Active Taylor Rules Still Breed Sunspots: Sunspot Volatility, Risk-Premium, and the Business Cycle

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HKUST

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Research interest: macroeconomics (macro-finance, monetary), asset pricing, and contract theory. Among my macro works:

With Marc Dordal Carreras (HKUST):
- A Unified Theory of the Term-Structure and Monetary Stabilization
- A Higher-Order Forward Guidance
- Empirical Estimation of Bond Market Segmentation (with Anna Carruthers)
- Entangled AD-AS through firm entry (with Zhenghua Qi)

With other people:
- Heterogeneous Beliefs, Risk Amplification, and Asset Returns (with Goutham Gopalakrishna (EFPL and SFI) and Theofanis Papamichalis (Cambridge))
- Risky Growth with Short-Term Debt (with Artur Doshchyn (Oxford))
“One of the central and most widely shared ideas in the academic finance literature is the importance of time variation in the risk premiums (or expected returns) on a wide range of assets. At the same time, canonical macro models in the New Keynesian genre of the sort that are often used to inform monetary policy tend to exhibit little or no meaningful risk premium variation.”

Jeremy Stein, Governor of the Federal Reserve System (2014)

“Monetary policy should not be the first line of defense on financial stability. We look to more appropriate tools in the first instance, as a first line of defense. And those would be regulation, supervision, high-capital, high-liquidity stress testing, all of those things, macroprudential tools”

Jerome Powell, Chair of the Federal Reserve (2020)
To study a monetary policy’s financial stability concern, turn our eyes to the first statement by Stein (2014) and reframe it into:

1. Canonical finance: risk-premium $\propto$ volatility$^2$ (e.g., Merton (1971))
   - Usually overlooked in a textbook macroeconomic model
   - **Reason**: log-linearized $\implies$ no price of risk ($\simeq$ risk-premium)

2. We study these components seriously in monetary frameworks
   - Need analytical global solutions
What we do + findings

Standard non-linear New-Keynesian model

1. **Show**: proper accounting of a price of risk changes dynamics

   Aggregate volatility↑ ⇐⇒ precautionary saving↑ ⇐⇒ aggregate demand↓

- **Conventional Taylor rules** \(\Rightarrow\) \(\exists\) new indeterminacy (aggregate volatility)
- **Sunspot equilibria**: \(\exists\) sunspot in aggregate volatility: driving business cycles
What we do + findings

Standard non-linear New-Keynesian model

1. **Show**: proper accounting of a price of risk changes dynamics

   Aggregate volatility $\uparrow \iff$ precautionary saving $\uparrow \iff$ aggregate demand $\downarrow$

   - **Conventional Taylor rules** $\implies \exists$ new indeterminacy (aggregate volatility)
   - **Sunspot equilibria**: $\exists$ sunspot in aggregate volatility: driving business cycles

Non-linear New-Keynesian model with a stock market + portfolio

2. **Build** a parsimonious New-Keynesian framework where:

   Stock volatility $\uparrow \iff$ risk-premium $\uparrow \iff$ wealth $\downarrow \iff$ aggregate demand $\downarrow$

   - Asset price as endogenous shifter in aggregate demand (and vice-versa)
   - **VAR analysis**: financial vs real volatility
What we do + findings

Isomorphic structure between two frameworks

- **Conventional Taylor rules** $\Rightarrow$ **Sunspot equilibria** (in stock market volatility): (endogenous) stock market volatility and risk-premium driven business cycle

- **Risk-premium targeting** in a specific way $\Rightarrow$ determinacy again

**Takeaway** (*Ultra-divine coincidence*)

One monetary tool ($i_t$) $\Rightarrow$ (i) inflation, (ii) output, and (iii) risk-premium
- Generalization of the Taylor rule in a risk-centric environment with risk-premium
- Aggregate wealth management of the monetary policy

**Remember:** no bubble $\Rightarrow$ only fundamental asset pricing

Literature review
A non-linear textbook New-Keynesian model (demand block)
A textbook New-Keynesian model with rigid price ($\pi_t = 0, \forall t$)

The representative household’s problem (given $B_0$):

$$\max \{B_t, C_t, L_t\}_{t \geq 0} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left[ \log C_t - \frac{L_t^{\frac{1}{\eta} + \frac{1}{\eta}}}{1 + \frac{1}{\eta}} \right] dt \quad \text{s.t.} \quad \dot{B}_t = i_t B_t - \bar{p} C_t + w_t L_t + D_t$$

where

- $B_t$: nominal bond holding
- $D_t$ includes fiscal transfer + profits of the intermediate sector
- Rigid price: $p_t = \bar{p}$ for $\forall t$ (demand-determined)
A textbook New-Keynesian model with rigid price \((\pi_t = 0, \forall t)\)

The representative household’s problem (given \(B_0\)):

\[
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\]

where

- \(B_t\): nominal bond holding
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- Rigid price: \(p_t = \bar{p}\) for \(\forall t\) (demand-determined)

\[1\] A non-linear Euler equation (in contrast to textbook log-linearized one)

\[
\mathbb{E}_t \left( \frac{dC_t}{C_t} \right) = (i_t - \rho) dt + \text{Var}_t \left( \frac{dC_t}{C_t} \right)
\]

\[2\] (Aggregate) business cycle volatility \(\uparrow\) \(\Rightarrow\) precautionary saving \(\uparrow\) \(\Rightarrow\) recession now (thus the drift \(\uparrow\))

**Problem**: both variance and drift are endogenous, is monetary policy \(i_t\) (Taylor rule) enough for stabilization?
A textbook New-Keynesian model with rigid price ($\pi_t = 0, \forall t$)

**Firm** $i$: face monopolistic competition à la Dixit-Stiglitz with $Y_t^i = A_tL_t^i$ and

$$\frac{dA_t}{A_t} = g\,dt + \sigma dZ_t$$

- $dZ_t$: aggregate Brownian motion (i.e., only risk source)
- $(g, \sigma)$ are exogenous

**Flexible price economy** (benchmark): the ‘natural’ output $Y_t^n$ follows

$$\frac{dY_t^n}{Y_t^n} = \left( r^n - \rho + \sigma^2 \right) dt + \sigma dZ_t$$

$$= gdt + \sigma dZ_t = \frac{dA_t}{A_t}$$

where $r^n = \rho + g - \sigma^2$ is the ‘natural’ rate of interest
A textbook New-Keynesian model with rigid price ($\pi_t = 0, \forall t$)

With

$$\hat{Y}_t = \ln \frac{Y_t}{Y^n_t}, \quad (\sigma)^2 dt = \text{Var}_t \left( \frac{dY^n_t}{Y^n_t} \right),$$

Benchmark volatility

$$\left(\sigma + \sigma_s^t \right)^2 dt = \text{Var}_t \left( \frac{dY_t}{Y_t} \right),$$

Actual volatility

Exogenous

Endogenous

What is $r^T_t$? A risk-adjusted natural rate of interest ($\sigma_s^t \uparrow \Rightarrow r^T_t \downarrow$)
A textbook New-Keynesian model with rigid price ($\pi_t = 0, \forall t$)

With

$$
\dot{\hat{Y}}_t = \ln \frac{Y_t}{Y_t^n}, \quad (\sigma)^2 dt = \text{Var}_t \left( \frac{dY^n_t}{Y^n_t} \right),
$$

Exogenous

$$
(\sigma + \sigma_s^t)^2 dt = \text{Var}_t \left( \frac{dY_t}{Y_t} \right)
$$

Endogenous

Benchmark volatility

Actual volatility

A **non-linear IS equation** (in contrast to textbook linearized one)

$$
d\hat{Y}_t = \left( i_t - \left( r^n - \frac{1}{2}(\sigma + \sigma_s^t)^2 + \frac{1}{2}\sigma^2 \right) \right) dt + \sigma_s^t dZ_t
$$

(1)

- **What is $r^T_t$?** a risk-adjusted natural rate of interest ($\sigma_s^t \uparrow \implies r^T_t \downarrow$)

$$
r^T_t \equiv r^n - \frac{1}{2}(\sigma + \sigma_s^t)^2 + \frac{1}{2}\sigma^2
$$
Big Question: Taylor rule \( i_t = r^n + \phi_y \hat{Y}_t \) for \( \phi_y > 0 \) ⇒ full stabilization?

Up to a first-order (no volatility feedback): Blanchard and Kahn (1980)
- \( \phi_y > 0 \): Taylor principle \( \iff \hat{Y}_t = 0 \) (unique equilibrium)

- What about the higher-order economy?
Big Question: Taylor rule \( i_t = r^n + \phi_y \hat{Y}_t \) for \( \phi_y > 0 \) \( \Rightarrow \) full stabilization?

Up to a first-order (no volatility feedback): Blanchard and Kahn (1980)
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What about the higher-order economy?

<table>
<thead>
<tr>
<th>Proposition (Fundamental Indeterminacy)</th>
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<tbody>
<tr>
<td>For any ( \phi_y &gt; 0 ):</td>
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<tr>
<td>( \exists ) a rational expectations equilibrium that supports a sunspot ( \sigma^s_0 &gt; 0 ) satisfying:</td>
</tr>
<tr>
<td>1. ( \mathbb{E}_t (\hat{Y}_s) = \hat{Y}_t ) for ( \forall s &gt; t ) (martingale)</td>
</tr>
<tr>
<td>2. ( \sigma^s_t \xrightarrow{a.s} \sigma^s_\infty = 0 ) and ( \hat{Y}_t \xrightarrow{a.s} 0 ) (almost sure stabilization)</td>
</tr>
<tr>
<td>3. ( \mathbb{E}<em>0 (\max</em>{t \geq 0} (\sigma^s_t)^2) = \infty ) (0(^+)-possibility divergence)</td>
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</table>

Aggregate volatility↑ possible through the intertemporal coordination of agents
A textbook New-Keynesian model with rigid price ($\pi_t = 0, \forall t$)

**Key:** construct a path-dependent intertemporal consumption (demand) strategy

\[
\hat{Y}_t < 0 \quad \leftarrow \quad \text{Average path}
\]

\[
\hat{Y}_0 \quad \text{Stabilized}
\]

\[
\hat{Y}_t \quad \text{Sunspot}
\]

\[
\hat{Y}_t \quad \text{Attraction}
\]

\[
\hat{Y}_t \quad \text{Divergence}
\]

- **Stabilized as attractor:** $\sigma_t^s \xrightarrow{a.s.} \sigma^s_\infty = 0$ and $\hat{Y}_t \xrightarrow{a.s.} 0$ but $E_0(\max_{t \geq 0} (\sigma_t^s)^2) = \infty$
1. An endogenous aggregate risk arises and drives the business cycle.

2. Sunspots in \( \{\sigma^s_t\} \) act similarly to animal spirit?

3. New monetary policy

\[
i_t = r^n + \phi_y \hat{Y}_t - \frac{1}{2} \left( (\sigma + \sigma^s_t)^2 - \sigma^2 \right)
\]

Aggregate volatility targeting?
Animal spirit targeting?

- Restores a determinacy and stabilization, but what does it mean?

Next: open the stock market, and relate these terms to the risk-premium.
The model with a stock market + portfolio decision
Standard demand-determined environment

\[ \sigma_t^s \uparrow \implies \text{precautionary saving} \uparrow \implies \text{consumption (output)} \downarrow \]

We can build a **theoretical framework with explicit stock markets** where

\[ \text{Financial volatility} \uparrow \implies \text{risk-premium} \uparrow \implies \text{wealth} \downarrow \implies \text{output} \downarrow \]

- Wealth-dependent aggregate demand

- Now, sticky price so \( \pi_t \neq 0 \): Phillips curve à la Calvo (1983)

Skip the detail
Identical **capitalists** and **hand-to-mouth workers** (Two types of agents)

- **Capitalists**: consumption - portfolio decision (between stock and bond)
- **Workers**: supply labors to firms (hand-to-mouth)

1. **Technology**

   \[
   \frac{dA_t}{A_t} = g \cdot dt + \sigma \cdot dZ_t
   \]

   *Growth* \hspace{1cm} *Aggregate shock*

2. **Hand-to-mouth workers**: supply labors + solves the following problem

   \[
   \max_{C_t^w, N_t^w} \left( \frac{C_t^w}{A_t} \right)^{1-\varphi} - \left( \frac{N_t^w}{1+\chi_0} \right) \\
   \text{s.t. } p_t C_t^w = w_t N_t^w
   \]

   - Hand-to-mouth assumption can be relaxed, without changing implications

3. **Firms**: production using labors + pricing à la Calvo (1983)

4. **Financial market**: zero net-supplied risk-free bond + stock (index) market
**Capitalists**: standard portfolio and consumption decisions (very simple)

1. Total financial wealth $a_t = p_t A_t Q_t$, where (real) stock price $Q_t$ follows:

   $\frac{dQ_t}{Q_t} = \mu^q_t \cdot dt + \sigma^q_t \cdot dZ_t$

   - $\mu^q_t$ and $\sigma^q_t$ are both endogenous (to be determined)

2. Each solves the following optimization (standard)

   $$\max_{C_t, \theta_t} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \log C_t \, dt \quad \text{s.t.}$$

   $$da_t = (a_t(i_t + \theta_t(i^m_t - i_t)) - p_t C_t) \, dt + \theta_t a_t(\sigma + \sigma^q_t) \, dZ_t$$

   - Aggregate consumption of capitalists $\propto$ aggregate financial wealth
     $$C_t = \rho A_t Q_t$$
   - Equilibrium **risk-premium** is determined by the total risk
     $$i^m_t - i_t \equiv rp_t = (\sigma + \sigma^q_t)^2$$
Other equilibrium conditions

**Dividend yield:** dividend yield $= \rho$, as in Caballero and Simsek (2020)

- A positive feedback loop between asset price $\iff$ dividend (output)

**Determination of nominal stock return $dI^m_t$**

$$dI^m_t = \left[ \rho + \pi_t + g + \mu^q_t + \sigma\sigma^q_t \right] dt + (\sigma + \sigma^q_t) dZ_t$$

- Dividend yield
- Inflation
- Capital gain
- Drift
- Monetary policy
- Risk-premium

- Close the model with supply-side (Phillips curve) and $\{i_t\}$ rule
Flexible price economy allocations (benchmark)

- $\sigma_{t}^{q,n} = 0, Q_{t}^{n}, N_{W,t}^{n}, C_{t}^{n}, r^{n}$ (natural rate), $rp^{n}$ (natural risk-premium)

Gap economy (log deviation from the flexible price economy)

- With asset price gap $\hat{Q}_{t} \equiv \ln \frac{Q_{t}^{n}}{Q_{t}} = \hat{C}_{t}$ and $\pi_{t}$

Proposition (Dynamic IS)

A dynamic gap economy can be described with the following equations:

1. $\mathbb{E}_{t}d\pi_{t} = (\rho \pi_{t} - \kappa \hat{Q}_{t})dt$ with $\kappa > 0$

2. $d\hat{Q}_{t} = (i_{t} - \pi_{t} - \left( r_{T}^{T} \right))dt + \sigma_{t}^{q} dZ_{t}$ where $r_{T}^{T} = r^{n} - \frac{1}{2} (rp_{t} - rp^{n})$

$$\equiv r^{n} - \frac{1}{2} \hat{r}p_{t}$$

where $rp_{t} = (\sigma + \sigma_{t}^{q})^{2}$ and $rp^{n} = \sigma^{2} \implies \hat{r}p_{t} \equiv rp_{t} - rp^{n}$
Isomorphism: the same mathematical structure

Now, with asset (stock) price gap $\hat{Q}_t$:

$$d\hat{Q}_t = \left( i_t - \pi_t - \left( r^n - \frac{1}{2} \sigma^2 \right) \right) dt + \sigma^q_t dZ_t$$

(2)

Here

$$\sigma^q_t \uparrow \implies r_p \uparrow \implies \hat{Q}_t \downarrow \implies \hat{Y}_t \downarrow$$

Monetary policy: Taylor rule to Bernanke and Gertler (2000) rule

$$i_t = r^n + \phi_\pi \pi_t + \phi_y \hat{y}_t$$

$$= \zeta \hat{Q}_t$$

$$= r^n + \phi_\pi \pi_t + \phi_q \hat{Q}_t, \text{ where } \phi = \phi_q + \frac{\kappa(\phi_\pi - 1)}{\rho} > 0$$

Taylor principle
Bernanke and Gertler (2000) rule and indeterminacy

Multiple equilibria (risk-premium sunspot)

- How?: countercyclical risk-premium with conventional Taylor rules

1. a fear of $\sigma_q^t$, $r_p^t$ ↑
2. Demand for stock ↓
3. Asset price ↓
4. Actual $i_t^m$ ↑ (self-fulfilling $r_p^t$ ↑)
Is a sunspot $\sigma_0^q \neq 0$ supported by a rational expectations equilibrium? 
: with Bernanke and Gertler (2000) rule

Assume $\sigma_0^q > 0$ for some reason (initial disruption)

- The same **martingale equilibrium**

  ![Mathematical explanation](math_explanation.png)  ![Tree diagram](tree_diagram.png)
Is a sunspot $\sigma^q_0 \neq 0$ supported by a rational expectations equilibrium? 
with Bernanke and Gertler (2000) rule

Assume $\sigma^q_0 > 0$ for some reason (initial disruption)

- The same martingale equilibrium  

Proposition (Fundamental Indeterminacy)

For any $\phi > 0$:

$\exists$ a rational expectations equilibrium that supports a sunspot $\sigma^q_0 > 0$ satisfying:

1. $\sigma^q_t \xrightarrow{a.s.} \sigma^q_\infty = 0$, $\dot{Q}_t \xrightarrow{a.s.} 0$, and $\pi_t \xrightarrow{a.s.} 0$ (almost sure stabilization)

2. $\mathbb{E}_0(\max_{t \geq 0}(\sigma^q_t)^2) = \infty$ ($0^+$-possibility divergence)

(Almost surely) stabilized in the long run after sunspot $\sigma^q_0 > 0$

Meantime: crisis with financial volatility (risk-premium)↑, asset price↓, and business cycle↓

$\mathbb{E}_0(\max_t(\sigma^q_t)^2) = \infty$: an $\epsilon \to 0$ possibility of $\infty$-severity crisis ($\sigma^q_t \to \infty$)

- $\exists$ big crisis that supports $\sigma^q_0 > 0$ (e.g., Martin (2012) in asset pricing contexts)
(a) With $\phi_\pi = 1.5$
(a) With \( \phi_{\pi} = 1.5 \)

(b) With \( \phi_{\pi} = 2.5 \).

Figure: \( \{\sigma^q_t, \hat{Q}_t\} \) dynamics when \( \sigma^{q,n} = 0 \) and \( \sigma^q_0 = 0.9 \), with reasonable calibration

- As monetary policy responsiveness \( \phi \uparrow \)
  - Stabilization speed \( \uparrow \), \( \exists \) more severe crisis sample path
- \( \sigma^q_t \uparrow \) by \( \sigma \implies 2 - 10\% \downarrow \) in \( Q_t \) (depending on monetary responsiveness)
With $\phi = 1.5$

(a) With $\phi = 1.5$

(b) With $\phi = 2.5$

Figure: $\{\sigma^q_t, \hat{Q}_t\}$ dynamics when $\sigma_{n,t} = 0$ and $\sigma_0^q = 0.9$, with reasonable calibration

- As monetary policy responsiveness $\phi \uparrow$
  - Stabilization speed $\uparrow$, $\exists$ more severe crisis sample path
- $\sigma^q_t \uparrow$ by $\sigma \implies 2 - 10\% \downarrow$ in $Q_t$ (depending on monetary responsiveness)

Opposite case: with initial sunspot $\sigma_0^q < 0$

- Explains boom phase

Financial volatility (risk-premium) $\downarrow$, asset price $\uparrow$ and business cycle $\uparrow$
A modified monetary rule: targeting of risk-premium

New monetary policy $\implies$ financial + macro stabilities $\hat{Q}_t = \pi_t = \hat{r}p_t = 0$

New targeting

$$i_t = r^n + \phi\pi\pi_t + \phi_q\hat{Q}_t - \frac{1}{2} \hat{r}p_t,$$

where $\phi \equiv \phi_q + \frac{\kappa(\phi\pi - 1)}{\rho} > 0$

restores a determinacy with:

Takeaway (Ultra-divine coincidence)

One monetary tool ($i_t$) $\implies$ (i) inflation, (ii) output, and (iii) risk-premium
A modified monetary rule: targeting of risk-premium

Leading to:

\[ i_t + rp_t - \frac{1}{2} rp_t = i_{t}^{m} \]

\[ \rho + \mathbb{E}_t(\text{d} \log a_t) \]

\[ r^n + rp^n - \frac{1}{2} rp^n = i_{t}^{m,n} \]

\[ \rho + \mathbb{E}_t(\text{d} \log a^n_t) \]

\[ \phi \pi \pi_t + \phi_q \hat{Q}_t \]

Business cycle targeting

- \( i_{t}^{m} \), not \( i_t \), follows a Taylor rule?

- A rate of change of log-wealth follows a Taylor rule both in standard model (without risk-premium) and our framework (with risk-premium)
Thank you very much!
(Appendix)
1. Volatility↑
2. Risk premium↑
3. Wealth↓
4. Economy↓

1 → 2 comes from “non-linearity (not linearizing)”
2 → 3 comes from “portfolio decision” of each investor and externality
3 → 4 comes from the fact wealth drives aggregate demand
4 → 1 where business cycle has its own volatility (self-sustaining)
Financial volatility measures

The correlation between series is remarkably high and they all display positive spikes at the beginning and/or initial months following NBER-dated recessions

- Many of past recessions are, in nature, financial

**Figure:** Common measures of the financial volatility (left) and real vs. financial uncertainty decomposed by Ludvigson et al. (2015) (right)
In a similar manner to Bloom (2009), Ludvigson et al. (2015):

\[
\begin{bmatrix}
\log (\text{Industrial Production}) \\
\log (\text{Employment}) \\
\log (\text{Real Consumption}) \\
\log (\text{CPI}) \\
\log (\text{Wages}) \\
\text{Hours} \\
\text{Real Uncertainty (LMN)} \\
\text{Fed Funds Rate} \\
\log (\text{M2}) \\
\log (\text{S&P-500 Index}) \\
\text{Financial Uncertainty (LMN)}
\end{bmatrix}
\]

(3)

Financial uncertainty (LMN) is also replaced by the stock price volatility (following Bloom (2009)) and Baa 10-years bond premia.
Vector Autoregression (VAR) analysis

(a) Response: Industrial Production

(b) Industrial Production

Figure: Impulse-response of IP to one std.dev shock in financial uncertainty measures (left) and the historical decomposition of IP to various attributes (right)

1. IP falls by 2.5% after one standard deviation spike in the Ludvigson et al. (2015)’s financial uncertainty measure
   - Financial uncertainty has been important in driving IP boom-bust patterns

2. Other graphs: IRF and historical decomposition of S&P 500, FFR (monetary policy), FEVD
IRF and historical decomposition of S&P500 index

(a) Response: S&P-500 Index

(b) S&P-500 Index
IRF of FFR in response to financial and real uncertainty shocks

(a) Shock: Financial Uncertainty

(b) Shock: Real Uncertainty

With 3 different financial uncertainty measures: Ludvigson et al. (2015), Bloom (2009), Baa 10-years bond premia (left)
Forecast Error Variance Decomposition (FEVD) of IP, S&P500, FFR

(i) Industrial Production

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(ii) S&P-500 Index

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(iii) Fed Funds Rate

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Financial uncertainty shocks explain close to:

- 5% of the fluctuations in both IP and S&P-500 series

Real uncertainty explains:

- Additional 2-4% of movements in industrial activity in the medium run
Key previous works (only a few among many)

- Financial wealth (e.g., risk-intolerance) and aggregate demand: Mian and Sufi (2014), Caballero and Farhi (2017), Guerrieri and Lacoviello (2017), Caballero and Simsek (2020a, 2020b), Chodorow-Reich et al. (2021), Caballero et al. (2021)

- Financial disruption (volatility) and macroeconomy: Gilchrist and Zakrajšek (2012), Brunnermeir and Sannikov (2014), Guerrieri and Lorenzoni (2017), Di Tella and Hall (2020)

Our paper: a monetary framework that incorporates financial wealth, aggregate financial volatility, risk-premium, and business cycle (all endogenous)


Our paper: a monetary policy’s financial targeting (first and second-orders) in the world without bubble + lean against the stock market


- Indeterminacy with an idiosyncratic risk: Acharya and Dogra (2020)

Our paper: an analytical expression of time-varying risk-premium in a monetary model + new indeterminacy in aggregate volatility
1. Capitalists bear \((\sigma_t + \sigma_t^q)\) amount of risks when investing in stock market
   * Risk-premium \(r_{pt} = (\sigma_t + \sigma_t^q)^2\)
   * Natural risk-premium (in the flexible price economy) \(r_{pn} = (\sigma_t + \sigma_t^{q,n})^2 = 0\)

2. If a real return on stock investment is different from its natural level (return of stock investment in the flexible price economy), then \(\hat{Q}_t\) jumps

**Takeaway (Risk-adjusted natural rate)**

\(r_{Tt}\) is a real risk-free rate that makes:

stock market’s real return (with risk-premium \(r_{pt}\)) = natural economy’s (with risk-premium \(r_{pn}\))

\[
\left( r_{Tt} + r_{pt} \right) - \frac{1}{2} r_{pt} = \left( r_{pn} + r_{pn} \right) - \frac{1}{2} r_{pn}
\]
Is a sunspot $\sigma_0^q \neq \sigma^{q,n}$ supported by a rational expectations equilibrium? : with Bernanke-Gertler (2000) rule

Assume $\sigma_0^q > \sigma^{q,n} = 0$ for some reason (initial sunspot)

Blanchard and Kahn (1980) does not apply: we construct a rational expectations equilibrium (REE: not diverging on average) supporting an initial sunspot $\sigma_0^q$

\[
d\hat{Q}_t = \left( i_t - \pi_t - \left( r_t^n - \frac{1}{2}(rp_t - rp_t^n) \right) \right) dt + \sigma_t^q dZ_t
\]

\[
= \left( (\phi \pi - 1)\pi_t + \phi_q \hat{Q}_t + \frac{1}{2}(rp_t - rp_t^n) \right) dt + \sigma_t^q dZ_t
\]

\[
= 0, \quad \forall t
\]

- Called the ‘martingale equilibrium’: supporting an initial sunspot in financial volatility $\sigma_0^q$

- $\{\sigma_t^q\}$ has its own (endogenous) stochastic process, given initial $\sigma_0^q \neq 0$

\[
d\sigma_t^q = -\frac{\phi^2(\sigma_t^q)^2}{2(\sigma_t + \sigma_t^q)^3} dt - \phi \frac{\sigma_t^q}{\sigma_t + \sigma_t^q} dZ_t
\]
Again, the same structure
Asset price \( \{ q_t \} \) and the conditional volatility \( \{ \sigma^q_t \} \) are stochastic.

- Rational expectations equilibrium (REE): no divergence on expectation.
- As \( q_t \) approaches the stabilized path, then \( \sigma^q_t \downarrow \), and more likely stays there: convergence (\( \sigma^q_t \overset{a.s.}{\to} \sigma^q_\infty = \sigma^{q.n} = 0 \)).
- But in the worst scenario \( \sigma^q_t \) diverges (with \( 0^+ \)-probability).
What if central bank uses the following alternative rule, where $\phi_{rp} \neq \frac{1}{2}$?

$$i_t = r^n_t + \phi_\pi \pi_t + \phi_q \hat{Q}_t - \phi_{rp} \hat{r}_p, \text{ where } \phi \equiv \phi_q + \frac{\kappa(\phi_\pi - 1)}{\rho} > 0$$

- Then still $\exists$ martingale equilibrium supporting sunspot $\sigma_0^q \neq 0$
- As $|\phi_{rp} - \frac{1}{2}| \uparrow \implies$ (on average) longer time for $\sigma_t^q$ to vanish
- Especially, $\phi_{rp} < 0$ (Real Bills Doctrine) is a bad idea
When $\phi_{rp}$ deviates from $\frac{1}{2}$

<table>
<thead>
<tr>
<th>$\phi_{rp} &lt; 0$ (Real Bills Doctrine)</th>
<th>$0 &lt; \phi_{rp} &lt; \frac{1}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>As $\phi \uparrow$, convergence speed $\uparrow$ and $\exists$ more amplified paths</td>
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</table>
When $\phi_{rp}$ deviates from $\frac{1}{2}$

(a) With $\phi_{rp} = 1$

(b) With $\phi_{rp} = 1.5$.

Figure: $\{\sigma^q_t, \hat{Q}_t\}$ dynamics when $\sigma^{q,n} = 0$ and $\sigma^q_0 = 0.9$, with varying $\phi_{rp} > \frac{1}{2}$