1, a_2, b, \eta)l(\eta)dzd\eta - v(a_1), \tag{C.15}$$

where $q(\cdot)$ denotes the conditional density function of $z(0)$ given $(a_1, a_2, b, \eta)$ and $l(\cdot)$ denotes the density function of $\eta \sim N(0, 1)$.

Now, suppose the agent takes $(a_1, a_2, -b)$ under $w^*(z(0))$. Then, his expected utility becomes:

$$\int u(w^*(z(0)))g(z(0), \eta|a_1, a_2, -b)dzd\eta - v(a_1) = \int u(w^*(z(0)))q(z(0)|a_1, a_2, -b, \eta)l(\eta)dzd\eta - v(a_1). \tag{C.16}$$

Since

$$q(z(0)|a_1, a_2, b, \eta) = \frac{1}{\sqrt{2\pi}a_2} \exp\left(-\frac{(z(0) - \phi(a_1, a_2) - b\eta)^2}{2a_2^2}\right), \tag{C.17}$$

we have

$$q(z(0)|a_1, a_2, b, \eta) = q(z(0)|a_1, a_2, -b, -\eta). \tag{C.18}$$

Since $\eta \sim N(0, 1)$ is symmetrically distributed around 0 and $l(\eta) = l(-\eta)$, $\forall\eta$, we finally have

$$\int u(w^*(z(0)))g(z(0), \eta|a_1, a_2, b)dzd\eta - v(a_1) = \int u(w^*(z(0)))g(z(0), \eta|a_1, a_2, -b)dzd\eta - v(a_1). \tag{C.19}$$

■

**Proof of Proposition 12**: To prove this proposition, we start with the following Lemma 11. Our proof strategy here will be similar to Proposition 3, but now we have the project choice $a_2^o$ chosen by the manager.

**Lemma 11** *If $\mu_2^* > 0$ for contract $w^*(z(0))$ in equation (52), then the optimal contract $w^o(x,\eta)$ guaranteeing that the agent takes $a_1^o, a_2^o, a_3^o = R$ $(b = 0)$, i.e., $w^o(x,\eta)$ in equation (C.8), must satisfy*

*(1) $\mu_2^o \geq 0$*

*(2) $\mu_4^o(b) \neq 0$ $(> 0)$ for a positive Borel-measure of $b$.*[9]

*(3) $w^o(x,\eta) = w^o(x,-\eta)$ for all $x, \eta$ and $\mu_4^o(b) = \mu_4^o(-b)$ for all $b$.*

**Proof.** (1) $\underline{\mu_2^o \geq 0}$: Assume that $\mu_2^o < 0$, then under the contract $w^1(x,\eta)$ satisfying

$$\frac{1}{u'(w^1(x,\eta))} = \lambda + (\mu_1^o\phi_1^o + \mu_2^o\phi_2^o)\frac{x-\phi(a_1^o,a_2^o)}{(a_2^o)^2} + \frac{\mu_2^o}{a_2^o}\left(\frac{(x-\phi(a_1^o,a_2^o))^2}{(a_2^o)^2} - 1\right), \quad \text{(C.20)}$$

for $(x,\eta)$ satisfying $w^1(x,\eta) \geq k$ and $w^1(x,\eta) = k$, the agent voluntarily chooses $b = 0$ even though we did not consider the constraint (iii) in (C.7). Thus $w^1(x,\eta)$ becomes the solution of (C.7). However, it contradicts with our assumption of $\mu_2^* > 0$ for $w^*(z(0))$ since $(w^1(x,\eta), \mu_1^o, \mu_2^o)$ becomes $(w^*(z(0)), \mu_1^*, \mu_2^*)$ without the incentive constraint (iii) about $b$.

(2) $\underline{\mu_4^o(b) \neq 0 \text{ for a positive Borel-measure of } b}$: Assume $\mu_4^o(b) = 0$ $a.s.$ Then optimal contract $w^o(x,\eta)$ becomes:

$$\frac{1}{u'(w^o(x,\eta))} = \lambda + (\mu_1^o\phi_1^o + \mu_2^o\phi_2^o)\frac{x-\phi(a_1^o,a_2^o)}{(a_2^o)^2} + \frac{\mu_2^o}{a_2^o}\left(\frac{(x-\phi(a_1^o,a_2^o))^2}{(a_2^o)^2} - 1\right), \quad \text{(C.21)}$$

for $(x,\eta)$ satisfying $w^o(x,\eta) \geq k$ and $w^o(x,\eta) = k$.

Because we already know $(w^o(x,\eta), \mu_1^o, \mu_2^o, a_1^o, a_2^o)$ becomes $(w^*(z(0)), \mu_1^*, \mu_2^*, a_1^*, a_2^*)$ in this case and $\mu_2^* > 0$ holds, $(w^o(x,\eta), \mu_1^o, \mu_2^o)$ will induce $b = \pm\infty$ instead of $b = 0$ from the agent, a contradiction to the constraint (iii) in (C.7).

(3) $\underline{w^o(x,\eta) = w^o(x,-\eta) \text{ for all } x,\eta \text{ and } \mu_4^o(b) = \mu_4^o(-b) \text{ for all } b}$: We first see:[10]

$$g(x,\eta|b) = \frac{1}{2\pi a_2^o}\exp\left(-\frac{1}{2}\frac{(x-\phi(a_1^o,a_2^o)-b\eta)^2}{(a_2^o)^2} - \frac{1}{2}\eta^2\right), \quad \text{(C.22)}$$

[9] We already know $\mu_4^o(b) \geq 0$ for every $b$ (almost surely), since it is derived from the inequality constraint at each $b$.

[10] We suppress $a_1^o, a_2^o$ in $g(x,\eta|a_1^o,a_2^o,b)$ in (C.7) to make our expressions simpler.

where

$$\frac{g(x,\eta|b)}{g(x,\eta|b=0)} = \exp\left(\frac{b\eta(x-\phi(a_1^o,a_2^o))}{(a_2^o)^2}\right)\exp\left(-\frac{b^2\eta^2}{2(a_2^o)^2}\right). \tag{C.23}$$

From (C.22), and (C.23), we observe that $g(x,\eta|b=0)$, $g_1(x,\eta|b=0)$, and $g_2(x,\eta|b=0)$ are all even with $\eta$ where $g_1$ and $g_2$ are partial derivatives of $g$ with respect to $a_1$ and $a_2$: i.e., (i) $g(x,-\eta|b=0) = g(x,\eta|b=0)$; (ii) $g_1(x,-\eta|b=0) = g_1(x,\eta|b=0)$; (iii) $g_2(x,-\eta|b=0) = g_2(x,\eta|b=0)$. Also from (C.22), we acknowledge:

$$g(x,-\eta|b) = g(x,\eta|-b), \;\; \forall(x,\eta,b). \tag{C.24}$$

Our strategy is to prove that: (i) if $w^o(x,\eta)$ is an optimal contract, then $w^o(x,-\eta)$ satisfies all the constraints in (C.7); (ii) Related to (i), if $w^o(x,\eta)$ is an optimal contract, then $w^o(x,-\eta)$ also becomes an optimal contract; and (iii) $\mu_4^o(-b) = \mu_4^o(b)$ for $\forall b$ at the optimum.

**Step 1**. If $w^o(x,\eta)$ is an optimal contract, then $w^o(x,-\eta)$ satisfies all the constraints in (C.7).

(i) As $w^o(x,\eta)$ is optimal, note that it satisfies all of the constraints in (C.7). We start from the incentive compatibility in action $a_1$: based on that $g_1(x,\eta|b=0)$ is even in $\eta$,

$$\begin{aligned}\int u(w^o(x,-\eta))g_1(x,\eta|b=0)dxd\eta - v'(a_1^o) &= \int u(w^o(x,-\eta))g_1(x,-\eta|b=0)dxd\eta - v'(a_1^o)\\ &= \int u(w^o(x,\eta))g_1(x,\eta|b=0)dxd\eta - v'(a_1^o) = 0,\end{aligned}$$

where we use the change of variable (i.e., $-\eta$ to $\eta$) in the second equality.

(ii) Incentive compatibility in action $a_2$: based on that $g_2(x,\eta|b=0)$ is even in $\eta$,

$$\begin{aligned}\int u(w^o(x,-\eta))g_2(x,\eta|b=0)dxd\eta &= \int u(w^o(x,-\eta))g_2(x,-\eta|b=0)dxd\eta\\ &= \int u(w^o(x,\eta))g_2(x,\eta|b=0)dxd\eta = 0.\end{aligned}$$

(iii) Incentive compatibility in *after-hedging* risk exposure $b$: as $w^o(x,\eta)$ is optimal, we know it satisfies

$$\int u(w^o(x,\eta))\left(g(x,\eta|b=0)-g(x,\eta|b)\right)dxd\eta \geq 0,\ \forall \underline{b}. \tag{C.25}$$

From (C.24) and that $g(x,\eta|b=0)$ is even in $\eta$, we obtain for $\forall b$,

$$\begin{aligned}
&\int u(w^o(x,-\eta))\left(g(x,\eta|b=0)-g(x,\eta|b)\right)dxd\eta \\
&\quad= \int u(w^o(x,-\eta))\left(g(x,-\eta|b=0)-g(x,-\eta|-b)\right)dxd\eta \\
&\quad= \int u(w^o(x,\eta))\left(g(x,\eta|b=0)-g(x,\eta|-b)\right)dxd\eta \geq 0,
\end{aligned} \tag{C.26}$$

where the first equality is from (C.24) and the second equality is from the change of variable (i.e., $-\eta$ to $\eta$). Thus, we proved that if $w^o(x,\eta)$ is an optimal contract, then $w^o(x,-\eta)$ satisfies all the constraints in (C.7).

**Step 2**. Next, if $w^o(x,\eta)$ is an optimal contract, then $w^o(x,-\eta)$ also becomes an optimal contract.
From the above Step 1, $w^o(x,-\eta)$ satisfies all the constraints in (C.7). It is sufficient to show that $w^o(x,-\eta)$ achieves the same efficiency as $w^o(x,\eta)$. It follows from:

$$\begin{aligned}
&\int (x-w^o(x,-\eta))g(x,\eta|b=0)dxd\eta + \lambda\left(\int u(w^o(x,-\eta))g(x,\eta|b=0)dxd\eta - v(a_1^o)\right) \\
&\quad= \int (x-w^o(x,-\eta))g(x,-\eta|b=0)dxd\eta + \lambda\left(\int u(w^o(x,-\eta))g(x,-\eta|b=0)dxd\eta - v(a_1^o)\right) \\
&\quad= \int (x-w^o(x,\eta))g(x,\eta|b=0)dxd\eta + \lambda\left(\int u(w^o(x,\eta))g(x,\eta|b=0)dxd\eta - v(a_1^o)\right),
\end{aligned} \tag{C.27}$$

where the first equality is from that $g(x,\eta|b=0)$ is symmetric in $\eta$, and the second equality is from the change of variable (i.e., $-\eta$ to $\eta$). Therefore, if $w^o(x,\eta)$ is an optimal contract, then $w^o(x,-\eta)$ becomes an optimal contract and we obtain $w^o(x,-\eta)=w^o(x,\eta)$.[11]

**Step 3**. $\mu_4^o(-b)=\mu_4^o(b)$ for $\forall b$.
Note from the Lagrange duality theorem (see e.g., Luenberger (1969)) that the optimal

[11] We implicitly assume that the optimal contract is unique in this environment, following the literature (e.g., Jewitt et al. (2008)).

solution $(\mu_1^o, \mu_2^o, \{\mu_4^o(b)\}, w^o(\cdot))$ is the one that solves:

$$\begin{aligned}\min_{\mu_1,\mu_2,\{\mu_4(\cdot)\}} \max_{w(\cdot)} \mathcal{L} \equiv & \int (x - w(x,\eta))g(x,\eta|b=0)dxd\eta + \lambda\left(\int u(w(x,\eta))g(x,\eta|b=0)dxd\eta - v(a_1^o)\right) \\ & + \mu_1\left(\int u(w(x,\eta))g_1(x,\eta|b=0)dxd\eta - v'(a_1^o)\right) + \mu_2\left(\int u(w(x,\eta))g_2(x,\eta|b=0)dxd\eta\right) \\ & + \int_b \mu_4(b)\left(\int u(w(x,\eta))\left(g(x,\eta|b=0) - g(x,\eta|b)\right)dxd\eta\right)db,\end{aligned} \tag{C.28}$$

while satisfying $\mu_4^o(b) \geq 0$ for $\forall b$ and the following complementary slackness condition at the optimum:

$$\mu_4^o(b)\left(\int u(w^o(x,\eta))\left(g(x,\eta|b=0) - g(x,\eta|b)\right)dxd\eta\right) = 0, \quad \forall b. \tag{C.29}$$

The last term in (C.28) given the optimal contract $w^o(x,\eta)$ can be written as

$$\begin{aligned}&\int_b \mu_4(b)\left(\int u(w^o(x,\eta))\left(g(x,\eta|b=0) - g(x,\eta|b)\right)dxd\eta\right)db \\ &= \int_b \mu_4(-b)\left(\int u(w^o(x,-\eta))\left(g(x,\eta|b=0) - g(x,\eta|-b)\right)dxd\eta\right)db,\end{aligned} \tag{C.30}$$

where we use the change of variable (i.e., $b$ to $-b$) and $w^o(x,-\eta) = w^o(x,\eta)$. Now with (C.24) and that $g(x,\eta|b=0)$ is even in $\eta$, we know:

$$\begin{aligned}\int u(w^o(x,-\eta))\left(g(x,\eta|b=0) - g(x,\eta|-b)\right)dxd\eta &= \int u(w^o(x,-\eta))\left(g(x,-\eta|b=0) - g(x,-\eta|b)\right)dxd\eta \\ &= \int u(w^o(x,\eta))\left(g(x,\eta|b=0) - g(x,\eta|b)\right)dxd\eta,\end{aligned} \tag{C.31}$$

where we use the change of variable (i.e., $-\eta$ to $\eta$) for the second equality. With (C.30) and (C.31), the last term in (C.28) can be therefore written as

$$\begin{aligned}&\int_b \mu_4(b)\left(\int u(w^o(x,\eta))\left(g(x,\eta|b=0) - g(x,\eta|b)\right)dxd\eta\right)db \\ &= \int_b \mu_4(-b)\left(\int u(w^o(x,\eta))\left(g(x,\eta|b=0) - g(x,\eta|b)\right)dxd\eta\right)db.\end{aligned} \tag{C.32}$$

Plugging in (C.32) to the original Lagrangian in (C.28) yields $\mu_4^o(-b) = \mu_4^o(b)$.

**Step 4**. We have:

$$\int u(w^o(x,\eta))g(x,\eta|b)dxd\eta = \int u(w^o(x,\eta))g(x,\eta|-b)dxd\eta, \tag{C.33}$$

which implies that the agent's indirect utility is symmetric in $b$ around $b=0$.

It follows from:

$$\begin{aligned}\int u(w^o(x,\eta))g(x,\eta|-b)dxd\eta &= \int u(w^o(x,\eta))g(x,-\eta|b)dxd\eta = \int u(w^o(x,-\eta))g(x,-\eta|b)dxd\eta \\ &= \int u(w^o(x,\eta))g(x,\eta|b)dxd\eta,\end{aligned} \tag{C.34}$$

where we use (C.24) in the first equality, $w^o(x,-\eta)=w^o(x,\eta)$ in the second, and and the change of variable (i.e., $-\eta$ to $\eta$) in the third equality. ■

**Proof of Proposition 12**: Given $(a_1^o, a_2^o)$, we define $\widehat{Cov} \equiv (x-\phi(a_1^o,a_2^o))\eta$.[12] Since

$$\exp\left(\frac{b\eta(x-\phi(a_1^o,a_2^o))}{(a_2^o)^2}\right) = \exp\left(\frac{b}{(a_2^o)^2}\widehat{Cov}\right) = \sum_{k=0}^{\infty}\frac{1}{k!}\frac{b^k}{(a_2^o)^{2k}}\widehat{Cov}^k, \tag{C.35}$$

From equation (C.23), we obtain

$$\frac{g(x,\eta|b)}{g(x,\eta|b=0)} = \left(\sum_{k=0}^{\infty}\frac{1}{k!}\frac{b^k}{(a_2^o)^{2k}}\widehat{Cov}^k\right)\exp\left(-\frac{b^2\eta^2}{2(a_2^o)^2}\right), \tag{C.36}$$

and therefore, we attain

$$\begin{aligned}\int \mu_4^o(b)\left(1-\frac{g(x,\eta|b)}{g(x,\eta|b=0)}\right)db &= \int \mu_4^o(b)db - \int \mu_4^o(b)\left(\sum_{k=0}^{\infty}\frac{1}{k!}\frac{b^k}{(a_2^o)^{2k}}\widehat{Cov}^k\right)\exp\left(-\frac{b^2\eta^2}{2(a_2^o)^2}\right)db \\ &= \int \mu_4^o(b)db - \sum_{k=0}^{\infty}\left(\frac{1}{k!}\frac{1}{(a_2^o)^{2k}}\underbrace{\left(\int \mu_4^o(b)b^k\exp\left(-\frac{b^2\eta^2}{2(a_2^o)^2}\right)db\right)}_{\equiv C_k(\eta)}\right)\widehat{Cov}^k.\end{aligned} \tag{C.37}$$

When $k$ is odd, the coefficient $C_k(\eta)$ becomes 0 for $\forall\eta$, since $\mu_4^o(b)=\mu_4^o(-b)$ for all $b$ from

[12] This is a realized value of sample covariance between $x$ and $\eta$, as our framework is in single-period setting.

Lemma 11 implies

$$C_{k:odd}(\eta) = \int \mu_4^o(b) b^k \exp\left(-\frac{b^2\eta^2}{2(a_2^o)^2}\right) db = \int_{b\geq 0} \left( \underbrace{\mu_4^o(b) - \mu_4^o(-b)}_{=0} \right) b^k \exp\left(-\frac{b^2\eta^2}{2(a_2^o)^2}\right) db = 0. \tag{C.38}$$

When $k$ is even, the coefficient $C_k(\eta)$ becomes strictly positive for $\forall\eta$, since $\mu_4^o(b) \neq 0$ for the non-zero measure of $b$ from Lemma 11 implies

$$\begin{aligned} C_{k:even}(\eta) = \int \mu_4^o(b) b^k \exp\left(-\frac{b^2\eta^2}{2(a_2^o)^2}\right) db &= \int_{b\geq 0} (\mu_4^o(b) + \mu_4^o(-b)) b^k \exp\left(-\frac{b^2\eta^2}{2(a_2^o)^2}\right) db \\ &= 2\int_{b\geq 0} \mu_4^o(b) b^k \exp\left(-\frac{b^2\eta^2}{2(a_2^o)^2}\right) db > 0. \end{aligned} \tag{C.39}$$

Therefore, (C.37) becomes:

$$\begin{aligned} \int \mu_4^o(b)\left(1 - \frac{g(x,\eta|b)}{g(x,\eta|b=0)}\right) db = &\int \mu_4^o(b) db \\ &- 2\sum_{k:even}^{\infty} \left(\frac{1}{k!}\frac{1}{(a_2^o)^{2k}} \left(\int_{b\geq 0} \mu_4^o(b) b^k \exp\left(-\frac{b^2\eta^2}{2(a_2^o)^2}\right) db\right)\right) \widehat{Cov}^k. \end{aligned} \tag{C.40}$$

Finally, we can plug the expression (C.40) into our optimal contact $w^o(x,\eta)$ in (C.8) when $w^o(x,\eta) \geq k$ and obtain

$$\begin{aligned} \frac{1}{u'(w^o(x,\eta))} = &\lambda + (\mu_1^o\phi_1^o + \mu_2^o\phi_2^o)\frac{x - \phi(a_1^o, a_2^o)}{(a_2^o)^2} + \frac{\mu_2^o}{a_2^o}\left(\frac{(x - \phi(a_1^o, a_2^o))^2}{(a_2^o)^2} - 1\right) + \underbrace{\int \mu_4^o(b) db}_{>0} \\ &- 2\underbrace{\sum_{k:\text{even}}^{\infty} \frac{1}{k!}\frac{1}{(a_2^o)^{2k}} \underbrace{\left(\int_{b\geq 0} \mu_4^o(b) b^k \exp\left(-\frac{b^2\eta^2}{2(a_2^o)^2}\right) db\right)}_{\equiv C_{k:\text{even}}(\eta) > 0}}_{\equiv D_{k:\text{even}}(\eta) > 0} \widehat{Cov}^k. \end{aligned} \tag{C.41}$$

Since $D_{k:\text{even}}(\eta) > 0$ for all even numbers $k$, given $(x, \eta)$ a higher $\widehat{Cov}$ results in a lower compensation $w^o(x,\eta)$. Also as $D_{k:\text{even}}(\eta) > 0$ decreases in $\eta^2$, given $(x, \widehat{Cov})$, a higher $\eta^2$ results in a higher $w^o(x,\eta)$. In sum the principal punishes a sample covariance $|\widehat{Cov}|$ but becomes lenient when a high $|\widehat{Cov}|$ comes from the high $\eta$ realization, not from the agent's speculation activity ($b \neq 0$).

**Note**: Let $\rho(b) \equiv \int u(w^o(x,\eta))g(x,\eta|a_1^o, a_2^o, b)dxd\eta - v(a_1^o)$ be the agent's expected indirect utility as a function of $b$. Then from , we obtain $\rho(b) = \rho(0)$ holds for a positive measure of $b$[13] and $\rho(b)$ must be symmetric around $b = 0$ from (C.33) in Lemma 11.

As $b \to \pm\infty$, $\widehat{Cov} \to \pm\infty$ at any realization of $(\theta, \eta)$ since $\widehat{Cov} = b\eta^2 + a_2^o\theta\eta$ and $\eta^2 > 0$. The above optimal contract in (C.41) implies: as $b \to \pm\infty$, we have $w(x,\eta) = w(\phi(a_1^o, a_2^o) + b\eta + a_2^o\theta, \eta) < w(\phi(a_1^o, a_2^o) + a_2^o\theta, \eta)$ uniformly on $(\theta, \eta)$.[14] Thus we have $\rho(b) < \rho(0)$ when $b \to \pm\infty$.

■

**Proof of Proposition 13**: Although we do not explicitly characterize $SW^N$ in (48), we at least see $SW^N$ is a continuous function of $\sigma_R^2$. On the other hand, $w^*(y)$ characterized in (42) and $w^o(x,\eta)$ in equation (C.8) are independent of $\sigma_R^2$, and so are $SW^*(a_1^*, a_2^*)$ and $SW^o(a_1^o, a_2^o, a_3^o = R)$. Thus, as the amount of uncertainty on the firm's risk exposure approaches zero (i.e., $\sigma_R^2 \to 0$), we have

$$SW^*(a_1^*, a_2^*) - SW^N \to 0, \tag{C.42}$$

since the reason $SW^N < SW^*(a_1^*, a_2^*)$ is that the shareholders do not observe the realized $R$ and this informational asymmetry disappears as $\sigma_R^2 \to 0$. As $SW^*(a_1^*, a_2^*) - SW^o(a_1^o, a_2^o, R) > 0$ remains unchanged as $\sigma_R^2 \to 0$, when $\sigma_R^2$ is very small, we have

$$SW^o(a_1^o, a_2^o, a_3^o = R) - SW^N < 0. \tag{C.43}$$

■

[13] Due to the complementary slackness condition (C.29) about the constraint (iii) of the optimization in equation (C.7), $\mu_4^o(b) > 0$ for a positive measure of $b$ in Lemma 11 means $\rho(b) = \rho(0)$ for a positive measure of $b$.

[14] Actually $b \to \pm\infty$ also affects the output $x$ in (C.41). While terms up to a second-order of the output $x$ enter in the optimal contract in (C.41), higher-order terms of $\widehat{Cov}$ clearly dominates the first and second order terms of $x$.