Self-fulfilling Volatility and a New Monetary Policy

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What we do

Standard non-linear New Keynesian model

$\exists$a price of risk coming from

$$\underbrace{\text{Aggregate volatility}\uparrow}_{\text{Endogenous}} \Longrightarrow \underbrace{\text{precautionary saving}}_{\text{Endogenous}}\uparrow \Longrightarrow \underbrace{\text{aggregate demand}\downarrow}_{\text{Endogenous}}$$

Takeaway (**Self-fulfilling volatility**)

In macroeconomic models with nominal rigidities, $\exists$global solution where:

- Taylor rules (targeting inflation and output) $\longrightarrow$ $\exists$self-fulfilling apparition of aggregate volatility
- Only direct volatility (e.g., risk premium) targeting can restore determinacy

Why it's important

New Keynesian models are widely used for policy purposes:

- New equilibria with endogenous aggregate volatility processes – implications for policymaking (growth targeting)
- Can generate extremely persistent processes for output gap deviations
- How? Strong complementarity in household actions, e.g., paradox of thrift

Welfare costs of the business cycle:

- Additional volatility costs
- **First-order costs:** stationary mean of output gap *can be* below its natural counterpart (in the global solution)

A textbook New-Keynesian model with rigid price

The representative household's problem (given B_0):

$$\Gamma_t \equiv \max_{\{B_t\}_{t>0}, \{C_t, L_t\}_{t\geq 0}} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left[\log C_t - \frac{L_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right] dt \quad \text{s.t.} \quad \dot{B}_t = i_t B_t - \bar{p} C_t + w_t L_t + D_t$$

where

- B_t: nominal bond holding, D_t includes fiscal transfer + profits
- Rigid price: $p_t = \bar{p}$ for $\forall t$ (i.e., purely demand-determined)

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Endogenous volatility

A non-linear Euler equation (in contrast to log-linearized one)

$$\mathbb{E}_t \left(\frac{dC_t}{C_t} \right) = (i_t - \rho) dt + \underbrace{\mathrm{Var}_t \left(\frac{dC_t}{C_t} \right)}_{\text{Precautionary premium}}$$

Endogenous drift

Aggregate volatility$\uparrow$ $\Longrightarrow$ precautionary saving$\uparrow$ $\Longrightarrow$ recession (the drift$\uparrow$)

Problem: both **variance** and **drift** are endogenous, is Taylor rule enough?

A textbook New-Keynesian model with rigid price

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where

- B_t: nominal bond holding, D_t includes fiscal transfer + profits
- Rigid price: $p_t = \bar{p}$ for $\forall t$ (i.e., purely demand-determined)

Intra-temporal optimality:

$$\frac{1}{\bar{p} C_t} = \frac{L_t^{\frac{1}{\eta}}}{w_t}$$

Transversality condition:

$$\lim_{t\to\infty} \mathbb{E}_0 \left[e^{-\rho t} \Gamma_t \right] = 0 \tag{1}$$

A textbook New-Keynesian model with rigid price

Firm i: face monopolistic competition à la Dixit-Stiglitz with $Y_t^i = A_t L_t^i$ and

$$\frac{dA_t}{A_t} = g\,dt + \underbrace{\sigma}_{\text{Fundamental risk}} dZ_t$$

- dZ_t: aggregate Brownian motion (i.e., only risk source)
- (g, σ) are exogenous

Flexible price economy as benchmark: the 'natural' output Y_t^n follows

$$\frac{dY_t^n}{Y_t^n} = \left(r^n - \rho + \sigma^2\right) dt + \sigma dZ_t$$
$$= g\,dt + \sigma dZ_t = \frac{dA_t}{A_t}$$

where $r^n = \rho + g - \sigma^2$ is the 'natural' rate of interest

Non-linear IS equation

$$\hat{Y}_t = \ln \frac{Y_t}{Y_t^n}, \quad \underbrace{\left(\sigma \right)^2 dt = \mathrm{Var}_t \left(\frac{dY_t^n}{Y_t^n} \right)}_{\text{Benchmark volatility}}, \quad \underbrace{\left(\sigma + \sigma_t^s \right)^2 dt = \mathrm{Var}_t \left(\frac{dY_t}{Y_t} \right)}_{\text{Actual volatility}}$$

Exogenous

Endogenous

Non-linear IS equation

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Exogenous   Endogenous

A non-linear IS equation (in contrast to textbook linearized one)

$$d\hat{Y}_t = \left(i_t - \underbrace{\left(r^n \overbrace{-\frac{1}{2}(\sigma + \sigma_t^s)^2 + \frac{1}{2}\sigma^2}^{\text{New terms}} \right)}_{\equiv r_t^T} \right) dt + \sigma_t^s \, dZ_t \qquad (2)$$

What is r_t^T?: a risk-adjusted natural rate of interest ($\sigma_t^s \uparrow \Longrightarrow r_t^T \downarrow$)

$$r_t^T \equiv r^n - \frac{1}{2} \underbrace{(\sigma + \sigma_t^s)^2}_{\substack{\text{Precautionary} \\ \text{premium}}} + \frac{1}{2}\sigma^2$$

Non-linear IS equation

Big Question

Taylor rule $i_t = r^n + \phi_y \hat{Y}_t$ for $\phi_y > 0 \Longrightarrow$ perfect stabilization?

Up to a first-order (no volatility feedback): Blanchard and Kahn (1980)

- $\phi_y > 0$: Taylor principle $\Longrightarrow$ $\hat{Y}_t = 0$ with $\sigma_t^s = 0$ for $\forall t$ (unique equilibrium)

Why? (recap): without the volatility feedback:

$$d\hat{Y}_t = (i_t - r^n)\,dt + \sigma_t^s dZ_t \underbrace{=}_{\substack{\text{Under}\\ \text{Taylor rule}}} \phi_y \hat{Y}_t dt + \sigma_t^s dZ_t$$

Then,

$$\mathbb{E}_t\left(d\hat{Y}_t\right) = \phi_y \hat{Y}_t.$$

If $\hat{Y}_t \neq 0$,

$$\lim_{s\to\infty} \mathbb{E}_t\left(\hat{Y}_s\right) \to \pm\infty$$

- Foundation of modern central banking

Now, with the non-linear effects in (2)

Proposition (Fundamental Indeterminacy)

For any $\phi_y > 0$, $\exists$an equilibrium supporting a volatility $\sigma_0^s > 0$ satisfying:

1. $\mathbb{E}_t \left(d\hat{Y}_t \right) = 0$ for $\forall t$ (i.e., local martingale)
2. $\sigma_t^s \xrightarrow{a.s} \sigma_\infty^s = 0$ and $\hat{Y}_t \xrightarrow{a.s} 0$ (i.e., almost sure stabilization)
3. 0^+-possibility divergence or non-uniform integrability given by

$$\mathbb{E}_0 \left(\sup_{t \geq 0} \left(\sigma + \sigma_t^s \right)^2 \right) = \infty$$

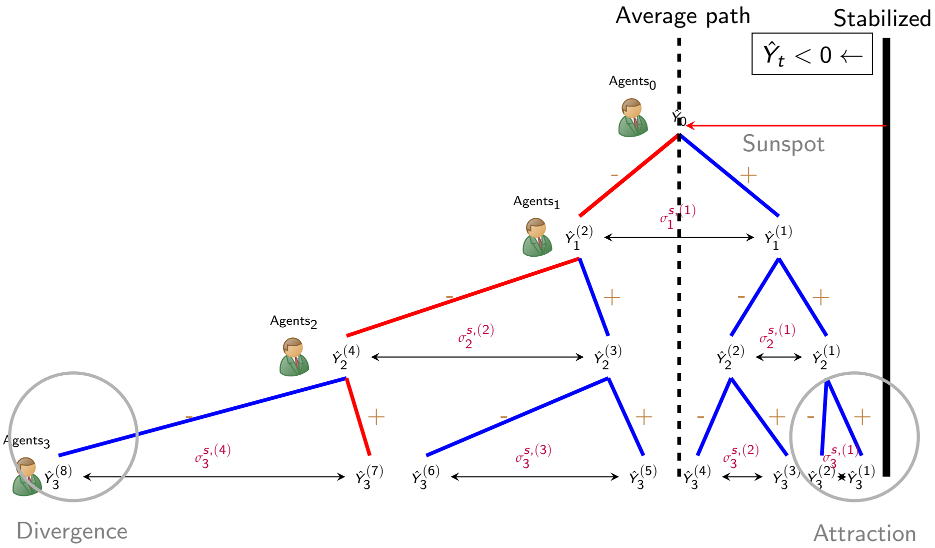
with

$$\lim_{K \to \infty} \sup_{t \geq 0} \left(\mathbb{E}_0 \left(\sigma + \sigma_t^s \right)^2 1_{\left\{ \left(\sigma + \sigma_t^s \right)^2 \geq K \right\}} \right) > 0.$$

Aggregate volatility↑ possible through the intertemporal coordination of agents

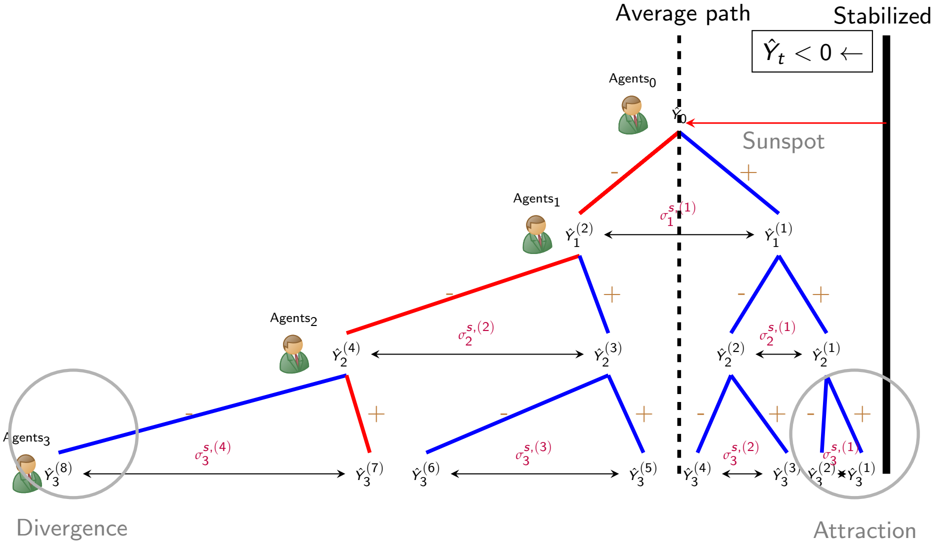
- Called a "martingale equilibrium" - non-stationary equilibrium
- Satisfies the transversality condition (1)

Key: a path-dependent intertemporal aggregate demand strategy



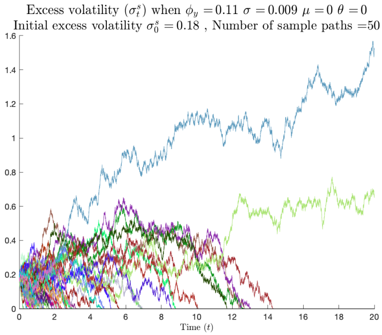
Stabilized as **attractor**: $\sigma_t^s \xrightarrow{a.s} \sigma_\infty^s = 0$ and $\hat{Y}_t \xrightarrow{a.s} 0$

Key: a path-dependent intertemporal aggregate demand strategy

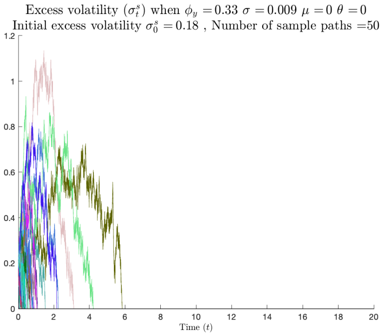


But divergence with 0^+-probability: $\mathbb{E}_0 \left(\sup_{t \geq 0} \left(\sigma + \sigma_t^s \right)^2 \right) = \infty$

Simulation results - martingale equilibrium



(a) With Taylor coefficient $\phi_y = 0.11$



(b) With Taylor coefficient $\phi_y = 0.33$

Figure: Martingale equilibrium: with $\phi_y = 0.11$ (Figure 1a) and $\phi_y = 0.33$ (Figure 1b)

Potential stationary equilibria?

Conjecture: Ornstein-Uhlenbeck process with endogenous volatility $\{\sigma_t^s\}$

$$d\hat{Y}_t = \left(i_t - \underbrace{\left(r^n \overbrace{- \frac{1}{2}(\sigma + \sigma_t^s)^2 + \frac{1}{2}\sigma^2}^{\text{New terms}} \right)}_{\equiv r_t^T} \right) dt + \sigma_t^s \, dZ_t \tag{3}$$

$$= \underbrace{\theta}_{>0} \cdot \left[\underbrace{\mu}_{\gtreqless 0} - \hat{Y}_t \right] dt + \sigma_t^s dZ_t$$

- μ as an *approximate* average of $\hat{Y}_t$
- θ as a speed of mean reversion
- $i_t = r^n + \phi_y \hat{Y}_t$ (i.e., Taylor rule) stays the same

Proposition (Fundamental Indeterminacy)

For $\theta > 0$, $\mu < \frac{\sigma^2}{2\phi_y}$ with $\mu \neq 0$:

1. $\{\sigma_t^s\}$ process satisfying (3) is stable, and admits a unique stationary distribution: with $\sigma \to 0$ and $\mu < 0$, the stationary distribution coincides with the "generalized gamma distribution" $GGD(a, d, p)$, given by

$$a = \sqrt{\frac{2(\theta + \phi_y)^2}{\theta}}, \quad d = -\frac{2\theta\mu\phi_y}{(\theta + \phi_y)^2}, \quad \text{and} \quad p = 2, \tag{4}$$

   where a is the scale parameter, d is the power-law shape parameter, p is the exponential shape parameter.

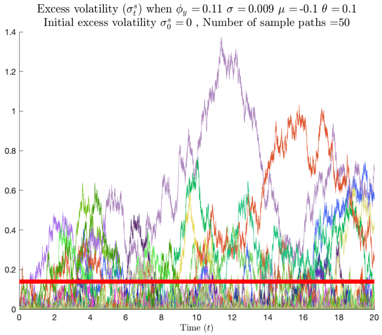
2. For $\theta > 0$ and $\mu = 0$, the σ_t^s process is again non-stationary (degenerate distribution at $\sigma_\infty^s = 0$).

3. The long-run expectations of the output gap $\hat{Y}_t$ and excess variance $(\sigma + \sigma_t^s)^2 - \sigma^2$ are given by

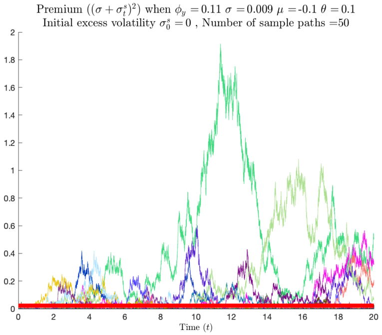
$$\lim_{t\to\infty} \mathbb{E}_0\left[\hat{Y}_t\right] = \mu, \quad \text{and} \quad \lim_{t\to\infty} \mathbb{E}_0\left[(\sigma + \sigma_t^s)^2 - \sigma^2\right] = -2\mu\phi_y.$$

Simulation results - Ornstein-Uhlenbeck equilibrium

With $\theta > 0$, $\mu < 0$



(a) Endogenous volatility σ_t^s



(b) Precautionary premium $(\sigma + \sigma_t^s)^2$

Figure: Ornstein-Uhlenbeck equilibrium: endogenous volatility $\{\sigma_t^s\}$ (Figure 2a) and the precautionary premium$\{(\sigma + \sigma_t^s)^2\}$ (Figure 2b)

- Even with $\sigma_0^s = 0$ (no initial volatility) $\Longrightarrow$ stationary $\{\sigma_t^s\}$ process

Another case

Simulation results - Ornstein-Uhlenbeck equilibrium

With $\theta > 0$, $\mu = 0$

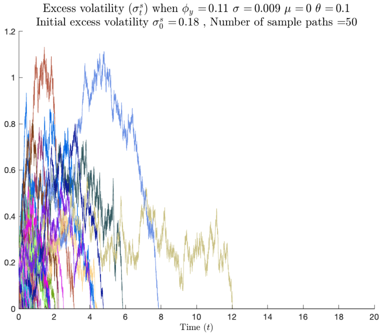


Figure: Endogenous volatility σ_t^s

- Again, degenerate distribution at $\sigma_\infty^s = 0$
- Faster convergence than the martingale equilibrium $(\theta = 0)$

A new monetary policy with volatility targeting

New monetary policy:

$$i_t = r^n + \phi_y \hat{Y}_t - \boxed{\frac{1}{2}\left(\underbrace{(\sigma + \sigma_t^s)^2}_{\equiv \mathrm{pp}_t} - \underbrace{\sigma^2}_{\equiv \mathrm{pp}^n}\right)}$$

Aggregate volatility targeting?

- Restores a **determinacy** and **stabilization**, but what does it mean?

A new monetary policy with volatility targeting

Leading to:

Ito term

Ito term

$$\boxed{i_t + \text{pp}_t - \frac{1}{2}\text{pp}_t} = \boxed{r^n + \text{pp}^n - \frac{1}{2}\text{pp}^n} + \underbrace{\phi_y \hat{Y}_t}_{\text{Business cycle targeting}}$$

$$i_t + \text{pp}_t - \frac{1}{2}\text{pp}_t = \rho + \frac{\mathbb{E}_t(d \log Y_t)}{dt}$$

$$r^n + \text{pp}^n - \frac{1}{2}\text{pp}^n = \rho + \frac{\mathbb{E}_t(d \log Y_t^n)}{dt}$$

- A % change of (i.e., return on) aggregate output (i.e., demand), not just the policy rate, follows Taylor rules

Key issue: monetary policy tool available $\neq$ objective

Model with inflation

Nominal rigidities à la Rotemberg (1982)

$$dp_t^i = \pi_t^i p_t^i \, dt,$$

with adjustment cost of inflation rate π_t^i:

$$\Theta(\pi_t^i) = \frac{\tau}{2}(\pi_t^i)^2 p_t Y_t,$$

New Keynesian Phillips curve:

Volatility of inflation growth

$$d\pi_t = \left[\left[2(\rho + \pi_t) - i_t - (\sigma + \sigma_t^s)(\sigma + \sigma_t^s + \sigma_t^\pi)\right]\pi_t - \left(\frac{\epsilon - 1}{\tau}\right)\left(e^{\left(\frac{\eta+1}{\eta}\right)\hat{Y}_t} - 1\right)\right] dt$$
$$+ \sigma_t^\pi \, \pi_t \, dZ_t,$$

The IS equation then becomes:

$$d\hat{Y}_t = \left[i_t - \pi_t - r_t^T\right] dt + \sigma_t^s \, dZ_t, \tag{5}$$

Taylor rule:

$$i_t = r^n + \phi_y \hat{Y}_t \tag{6}$$

- Transversality given by the same equation (1)

Model with inflation

Proposition (Fundamental Indeterminacy)

The model with sticky prices à la Rotemberg (1982) admits an alternative solution to the benchmark equilibrium given by:

$$\begin{aligned} d\hat{Y}_t &= \theta \left[\mu - \hat{Y}_t\right] \, dt + \sigma_t^s \, dZ_t, \\ \pi_t &= f(\sigma_t^s), \end{aligned} \tag{7}$$

where $f(\cdot)$ is a smooth function of excess volatility σ_t^s. This alternative equilibrium solution exists for any positive degree of price stickiness, as captured by the adjustment rate parameter $\tau > 0$.

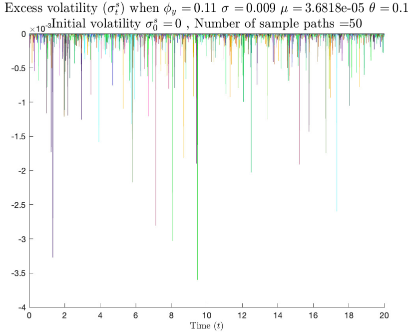
- Similar structure to the Ornstein-Uhlenbeck equilibrium, with π_t as a smooth function of σ_t^s
- Similar in the case of pricing à la Calvo (1983): see Online Appendix G

Thank you very much!

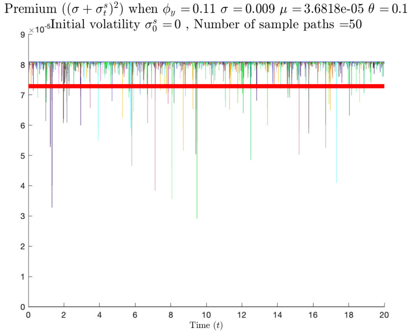
(Appendix)

Simulation results - Ornstein-Uhlenbeck equilibrium

With $0 < \mu < \dfrac{\sigma^2}{2\phi_y}$



(a) Endogenous volatility σ_t^s



(b) Precautionary premium $(\sigma + \sigma_t^s)^2$

Figure: Ornstein-Uhlenbeck equilibrium: endogenous volatility $\{\sigma_t^s\}$ (Figure 4a) and the precautionary premium$\{(\sigma + \sigma_t^s)^2\}$ (Figure 4b)

- Even with $\sigma_0^s = 0$ (no initial volatility) $\Longrightarrow$ stationary $\{\sigma_t^s\}$ process

Go back