Endogenous Technology Adoption in a Search Economy*

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Abstract

We develop a general-equilibrium model of endogenous technology adoption in an economy with product-market search frictions. Under frictional search, the static pricing game among heterogeneous firms producing the same variety yields variable markups represented by a gamma hazard function, resulting in within-variety profit dispersion that drives adoption incentives and economic growth—the mechanism absent under frictionless search. Strikingly, when imperfect search is the only friction, its degree does not affect growth, as the partial-equilibrium effect on markups is exactly offset by the general-equilibrium effect on demand. Incorporating additional features such as entry, search effort, and creative destruction creates a wedge between the two effects and allows search frictions to have dynamic implications. Our quantitative analysis of the U.S. economy shows that structural changes in search efficiency and the right tail of the productivity distribution over the past three decades have substantially raised welfare through technology adoption. The current U.S. productivity growth slowdown thus implies a sizable increase in adoption barriers.

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1 Introduction

Since the seminal work by Aghion and Howitt (1992), creative destruction has become a key framework for understanding economic growth fueled by technological progress:[1] The monopoly rents of an incumbent motivate potential entrants to pursue breakthrough innovation, thereby advancing the technology frontier. However, in this winner-take-all environment, firms that fall behind remain inactive in the market and have little incentive to *narrow* the gap with the leader through costly technology adoption. As a result, models grounded in creative destruction have difficulty accounting for endogenous adoption decisions of firms. Technology adoption, however, is increasingly recognized as a crucial driver of economic growth.[2]

This paper studies a setting in which adoption incentives of within-variety firms arise naturally: an economy with search frictions between buyers and sellers. Frictional search allows heterogeneous firms producing an identical variety to coexist in the market while charging different prices, thereby generating profit dispersion across firms with different productivity levels,[3] which in turn incentivizes technology laggards to engage in costly adoption. We characterize this process in a general-equilibrium growth model that integrates search frictions and endogenous technology adoption decisions. Our framework is parsimonious and tractable, yet flexible enough to accommodate rich heterogeneity. We then take U.S. data to different model variants and quantify changes in welfare over the past three decades attributable to structural changes in search efficiency and the right tail of the productivity distribution.

Our model comprises a static and a dynamic block. In the static block, there is a unit continuum of differentiated varieties, where *each* variety is produced by a continuum of firms with Pareto-distributed productivity. For each variety, buyers and sellers face search frictions characterized by Poisson arrivals of encounters, restricting each buyer to a randomly drawn subset of sellers in the economy. Given the subset of sellers encountered, the buyer matches with and purchases from the seller offering the lowest price, under

[1] For recent works on creative destruction and its implications on growth, see, e.g., Klette and Kortum (2004), Aghion et al. (2005), Lentz and Mortensen (2008), Aghion et al. (2019), Garcia-Macia et al. (2019), Peters (2020), and De Ridder (2024).

[2] See, e.g., Perla et al. (2021), Akcigit and Ates (2023), and Kalyani et al. (2025).

[3] For foundational theoretical works on market structures with search frictions, see, e.g., Stigler (1961), Butters (1977), Varian (1980), and Burdett and Judd (1983). For empirical evidence on within-variety price dispersion, see, e.g., Sorensen (2000), Baye et al. (2004), and Hortaçsu and Syverson (2004).

constant elasticity of substitution (CES) preferences.

Firms engage in price competition à la Bertrand, choosing optimal prices to maximize expected profits before the realization of encounters and matches. For analytical tractability, we follow the creative destruction literature in assuming unit elasticity of substitution across varieties.[4] Under this assumption, the pricing problem of firms reduces to a Riccati equation that characterizes the trade-off between profit margin and the expected number of buyers. Solving this equation yields variable markups represented by a gamma hazard function. We show that this markup representation is a strictly decreasing function of marginal cost, translates into an easily interpretable decomposition of incomplete pass-through, and generates firm profits that strictly increase with productivity.

The static results naturally give rise to a distinct *within-variety* technology adoption incentive for less productive firms in the search economy. At the extensive margin, search frictions result in strictly positive market power and profits for all firms producing the same variety. Meanwhile, at the intensive margin, buyer-seller matching based on the lowest price preserves the importance of technological advantage in determining firm profits.

The dynamic block of our model characterizes a firm's adoption decision as a trade-off between continuing production and paying a fixed labor cost to draw a new technology from the current productivity distribution. The monotonicity of firm profits reduces this decision to an optimal stopping problem, in which only firms at the lower bound of the within-variety productivity distribution choose to adopt. Specifically, the expected gain in production value stimulates the adoption incentive of the least productive firms. Their adoption, in turn, shifts the lower bound of the productivity distribution upward. As the lower bound rises, firms that continue production face diminishing technological advantages. The resulting decline in their production value then feeds back into the expected payoff from adoption and reshapes the adoption incentives. The economy's aggregate growth rate along the balanced growth path (BGP) is determined by this process.

Closing the model in general equilibrium delivers the central insight of the paper. In particular, with only exogenous search frictions, our baseline model exhibits a sharp *discontinuity* in the effects of search frictions on technology adoption and growth: While

[4] For previous works that impose this assumption, see, e.g., Aghion et al. (2005), Peters (2020), Aghion et al. (2023), Akcigit and Ates (2023), and De Ridder (2024).

the presence of search frictions gives rise to adoption incentives, the magnitude of those frictions does not affect the adoption decisions of firms as long as it is strictly positive. The tractability of our model makes it possible to pinpoint this result as the consequence of the offsetting partial- and general-equilibrium forces. On the one hand, higher search frictions increase overall market power and raise variable markups, thereby strengthening adoption incentives for low-productivity firms. On the other hand, they exacerbate within-variety labor misallocation and depress aggregate demand, thereby discouraging adoption.

Therefore, for search frictions to affect technology adoption and economic growth, it is essential to incorporate additional features that create a *wedge* between the two forces. We explore three different scenarios. The first scenario allows potential firms to decide whether to enter a product market, factoring in a fixed entry cost. The second allows sellers of each variety to endogenously choose their optimal search effort, subject to a convex search cost function. The third introduces a Pareto productivity distribution bounded both above and below, which allows technology adoption and creative destruction to coexist. Our analytical results reveal that search frictions have dynamic implications across all three extensions, with the underlying mechanisms closely tied to the particular scenario in question.

As a final step, we take U.S. data to different model variants to quantify the BGP welfare effects of two notable structural changes over the past three decades: the improving search efficiency and the thickening right tail of the productivity distribution. The closed-form representation of variable markups, together with its isomorphic structure across all model extensions, allows us to efficiently estimate these parameter changes using the generalized method of moments (GMM).

Consistent with the analytical results, our quantitative analysis shows that the welfare effect of the secular reduction in search frictions largely depends on the mechanisms driving its dynamic impact on technology adoption and growth. In the baseline economy where search frictions have no dynamic impact, the welfare effect is only 3.03%, entirely driven by static consumption gains. When entry is endogenous, the decline in search frictions discourages within-variety firm entry, raising demand faced by incumbent firms. This channel, in turn, incentivizes low-productivity firms to adopt better technologies, leading to faster economic growth and a much larger welfare gain of 30.91%. By contrast, when search comes at a labor cost, the improvement in search technology induces all firms to allocate more labor to search, including those with low productivity. This channel diverts labor away from production, contracts demand, and weakens adoption incentives, resulting in a 15.85% *decline* in BGP welfare. Finally, when creative destruction is introduced, the decrease in search frictions only slightly weakens incentives to adopt. Combined with the static impact, the implied welfare gain is 1.10%.

As for the fatter right tail of the productivity distribution, it raises aggregate demand by reallocating within-variety production labor from low- to high-productivity firms. We find that this general-equilibrium effect substantially strengthens the adoption incentives of low-productivity firms, generating quantitatively large increases in aggregate growth and welfare across all model variants. To reconcile this effect with the observed slowdown in U.S. productivity growth, our model implies a sizable increase in the technology adoption cost over the past three decades, ranging from 44.73% to 120.61%.

Related literature. This paper contributes to the theoretical literature on search frictions in product markets. This literature typically relies on search frictions between buyers and sellers to microfound imperfectly competitive market structures and characterize the resulting static equilibrium price dispersion (Stigler, 1961; Butters, 1977; Varian, 1980; Burdett and Judd, 1983; Janssen and Moraga-González, 2004; Ellison and Ellison, 2009; Choi et al., 2018; Menzio, 2024a,b). Recent studies that incorporate market dynamics focus on product design and targeted advertising driven by consumers' idiosyncratic preferences (Cavenaile et al., 2023; Menzio, 2023). We instead shift the focus to production technology, highlighting how search frictions give rise to endogenous technology adoption decisions of heterogeneous firms within the same product market.

The static block of our model builds on Menzio (2024a,b), who studies how search frictions shape the equilibrium markup distribution for a single good in a partial-equilibrium framework. We differ from Menzio (2024a,b) in four aspects. First, we allow for a continuum of imperfectly substitutable varieties, with each variety served by a continuum of firms. Second, we parameterize firm heterogeneity using a Pareto productivity distribution. In particular, by embedding this Pareto assumption, together with unit elasticity of substitution across varieties, into Poisson search frictions, we deliver a closed-form solution for variable markups characterized by a gamma hazard function. This representation yields many desirable properties and greatly simplifies structural estimation. Third, we study general equilibrium and derive analytical solutions for all aggregate variables in our economy, relying on a microfounded Weibull distribution for the buyer-side minimum encountered price. More importantly, our results show that the general-equilibrium effect of search frictions on aggregate demand is a key determinant of adoption incentives. Finally, in one of our model variants, we extend Menzio (2024a,b) by allowing firms to choose their optimal search effort endogenously, rather than taking search frictions as given.

We also contribute to the theoretical literature on technology diffusion and adoption. While a large body of work treats agents as passively exposed to technological externalities (Kortum, 1997; Luttmer, 2007, 2011; Bloom et al., 2013; Buera and Oberfield, 2020), our model speaks to active decisions of firms to adopt better technology at a cost. This strand of literature starts with stylized models of Lucas and Moll (2014) and Perla and Tonetti (2014), and is incorporated into the open-economy monopolistic-competition framework à la Melitz (2003) by Sampson (2016) and Perla et al. (2021).

The dynamic block of our model builds on Perla et al. (2021), who characterize technology adoption as paying a fixed cost to draw a higher productivity level from the current productivity distribution. We differ from Perla et al. (2021) in two key aspects. Conceptually, under the implicit assumption of perfect search, monopolistic competition implies that each variety is produced only by its most efficient producer. Therefore, Perla et al. (2021) in fact study an adoption process in which the adopter randomly draws and learns from the frontier technology of a different variety, while continuing to produce its original variety. In contrast, by introducing search frictions, we allow firms with different productivity levels to coexist in the same product market. Technology adoption can therefore be interpreted more naturally as learning and imitation among producers within a variety. Analytically, connecting a search-theoretic market structure to endogenous adoption decisions yields new and surprising insights. For example, the offsetting partial- and general-equilibrium effects of search frictions on adoption incentives, as well as the mechanisms in our model extensions that allow search frictions to have dynamic implications, are all novel to the literature on technology adoption.

Insights from our model extensions connect to several strands of the literature. The mechanism that a reduction in search frictions indirectly stimulates adoption incentives of incumbents, by discouraging within-variety firm entry, contributes to the literature on endogenous entry (e.g., Hopenhayn, 1992; Melitz, 2003). When search is costly, the negative general-equilibrium effects of improved search technology on the allocative efficiency of search labor and adoption incentives add to recent theoretical works that incorporate endogenous search efforts in product markets (e.g., Allen, 2014; Arkolakis et al., 2025). By combining search frictions, technology adoption, and creative destruction in a tractable way, we also speak to the literature on the interaction between adoption and innovation (e.g., Benhabib et al., 2021; Trouvain, 2024).

Finally, our structural estimation indicates a secular decline in the shape parameter of the Pareto productivity distribution, implying a thickening right tail consistent with empirical evidence (Autor et al., 2020; Chen, 2023; Kwon et al., 2024). Across all model variants, this structural change substantially promotes technology adoption and economic growth. Given the well-documented slowdown in U.S. productivity growth (Bloom et al., 2020; Akcigit and Ates, 2021, 2023; Goldin et al., 2024), our quantitative analysis points to a sizable increase in barriers to technology adoption, in tandem with the rise of superstar firms. Recently, a growing body of empirical evidence shows that large incumbents use various tactics to stifle technology adoption by their smaller competitors, including strategic patenting (Hall et al., 2021; Akcigit and Ates, 2023; Argente et al., 2025), litigation (Galasso and Schankerman, 2015), and restrictions on labor mobility (Akcigit and Ates, 2023; Akcigit and Goldschlag, 2023; Fernández-Villaverde et al., 2025). Our result is consistent with this literature.

Layout The remainder of the paper is organized as follows. Section 2 introduces the baseline model and defines the BGP equilibrium. Section 3 solves the static expected profit maximization problem of firms and characterizes equations for variable markups and firm profits. Section 4 solves the dynamic technology adoption problem of firms and derives firm value. Section 5 closes the baseline model by characterizing aggregate variables, real GDP growth rate, and welfare. Section 6 incorporates additional features into the baseline model, including endogenous entry, endogenous search effort, and creative destruction. Section 7 takes U.S. data to different model variants for quantitative welfare analysis. Section 8 concludes and discusses the future research agenda. Appendix A contains proofs and derivations. Appendix B provides detailed data description, methodology, and discussion of our markup estimation in Section 7.

2 Model

In this section, we introduce the baseline search economy with endogenous technology adoption and define the BGP equilibrium.

Our model consists of a static block and a dynamic block. The static block builds on Menzio (2024a,b), who focuses on the Poisson search frictions between buyers and sellers of a given good. Unlike the single-good partial-equilibrium environment of Menzio (2024a,b), our framework allows for *imperfect substitutability* across varieties and examines *general equilibrium*.

The dynamic block characterizes endogenous technology adoption decisions of firms, following Perla et al. (2021). In contrast to Perla et al. (2021) who focus on monopolistic competition, the market structure in our search economy allows heterogeneous firms producing the same variety to coexist, giving rise to a distinct *within-variety* incentive for technology adoption.

2.1 Setup

Preferences The economy is endowed with a unit measure of homogeneous consumers who supply one unit of labor inelastically and whose utility function is given by:

$$U_t = \int_t^{\infty} \exp\left(-\rho\left(s-t\right)\right) \ln C_s \,\mathrm{d}s, \tag{1}$$

where C_t denotes the final good consumption at time t and ρ is the discount rate.

The final good is the CES aggregate of intermediate varieties indexed by ω. Although there exists a unit continuum of varieties, a key feature of our search economy is that consumers only have access to a fraction of varieties due to search frictions. Let $\mathcal{S}_t \subseteq [0,1]$ denote a random subset of *accessible* varieties to a consumer at time t with norm $|\mathcal{S}_t| = \Omega_t$, the final good consumption is given by:

$$C_t = \left[\int_{\mathcal{S}_t} C_t\left(\omega\right)^{\frac{\sigma-1}{\sigma}} \,\mathrm{d}\omega\right]^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where $C_t(\omega)$ is the consumption of variety ω at time t and σ is the elasticity of substitution across varieties.

In addition to consuming the final good C_t, consumers can invest in a risk-free bond A_t, which is in zero net supply, implying $A_t = 0$ in equilibrium. The budget constraint is thus:

$$P_t C_t + \dot{A}_t = w_t + \Pi_t + r_t A_t. \tag{3}$$

The final good price index P_t at each time t will be normalized to one to interpret all variables as real. w_t, Π_t, and r_t thus denote real wage, aggregate real profit redistributed to consumers, and the real interest rate, respectively.

Market structure The market consists of a unit continuum of varieties $\omega \in [0, 1]$. With frictionless search, firms producing perfect substitutes with heterogeneous productivity cannot coexist since the most efficient firms for a given variety will serve the entire market using limit pricing. With search friction, however, firms producing the same variety with different productivity levels can coexist because buyers have access only to a subset of firms in the economy. Therefore, in the baseline search economy, we allow *each* variety to be produced by an exogenous measure S of firms that differ in productivity.

Production technology We assume that the productivity distribution at time 0 is Pareto with shape parameter $\theta > 1$ and lower bound $\underline{z}_0 > 0$:[5]

$$G_0(z) = 1 - \left(\frac{z}{\underline{z}_0}\right)^{-\theta}. \tag{4}$$

A firm with productivity z possesses the following Cobb-Douglas technology:

$$y_t(z) = z \left[\frac{Q_t(z)}{\alpha}\right]^{\alpha} \left[\frac{l_t(z)}{1-\alpha}\right]^{1-\alpha},$$

where $y_t(z)$ is the output at time t, $l_t(z)$ denotes labor input, and

$$Q_t(z) = \left[\int_{\mathcal{S}_t} q_t(\omega; z)^{\frac{\sigma-1}{\sigma}} \, \mathrm{d}\omega\right]^{\frac{\sigma}{\sigma-1}} \tag{5}$$

[5] The Pareto distribution assumption for technology has been widely used in the literature, e.g., Kortum (1997), Chaney (2008), etc.

is the CES aggregate of accessible intermediate varieties. We assume that the intermediate varieties used by firms share the same elasticity of substitution as for consumers and are subject to identical search friction.[6]

Cost minimization yields the firm's marginal cost $c_t(z) = \frac{w_t^{1-\alpha}}{z}$. Given the initial Pareto productivity distribution $G_0(z)$, we have the following initial cost distribution:

$$H_0(c) = \left(\frac{c}{\bar{c}_0}\right)^{\theta}, \tag{6}$$

where $\bar{c}_0 = \frac{w_0^{1-\alpha}}{z_0}$ represents the cost upper bound at time 0.

Search and matching Search friction is modeled as Poisson arrivals of random encounters between buyers and sellers, à la Menzio (2024a,b) and Miyauchi (2024). For any given variety, the Poisson arrival rate of an encounter is given by:

$$M = \lambda S^{\gamma^s} B^{\gamma^b}, \tag{7}$$

where $\lambda > 0$ denotes search efficiency, S and B represent the measures of sellers and buyers for that variety, and γ^s and γ^b are their respective elasticities. Since both consumers and firms are buyers of intermediate varieties, we have $B = S + 1$. In our baseline search economy, we take both λ and S as given, so the arrival rate M is exogenous.

We define a *match* as the realization of a transaction between a buyer and a seller. Buyers of a given variety purchase from the seller offering the lowest price among all sellers they encounter. Following Menzio (2024a), with a continuous, differentiable, and strictly increasing price distribution $F_t(\cdot)$,[7] the probability that a buyer with n other encounters of a given variety matches with a seller offering price p is:

$$\left[1 - F_t(p)\right]^n. \tag{8}$$

Technology adoption Following Perla et al. (2021), we assume that firms at time t can either produce at current productivity or pay a fixed cost $\kappa > 0$ in units of labor to *adopt*

[6] With identical search friction, consumers and firms thus encounter the same measure of accessible intermediate varieties, i.e., $|\mathbb{S}_t| = \Omega_t$.

[7] Menzio (2024a) proves the continuity, differentiability, and strict monotonicity of the price distribution $F_t(\cdot)$ in an economy with Poisson search frictions similar to (7).

a new technology drawn from the current productivity distribution $G_t(\cdot)$. Let $V_{p,t}(z)$ denote the value of production for a firm with productivity z. The expected net value of technology adoption is then given by:

$$V_{a,t} = \int_{z_t}^{\infty} V_{p,t}(z)\, \mathrm{d}G_t(z) - \kappa w_t. \tag{9}$$

We assume that a firm can repeatedly redraw its productivity within time t until it has no incentive to do so. Firms' technology adoption collectively shapes the evolution of the productivity distribution and, in turn, the aggregate growth of the economy.

2.2 Balanced growth path equilibrium

For analytical tractability, we focus on the BGP equilibrium defined as follows:

Definition 1 (Balanced Growth Path Equilibrium)**.** *A balanced growth path (BGP) equilibrium is such that:*

- *Consumers choose final good consumption path $\{C_s\}_{s\in[t,\infty)}$ to maximize lifetime utility U_t in (1), subject to the budget constraint (3).*
- *Firms at each time t take the price distribution $F_t(\cdot)$ as given and set price p to maximize the expected profit $\pi_t(p; z)$ prior to the realization of encounters (7) and matches (8). The optimal prices of all firms are, in turn, consistent with the overall price distribution $F_t(\cdot)$.*
- *Firms at each time t adopt new technologies randomly drawn from the current productivity distribution $G_t(\cdot)$ whenever $V_{a,t} \geq V_{p,t}(z)$, where $V_{a,t}$ is given by (9). Technology adoption drives aggregate growth, which in turn determines firms' value of production $V_{p,t}(z)$. All aggregate variables grow at constant rates, with the real GDP growth rate denoted by g.*
- *The cross-sectional distribution of inverse relative productivity $\hat{z} = \frac{z_t}{z} \in (0, 1]$ is stationary, denoted by $\hat{G}(\cdot)$. The cross-sectional distribution of marginal cost c is also stationary, such that $H_t(c) = H(c) = \left(\frac{c}{\bar{c}}\right)^{\theta}$ at each time t.*
- *All markets (i.e., labor, intermediate and final goods, and risk-free bond) clear at each time t.*

3 Optimal Pricing, Markups, and Profits

In this section, we solve the firms' expected profit maximization problem in the baseline search economy and discuss the properties of optimal pricing, markups, and profits.

Expected profit maximization The following lemma provides the expression for a firm's expected profit before the realization of encounters and matches, as a function of price p and productivity z:

Lemma 1 (Expected Profit)**.** *The expected profit of a firm with price p and productivity z is given by:*

$$\pi_t\left(p; z\right) = \overline{b}_t\left(p\right) \overline{q}_t\left(p\right) \overline{\pi}_t\left(p; z\right), \tag{10}$$

where

$$\overline{b}_t\left(p\right) = \lambda S^{\gamma^s - 1} B^{\gamma^b} \exp\left(-\lambda S^{\gamma^s} B^{\gamma^b - 1} F_t\left(p\right)\right) \tag{11}$$

is the expected number of buyers matched with the firm,[8]

$$\overline{q}_t\left(p\right) = \overline{Q}_t p^{-\sigma} \tag{12}$$

is the firm's expected demand from a random buyer (i.e., a consumer or a firm), with $\overline{Q}_t$ denoting the expected real expenditure of a random buyer on the final good, and

$$\overline{\pi}_t\left(p; z\right) = p - c_t\left(z\right) = p - \frac{w_t^{1-\alpha}}{z}$$

is the firm's profit margin.

Proof. See Appendix A.1. □

Discussion Lemma 1 highlights two differences between the expected profit (10) and the profit in an economy with frictionless search. The first difference is straightforward: The *expected number of buyers matched with the firm*, $\overline{b}_t\left(p\right)$, emerges as a key determinant of expected profit. Without search frictions, all buyers of a variety would flow to the global technology leader for that variety. In contrast, the randomness introduced by search and

[8] Menzio (2024b) derives $\overline{b}_t\left(p\right)$ under a similar search and matching environment, but without externalities from the masses of buyers and sellers.

matching generates an intensive margin in the expected number of buyers, allowing heterogeneous firms producing the same variety to coexist.

What determines the intensive margin in the expected number of buyers? Taking logs of equation (11) and totally differentiating yields:

$$\mathrm{d}\ln \bar{b}_t(p) = \left[1 - \frac{M}{B}F_t(p)\right]\mathrm{d}\ln\frac{M}{B} + \mathrm{d}\ln\frac{B}{S} - \frac{M}{B}F_t'(p)\,\mathrm{d}p, \tag{13}$$

where $\frac{M}{B} = \lambda S^{\gamma^s} B^{\gamma^b - 1}$ represents the Poisson arrival rate of a random seller (i.e., a firm) for a buyer of any given variety. Equation (13) reveals three factors that affect a firm's expected number of buyers: the Poisson arrival rate of sellers $\frac{M}{B}$, the relative mass of buyers compared with sellers $\frac{B}{S}$, and the price p set by the firm. In the absence of matching, variations in the arrival rate of sellers $\frac{M}{B}$ pass through one-for-one to the expected number of buyers per seller. However, since buyers only match with the lowest-priced seller they encounter, the simultaneous arrival of sellers charging prices below the focal firm's price p, captured by $\frac{M}{B}F_t(p)$, limits this pass-through. The relative mass of buyers $\frac{B}{S}$, in comparison, affects all sellers equally, with unit elasticity. Finally, a marginal increase in price reduces the expected number of buyers by lowering the matching probability, with the semi-elasticity determined by the arrival of sellers charging the previous price p, $\frac{M}{B}F_t'(p)$.

Ignoring $\bar{b}_t(p)$, the remaining component in expected profit (10), $\bar{q}_t(p)\,\overline{\pi}_t(p;z)$, takes a form similar to that in an economy with frictionless search and CES preferences.[9] However, the second and more subtle difference is that the equilibrium object $\overline{Q}_t$ in (12) is no longer the buyers' total expenditure on the final good, but rather the *expected expenditure per buyer*. As a buyer may encounter and match with any seller in the economy regardless of the seller's position in the *within-variety* productivity distribution, $\overline{Q}_t$ aggregates the expected expenditure over all sellers of each variety.

Optimal pricing and markups A firm with productivity z sets price p to maximize the expected profit $\pi_t(p;z)$ in (10). The optimal price p becomes a function of productivity z and is denoted by $\tilde{p}_t(z) = \arg\max_p \pi_t(p;z)$. Based on Lemma 1, the first-order condition

[9] Note that the final good price index P_t is normalized to one.

of the problem is given by:

$$F_t'\left(\tilde{p}_t\left(z\right)\right)=\frac{\left(1-\sigma\right)\tilde{p}_t\left(z\right)+\sigma c_t\left(z\right)}{\lambda S^{\gamma^s}B^{\gamma^b-1}\tilde{p}_t\left(z\right)\left[\tilde{p}_t\left(z\right)-c_t\left(z\right)\right]},\tag{14}$$

which is a non-linear first-order ordinary differential equation (ODE) in $F_t\left(\cdot\right)$. The non-linearity and the nested endogenous mapping between productivity and price make (14) analytically intractable in most cases. However, as shown in Appendix A.2, equation (14) can be greatly simplified under the following regularity assumption:

Assumption 1 (Regularity Assumption)**.** *At each time t, a firm at the technology frontier sets its price as:*

$$\lim_{z\to\infty}\tilde{p}\left(z\right)=\frac{\left(\lambda S^{\gamma^s}B^{\gamma^b-1}\right)^{-\frac{1}{\theta}}}{\Gamma\left(\frac{\theta-1}{\theta}\right)}\bar{c},\tag{15}$$

where $\Gamma\left(a\right)=\int_0^\infty x^{a-1}\exp\left(-x\right)\mathrm{d}x$ denotes the gamma function.

Equation (15) serves two purposes. First, it provides a boundary condition required to pin down the solution to the ODE (14). Second, as shown in Appendix A.2, it is the *only* condition that can guarantee that the optimal price $\tilde{p}_t\left(z\right)$ is strictly increasing in marginal cost $c_t\left(z\right)$ for all parameter choices. In this case, we can apply the change of variables $\tilde{p}_t\left(z_t\left(c\right)\right)=p_t\left(c\right)$ and $F_t\left(p_t\left(c\right)\right)=H\left(c\right)$ to simplify (14) to the following ODE:[10]

$$p'\left(c\right)=\frac{\lambda S^{\gamma^s}B^{\gamma^b-1}H'\left(c\right)p\left(c\right)\left[p\left(c\right)-c\right]}{\left(1-\sigma\right)p\left(c\right)+\sigma c}.\tag{16}$$

It turns out that the optimal pricing function becomes time-invariant, i.e., $p_t\left(c\right)=p\left(c\right)$, and admits a closed-form solution in the special case of $\sigma=1$, in which the final good is a Cobb-Douglas aggregator. For analytical tractability, we focus on this particular case from now on. The following proposition characterizes firms' optimal pricing and markups in this special case:

Proposition 1 (Optimal Pricing and Markups)**.** *Suppose that $\sigma=1$ and Assumption 1 holds.*

10 Since $c_t\left(z\right)=\frac{w_t^{1-\alpha}}{z}$, inverting this expression yields $z_t\left(c\right)=\frac{w_t^{1-\alpha}}{c}$. For the detailed derivation of ODE (16), see Appendix A.2.

The optimal price of a firm with marginal cost c is the solution to the following Riccati equation:

$$p'(c) = \lambda S^{\gamma^s} B^{\gamma^b - 1} H'(c) \left[\frac{p(c)^2}{c} - p(c) \right]. \tag{17}$$

The markup of a firm with marginal cost c, in turn, is given by:

$$m(c) = \frac{p(c)}{c} = \frac{\left[\lambda S^{\gamma^s} B^{\gamma^b - 1} H(c)\right]^{-\frac{1}{\theta}} \exp\left(-\lambda S^{\gamma^s} B^{\gamma^b - 1} H(c)\right)}{\int_{\lambda S^{\gamma^s} B^{\gamma^b - 1} H(c)}^{\infty} x^{-\frac{1}{\theta}} \exp(-x)\, \mathrm{d}x}, \tag{18}$$

which is monotonically decreasing in c.

Proof. See Appendix A.2. □

Discussion The Riccati equation (17) can be written in the following easily interpretable form:

$$\frac{\mathrm{d}\ln p(c)}{\mathrm{d}\ln c} = \lambda S^{\gamma^s} B^{\gamma^b - 1} H'(c) \left[p(c) - c\right]. \tag{19}$$

Suppose that a firm with marginal cost c increases its price to the level charged by a firm with infinitesimally higher marginal cost $c + \mathrm{d}c$. Holding expected demand constant, the left-hand side of (19) represents the percentage increase in expected profits resulting from the rise in profit margin. On the other hand, raising the price at the margin exposes the firm to an additional price disadvantage relative to competitors whose marginal costs lie in $[c, c + \mathrm{d}c]$. In the search economy, this reduces the firm's expected demand, as captured by the arrival rate of such additional competitors, i.e., $\lambda S^{\gamma^s} B^{\gamma^b - 1} H'(c)$. The right-hand side of (19) thus measures the percentage decrease in expected profits resulting from the decline in expected demand, holding profit margin constant at its original level. When the expected gain equals the expected loss, the firm has no incentive to change its price, and $p(c)$ is therefore pinned down by equation (19).

How should we understand the markup function (18)? Qualitatively, it implies that in the search economy, even *atomistic* firms producing a *homogeneous* good will charge *strictly positive and variable* markups as long as they differ in marginal costs (i.e., productivity). Unlike standard monopolistic competition and oligopolistic competition models (e.g., Melitz, 2003; Atkeson and Burstein, 2008)—where markups arise from imperfect substitutability across varieties—search and matching between buyers and sellers give rise to within-variety markups that are directly tied to technology gaps. Unlike standard quality ladder and creative destruction models (e.g., Grossman and Helpman, 1991; Aghion and Howitt, 1992)—where the markup on a variety is solely charged by its global technology leader—search frictions introduce uncertainty so that each buyer faces a different local technology leader of their own, which gives every seller strictly positive market power.

Quantitatively, the markup function (18) can be written as the following *gamma hazard rate*:

$$m(c) = m(\Lambda(c)) = \frac{\widetilde{\Gamma}'\left(\frac{\theta-1}{\theta}, \Lambda(c)\right)}{1-\widetilde{\Gamma}\left(\frac{\theta-1}{\theta}, \Lambda(c)\right)}, \tag{20}$$

where $\widetilde{\Gamma}(a,x) = \frac{\int_0^x u^{a-1}\exp(-u)\mathrm{d}u}{\Gamma(a)}$ denotes the cumulative distribution function (CDF) of the gamma distribution and $\Lambda(c) = \lambda S^{\gamma^s} B^{\gamma^b - 1} H(c)$ represents the arrival rate of sellers with marginal costs below c. As proved by Glaser (1980), the monotonicity of the gamma hazard rate (20) is determined by the sign of $\frac{\theta-1}{\theta}$, with $0 < \frac{\theta-1}{\theta} < 1$ (i.e., $\theta > 1$) implying a strictly decreasing function of $\Lambda(c)$ (i.e., $m'(\Lambda(c)) < 0$).[11] Together with $\Lambda'(c) > 0$, markups therefore decrease monotonically in the marginal cost c.

To dissect the relationship between markup and marginal cost from an economic perspective, we derive the following elasticity from (20):

$$\frac{\mathrm{d}\ln m(c)}{\mathrm{d}\ln c} = \frac{\mathrm{d}\ln p(c)}{\mathrm{d}\ln c} - 1 = \theta\Lambda(c)\left[m(c)-1\right] - 1 < 0. \tag{21}$$

A direct implication of (21) is the *incomplete pass-through* from marginal costs to prices. Pass-through for a firm with marginal cost c is jointly determined by three factors. First, a larger Pareto shape parameter θ implies a thinner right tail of the productivity distribution and weaker competitive pressure from top firms with low marginal costs, which directly raises pass-through.[12]

The other two determinants of pass-through are the arrival rate of competitors with marginal costs below c, $\Lambda(c)$, and market power as captured by the net markup $m(c) - 1$. Holding market power fixed, an increase in marginal cost makes it more likely that the firm's potential buyers simultaneously encounter alternative sellers with lower marginal costs, i.e., an increase in $\Lambda(c)$. This intensified competitive disadvantage forces the firm

[11] See Appendix A.2 for a detailed proof.

[12] Note that θ also affects pass-through indirectly by entering $\Lambda(c)$ and $m(c) - 1$.

to attach greater weight to profits from buyers over whom it retains a competitive advantage, i.e., buyers who encounter only alternative sellers with higher marginal costs. Using the same terminology as Aghion et al. (2005) (with a different interpretation), this *static escape-competition effect* pushes up pass-through and is a distinctive feature of the search economy.

On the other hand, holding $\Lambda(c)$ fixed, increased marginal cost also directly reduces market power $m(c) - 1$, which in turn pulls down pass-through. This *market power effect* is widely documented in the variable markup literature (e.g., Atkeson and Burstein, 2008; Edmond et al., 2015, 2023). The key difference is that the search economy captures *within-variety* market power. Taken together, the opposing push-pull effects keep the pass-through incomplete.

Profits By first substituting $c_t(z) = \frac{w_t^{1-\alpha}}{z}$ back into the markup equation (18) and then plugging the resulting optimal price $\tilde{p}_t(z)$ into the expected profit (10), we can recover the time-t flow profit $\pi_t(z)$ as a function of productivity z:

$$\pi_t(z) = D_t \left[\left(\lambda S^{\gamma^s} B^{\gamma^b - 1} \right)^{-\frac{1}{\theta}} \exp\left(-\lambda S^{\gamma^s} B^{\gamma^b - 1} \left(\frac{z_t}{z} \right)^{\theta} \right) - \Gamma\left(\frac{\theta - 1}{\theta}, \lambda S^{\gamma^s} B^{\gamma^b - 1} \left(\frac{z_t}{z} \right)^{\theta} \right) \frac{z_t}{z} \right], \tag{22}$$

where $D_t = \lambda^{\frac{\theta+1}{\theta}} S^{\frac{\theta+1}{\theta}\gamma^s - 1} B^{\frac{\theta+1}{\theta}\gamma^b - \frac{1}{\theta}} \overline{Q}_t$ denotes the aggregate demand shifter at time t, determined in equilibrium, and $\Gamma(a, x) = \int_x^{\infty} u^{a-1} \exp(-u)\, \mathrm{d}u$ denotes the upper incomplete gamma function (Abramowitz and Stegun, 1965).[13]

Discussion Differentiating the flow profit equation (22) with respect to productivity z yields:

$$\pi_t'(z) = D_t \Gamma\left(\frac{\theta - 1}{\theta}, \lambda S^{\gamma^s} B^{\gamma^b - 1} \left(\frac{z_t}{z} \right)^{\theta} \right) \frac{z_t}{z^2} > 0, \tag{23}$$

which indicates that flow profits are strictly increasing in productivity. While, at the extensive margin, search frictions imply strictly positive market power and profits for all firms producing the same variety, buyer-seller matching based on the lowest price preserves the importance of technological advantage at the intensive margin of profits. It is this intensive-margin heterogeneity that generates a distinct *within-variety* technology

[13] In parallel, the lower incomplete gamma function $\gamma(a, x)$ is defined as $\gamma(a, x) = \Gamma(a) - \Gamma(a, x) = \int_0^x u^{a-1} \exp(-u)\, \mathrm{d}u$.

adoption incentive for less productive firms in the search economy. In contrast, standard models of creative destruction à la Aghion and Howitt (1992) focus on innovation activities that advance the technology frontier, since inter-firm profit differences arise only at the extensive margin in a winner-take-all manner. We therefore proceed to the dynamic block in the next section to characterize firms' endogenous technology adoption decisions in the search economy.

4 Technology Adoption and Firm Value

In this section, we characterize technology adoption decisions of firms, derive firms' value functions, and show how to solve for the aggregate growth rate of the economy.

Technology adoption First, the following lemma characterizes firms' technology adoption decisions and characterizes dynamic impacts on the productivity distribution:

Lemma 2 (Technology Adoption). *In the BGP equilibrium, at each time t, only firms at the lower bound $\underline{z}_t$ of the productivity distribution $G_t(\cdot)$ choose to adopt new technology, with the associated value-matching and smooth-pasting conditions given by:*

$$V_{p,t}(\underline{z}_t) = V_{a,t} \tag{24}$$

and

$$\frac{\partial V_{p,t}(z)}{\partial z}\Big|_{z=\underline{z}_t} = \frac{\partial V_{a,t}}{\partial z}\Big|_{z=\underline{z}_t} = 0, \tag{25}$$

respectively. Along the BGP, the productivity distribution $G_t(\cdot)$ remains Pareto with shape parameter θ, while its lower bound $\underline{z}_t$ grows at a constant rate over time, denoted by g_z.

Proof. See Appendix A.3. □

Discussion Equation (24) indicates that, at any point in time, *only* firms at the lower bound $\underline{z}_t$ of the productivity distribution choose to adopt new technology, while all other firms continue to produce. If instead a firm with productivity $z > \underline{z}_t$ had an incentive to adopt, the monotonicity of the profit function shown by (23) would imply that firms with lower productivity levels $z' \in [\underline{z}_t, z)$ also adopt and thus exit this region, contradicting the fact that $\underline{z}_t$ is the productivity lower bound. Technology adoption is therefore equivalent to an optimal stopping problem with $\underline{z}_t$ as the cutoff, where (24) and (25) correspond to the standard value-matching and smooth-pasting conditions, respectively (Stokey, 2009).

Firm value With the flow profit $\pi_t(z)$ given by (22), the Bellman equation for a firm with productivity z is:

$$r_t V_{p,t}(z) = \pi_t(z) + \frac{\mathrm{d}V_{p,t}(z)}{\mathrm{d}t}. \tag{26}$$

Let $\hat{z} = \frac{\underline{z}_t}{z} \in (0,1]$ denote the *inverse relative productivity*. Since the flow profit function (22) only depends on D_t and $\hat{z}$, we can define $v(\hat{z}) = \frac{V_{p,t}(z)}{D_t}$ as the detrended value function. We now have the following proposition that characterizes $v(\hat{z})$:

Proposition 2 (Detrended Value Function)**.** *In the BGP equilibrium, the detrended value function $v(\hat{z})$ of a firm with inverse relative productivity $\hat{z}$ is the solution to the following first-order linear ODE:*

$$v'(\hat{z}) = \frac{\rho}{g_z \hat{z}} v(\hat{z}) - \frac{\hat{\pi}(\hat{z})}{g_z \hat{z}} \tag{27}$$

where $\hat{\pi}(\hat{z}) = \frac{\pi_t(z)}{D_t}$ is the detrended flow profit.[14] *Solving the above ODE yields:*

$$v(\hat{z}) = \frac{1}{\rho}\left[\hat{\pi}(\hat{z}) + \int_{\hat{z}}^{1} \left(\frac{\hat{z}}{x}\right)^{\frac{\rho}{g_z}} \mathrm{d}\hat{\pi}(x)\right].^{15} \tag{29}$$

Proof. See Appendix A.4. □

[14] Specifically,

$$\hat{\pi}(\hat{z}) = \left(\lambda S^{\gamma^s} B^{\gamma^b - 1}\right)^{-\frac{1}{\theta}} \exp\left(-\lambda S^{\gamma^s} B^{\gamma^b-1} \hat{z}^{\theta}\right) - \Gamma\left(\frac{\theta-1}{\theta}, \lambda S^{\gamma^s} B^{\gamma^b-1}\hat{z}^{\theta}\right)\hat{z}. \tag{28}$$

[15] Suppose that $\sigma = 1$ and Assumption 1 holds. In this case, we can expand the detrended value function (29) to obtain the following expression:

$$\begin{aligned}\rho v(\hat{z}) = {} & \left(\lambda S^{\gamma^s} B^{\gamma^b-1}\right)^{-\frac{1}{\theta}} \exp\left(-\lambda S^{\gamma^s} B^{\gamma^b-1}\hat{z}^{\theta}\right) - \Gamma\left(\frac{\theta-1}{\theta}, \lambda S^{\gamma^s} B^{\gamma^b-1}\hat{z}^{\theta}\right)\hat{z} \\ & + \Gamma\left(\frac{\theta-1}{\theta}\right)\frac{g_z}{g_z-\rho}\left(\hat{z} - \hat{z}^{\frac{\rho}{g_z}}\right) - \sum_{n=0}^{\infty} \frac{(-1)^n \left(\lambda S^{\gamma^s} B^{\gamma^b-1}\right)^{n+\frac{\theta-1}{\theta}}}{n!\left(n+\frac{\theta-1}{\theta}\right)\left(n\theta+\theta-\frac{\rho}{g_z}\right)}\left(\hat{z}^{n\theta+\theta} - \hat{z}^{\frac{\rho}{g_z}}\right).\end{aligned} \tag{30}$$

For the detailed derivation of (30), see Appendix A.4.

Discussion The detrended value function $v(\hat{z})$ in (29) is decomposed into two components. The first component, captured by $\frac{\hat{\pi}(\hat{z})}{\rho}$, represents the *detrended value of current profits* given a constant inverse relative productivity $\hat{z}$. However, due to technology adoption by the least productive firms (i.e., firms with $\hat{z} = 1$) and the resulting growth in the productivity lower bound $\underline{z}_t$, $\hat{z}$ for a given firm increases over time. This mechanism gives rise to the second component—the *detrended value of future profit losses* arising from competitors' technology adoption—as captured by $\int_{\hat{z}}^{1} \left(\frac{\hat{z}}{x}\right)^{\frac{\rho}{g_z}} \mathrm{d}\frac{\hat{\pi}(x)}{\rho}$.[16] Clearly, this value loss is larger as the productivity lower bound $\underline{z}_t$ rises more rapidly (i.e., when g_z is higher).

To better understand the value-loss component, suppose the firm has inverse relative productivity $\hat{z}$ at time 0. We can then use a change of variables $x = \exp\left(g_z t_x\right)\hat{z}$ to rewrite it as:

$$\int_0^{-\frac{\ln \hat{z}}{g_z}} \exp\left(-\rho t_x\right) \mathrm{d}\frac{\hat{\pi}\left(\exp\left(g_z t_x\right)\hat{z}\right)}{\rho}. \tag{31}$$

Hence, (31) integrates the discounted *marginal* value loss $\exp\left(-\rho t_x\right)\mathrm{d}\frac{\hat{\pi}(\exp(g_z t_x)\hat{z})}{\rho}$ from time 0 to $-\frac{\ln \hat{z}}{g_z}$, during which inverse relative productivity moves from $\hat{z}$ to 1.

Future value losses in (31) are negligible for both the most (i.e., $\hat{z} \to 0$) and the least (i.e., $\hat{z} \to 1$) productive firms. The former are so far from the productivity lower bound that their profit flows are barely threatened by adopters: $\exp\left(-\rho t_x\right)\mathrm{d}\frac{\hat{\pi}(\exp(g_z t_x)\hat{z})}{\rho} \to 0$ when $\hat{z} \to 0$. The latter are very close to the lower bound and soon become adopters themselves: $-\frac{\ln \hat{z}}{g_z} \to 0$ when $\hat{z} \to 1$. In both cases, (31) approaches zero. Therefore, firms in the middle of the productivity distribution suffer the largest future value losses from competitors' technology adoption.

Finally, by plugging the detrended value function $v\left(\hat{z}\right)$ into the value-matching condition (24), the growth rate g_z of the productivity lower bound along the BGP solves:

$$\widetilde{D}\left(g_z\right)\left[\int_0^1 v\left(x; g_z\right)\mathrm{d}\hat{G}\left(x\right) - v\left(1; g_z\right)\right] = \kappa, \tag{32}$$

where $\widetilde{D}\left(g_z\right) = \frac{D_t}{w_t}$ is the detrended demand shifter that is determined in equilibrium and $\hat{G}\left(x\right) = x^{\theta}$ represents the distribution of inverse relative productivity $\hat{z}$. The growth rate

[16] It follows from equation (23) that:

$$\mathrm{d}\hat{\pi}\left(x\right) = -\frac{1}{D_t}\pi_t'\left(\frac{\underline{z}_t}{x}\right)\frac{\underline{z}_t}{x^2}\,\mathrm{d}x = -\Gamma\left(\frac{\theta-1}{\theta}, \lambda S^{\gamma^s} B^{\gamma^b - 1} x^{\theta}\right)\mathrm{d}x < 0,$$

which implies that the detrended flow profit decreases as x increases.

g_z of the productivity lower bound affects the detrended value function $v(\hat{z})$ in (29) by altering the value losses induced by competitors' technology adoption. In turn, the shape of $v(\hat{z})$ feeds back into g_z by changing the expected detrended value of adoption, i.e., $\int_0^1 v(x; g_z)\,\mathrm{d}\hat{G}(x) - v(1; g_z)$ in (32). Put differently, the intensity of technology adoption shapes the value of production, which in turn determines firms' incentives to adopt.

However, since the labor allocated to technology adoption depends on g_z,[17] the detrended demand shifter $\widetilde{D}$—directly linked to the labor used in production—is in turn a function of g_z. Pinning down g_z and the associated real GDP growth rate g thus requires solving for the general equilibrium, which is the focus of the next section.

5 General Equilibrium, Growth, and Welfare

In this section, we close the baseline model by solving for the aggregate variables in general equilibrium and characterizing the real GDP growth rate and consumer welfare.

Accessibility For any given variety, the probability that a buyer has access to it is equal to the probability of encountering at least one seller in that variety. The Poisson arrival of encounters implies that this probability is $1 - \exp\left(-\frac{M}{B}\right)$. Since there exists a unit continuum of varieties, the law of large numbers implies that this probability coincides with the fraction of varieties accessible to the buyer. That is:

$$\Omega_t = \Omega = 1 - \exp\left(-\lambda S^{\gamma^s} B^{\gamma^b - 1}\right). \tag{33}$$

Conditional on encountering at least one seller of a given variety, the following lemma characterizes the distribution of the minimum marginal cost accessible to a buyer:

Lemma 3 (Minimum Cost Distribution)**.** *For any buyer of any accessible variety, the minimum cost among encountered sellers follows a right-truncated Weibull distribution with shape*

[17] Specifically, the labor used in technology adoption is given by the product of the mass of adopters and the labor required per adopter:

$$L_a = \frac{G_t\left(\underline{z}_{t+\mathrm{d}t}\right) - G_t\left(\underline{z}_t\right)}{\mathrm{d}t} S\kappa = G_t'\left(\underline{z}_t\right)\underline{z}_t \frac{\mathrm{d}\ln \underline{z}_t}{\mathrm{d}t} S\kappa = S\kappa\theta g_z.$$

parameter θ, scale parameter $\lambda S^{\gamma^s} B^{\gamma^b-1}\overline{c}^{-\theta}$, and truncation point $\overline{c}$:

$$H_{min}(c) = \frac{1-\exp\left(-\lambda S^{\gamma^s} B^{\gamma^b-1}\overline{c}^{-\theta}c^{\theta}\right)}{\Omega}. \tag{34}$$

Proof. See Appendix A.5. □

Discussion Equation (33) and (34) jointly characterize the *accessibility* of buyers to low-marginal cost sellers, which plays a crucial role in determining the real wage w_t in general equilibrium. In particular, (33) captures the *extensive margin* of accessibility—whether a buyer can encounter a seller in a given variety. Clearly, Ω increases with the arrival rate of sellers, i.e., $\frac{\mathrm{d}\Omega}{\mathrm{d}\lambda S^{\gamma^s} B^{\gamma^b-1}} > 0$.

By contrast, (34) reflects the *intensive margin* of accessibility conditional on encountering a variety, i.e., the extent to which a buyer has access to low-cost sellers. Technically, the right-truncated Weibull distribution in (34) belongs to the Type-III extreme value family and arises from a Poisson-Pareto mixture, which is commonly used to characterize the *minimum* of a random variable—here, the minimum marginal cost among sellers encountered. By applying Jensen's inequality, we obtain $\frac{\partial H_{min}(c)}{\partial \lambda S^{\gamma^s} B^{\gamma^b-1}} \geq 0$,[18] which implies that the minimum marginal cost a buyer encounters is *stochastically* decreasing (i.e., in the sense of first-order stochastic dominance) in the arrival rate of sellers.

General equilibrium Given (33), (34), and the market-clearing conditions, the following proposition solves the aggregate variables in general equilibrium:

Proposition 3 (General Equilibrium)**.** *Suppose that $\sigma = 1$ and Assumption 1 holds. Along the*

[18] Taking the partial derivative of (34) with respect to $\lambda S^{\gamma^s} B^{\gamma^b-1}$ yields:

$$\frac{\partial H_{min}(c)}{\partial \lambda S^{\gamma^s} B^{\gamma^b-1}} = \frac{\exp\left(-\frac{M}{B}\left(1+\left(\frac{c}{\overline{c}}\right)^{\theta}\right)\right)}{\Omega^2}\left[\left(\frac{c}{\overline{c}}\right)^{\theta}\exp\left(\frac{M}{B}\right) - \exp\left(\left(\frac{c}{\overline{c}}\right)^{\theta}\frac{M}{B}\right) - \left(\frac{c}{\overline{c}}\right)^{\theta} + 1\right].$$

By Jensen's inequality and the convexity of the exponential function, we obtain:

$$\exp\left(\left(\frac{c}{\overline{c}}\right)^{\theta}\frac{M}{B}\right) = \exp\left(\left(\frac{c}{\overline{c}}\right)^{\theta}\frac{M}{B} + \left(1-\left(\frac{c}{\overline{c}}\right)^{\theta}\right)\cdot 0\right) \leq \left(\frac{c}{\overline{c}}\right)^{\theta}\exp\left(\frac{M}{B}\right) - \left(\frac{c}{\overline{c}}\right)^{\theta} + 1.$$

It follows that $\frac{\partial H_{min}(c)}{\partial \lambda S^{\gamma^s} B^{\gamma^b-1}} \geq 0$.

BGP equilibrium, real GDP is given by:

$$C_t = w_t + \Pi_t. \tag{35}$$

Real wage w_t is given by:

$$w_t = \left[\exp\left(1 - \frac{\lambda S^{\gamma^s} B^{\gamma^b - 1} \exp\left(-\lambda S^{\gamma^s} B^{\gamma^b-1}\right) - \mu\left(\lambda S^{\gamma^s} B^{\gamma^b-1}\right)}{\Omega}\right)\left(\lambda S^{\gamma^s} B^{\gamma^b-1}\right)^{\frac{1}{\theta}} \underline{z}_t\right]^{\frac{1}{1-\alpha}}, \tag{36}$$

where $\mu\left(\lambda S^{\gamma^s} B^{\gamma^b-1}\right) = \int_0^{\lambda S^{\gamma^s} B^{\gamma^b-1}} \frac{\ln \Gamma\left(\frac{\theta-1}{\theta}, x\right)}{\exp(x)}\,\mathrm{d}x$ is a constant. Aggregate real profit Π_t is given by:

$$\Pi_t = \left[\widetilde{D} S \int_0^1 \hat{\pi}(x)\,\mathrm{d}\hat{G}(x)\right] w_t, \tag{37}$$

where the detrended demand shifter $\widetilde{D}$ is:

$$\widetilde{D} = \left[(1-\alpha)\,\theta \int_0^1 \Gamma\left(\frac{\theta-1}{\theta}, \lambda S^{\gamma^s} B^{\gamma^b-1} x^\theta\right) x^\theta\,\mathrm{d}x\right]^{-1} \frac{1 - S\kappa\theta g_z}{S}. \tag{38}$$

Proof. See Appendix A.6. □

Discussion Proposition 3 provides a step-by-step decomposition of real GDP. First, the final-good market-clearing condition (35) implies that real GDP C_t equals the real wage w_t plus the aggregate real profit Π_t, which is lump-sum rebated to consumers.

Equation (36) gives a closed-form solution for the equilibrium real wage w_t. Note that the growth rate of w_t is given by $\frac{g_z}{1-\alpha}$, i.e., growth in the productivity lower bound $\underline{z}_t$ does not pass through one-for-one to real wage growth but instead generates an amplification effect through roundabout production, as summarized by the *Leontief inverse* $\frac{1}{1-\alpha}$.[19]

Besides $\underline{z}_t$ and $\frac{1}{1-\alpha}$, the equilibrium real wage w_t is affected by the Poisson arrival rate of sellers, $\lambda S^{\gamma^s} B^{\gamma^b-1}$, through three different channels: the extensive margin of accessibility Ω, the intensive margin of accessibility $H_{min}(c)$, and the optimal pricing $p(c)$. The first channel appears explicitly as Ω in (36), whereas the latter two channels are intertwined with each other and are summarized by the remaining terms involving $\lambda S^{\gamma^s} B^{\gamma^b-1}$. With a higher arrival rate of sellers, buyers gain access to a wider range of varieties (i.e., higher

[19] Specifically, $\frac{1}{1-\alpha} = \sum_{n=0}^{\infty} \alpha^n$, where n indexes the nth round through which the effects of productivity growth propagate via intermediate inputs.

Ω) and are more likely to encounter sellers with low marginal costs (i.e., higher $H_{min}(c)$), while sellers tend to charge lower prices due to intensified competition (i.e., lower $p(c)$). All these effects raise the real wage w_t.

Equilibrium aggregate real profit Π_t, given in (37), is proportional to the real wage w_t, implying that real GDP grows at the same rate as real wage along the BGP, i.e., $g = \frac{g_z}{1-\alpha}$. The multiplier $\widetilde{D}S\int_0^1 \hat{\pi}(x)\,\mathrm{d}\hat{G}(x)$ captures the *aggregate profitability* of firms in the economy, and is determined by the detrended demand shifter $\widetilde{D}$, the total measure of firms S, and the detrended flow profit of a representative firm, $\int_0^1 \hat{\pi}(x)\,\mathrm{d}\hat{G}(x)$. Holding $\widetilde{D}$ fixed, a higher arrival rate of sellers intensifies competition, lowers optimal prices, and consequently reduces the representative firm's detrended flow profit.[20]

Finally, given the productivity lower bound growth rate g_z, equation (38) provides an analytic expression for the detrended demand shifter $\widetilde{D}$. $\widetilde{D}$ is the product of average production labor per firm, $\frac{1-S\kappa\theta g_z}{S}$, and a multiplier that reflects *allocative efficiency* of production labor. The inverse of the multiplier, $(1-\alpha)\,\theta\int_0^1 \Gamma\left(\frac{\theta-1}{\theta}, \lambda S^{\gamma^s}B^{\gamma^b-1}x^\theta\right)x^\theta\,\mathrm{d}x$, is proportional to the labor demand of a representative firm, $\int_0^{\bar{c}} \tilde{l}_t(c)\,\mathrm{d}H(c)$, which captures economy-wide labor *misallocation*.[21] Without search frictions, labor becomes entirely concentrated in frontier firms for each variety. Now with search frictions, however, even relatively unproductive firms retain positive labor demand, which reduces allocative efficiency.

Beyond search frictions (i.e., lower $\lambda S^{\gamma^s}B^{\gamma^b-1}$), labor misallocation is also exacerbated by a higher labor share (i.e., higher $1-\alpha$). In contrast, the impact of the Pareto shape parameter θ is ambiguous: While a lower θ corresponds to a fatter right tail of the productivity distribution, implying a greater mass of high-productivity firms and a lower degree of labor misallocation, it also implies higher aggregate market power and markups, reducing allocative efficiency.

Growth and welfare Finally, by substituting (38) into (32) and applying integration by parts and the Fubini-Tonelli theorem (Royden and Fitzpatrick, 1988), the following propo-

[20] Specifically,

$$\frac{\partial\hat{\pi}(x)}{\partial\lambda S^{\gamma^s}B^{\gamma^b-1}} = -\frac{1}{\theta}\left(\lambda S^{\gamma^s}B^{\gamma^b-1}\right)^{-\frac{\theta+1}{\theta}}\exp\left(-\lambda S^{\gamma^s}B^{\gamma^b-1}x^\theta\right) < 0 \text{ for all } x\in(0,1].$$

[21] For the detailed derivation, see Appendix A.6.

sition characterizes the real GDP growth rate, as well as consumer welfare:

Proposition 4 (Growth and Welfare)**.** *Suppose that $(1-\alpha)\rho S\kappa\theta < 1$, $\sigma = 1$, and Assumption 1 holds. Along the BGP, the growth rate of real GDP is given by:*

$$g = \frac{g_z}{1-\alpha} = \begin{cases} \dfrac{1-(1-\alpha)\rho S\kappa\theta}{(1-\alpha)\left[(1-\alpha)\theta+1\right]S\kappa\theta}, & \text{for } \lambda \in \mathbb{R}_{++}, \\ 0, & \text{for } \lambda = \infty. \end{cases} \tag{39}$$

The welfare of consumers at time t is given by:

$$U_t = \frac{1}{\rho}\left(\ln C_t + \frac{g}{\rho}\right). \tag{40}$$

Proof. See Appendix A.7. □

Discussion Equation (39) delivers the central insight of the baseline model: Although the real GDP growth rate exhibits a sharp *discontinuity* when the economy shifts from a frictionless to a frictional search environment—i.e., g jumps from 0 to $\frac{1-(1-\alpha)\rho S\kappa\theta}{(1-\alpha)[(1-\alpha)\theta+1]S\kappa\theta}$ as λ falls from infinity to a finite value—as long as search friction exists, its magnitude does *not* affect the level of aggregate growth.

To understand why, we return to the value-matching condition (32) that pins down g_z. As shown in Appendix A.7, the expected detrended value of adoption $\int_0^1 v\left(x;g_z\right)\mathrm{d}\hat{G}\left(x\right) - v\left(1;g_z\right)$ can be rewritten as $-\frac{\int_0^1 \hat{G}(x)\mathrm{d}\hat{\pi}(x)}{\rho+\theta g_z}$, i.e., the value of expected cumulative marginal profits, discounted at the effective rate $\rho+\theta g_z$. For marginal profit $-\mathrm{d}\hat{\pi}\left(x\right)$,[22] it accrues to an adopter whenever the inverse relative productivity draw lies in $(0,x]$, which occurs with probability $\hat{G}\left(x\right)$. The term $-\int_0^1 \hat{G}\left(x\right)\mathrm{d}\hat{\pi}\left(x\right)$ thus integrates expected marginal profit gains over the full support $(0,1]$. The dampening effect of future adoption by competitors on the value of adopting today is captured by θg_z in the effective discount rate. Plugging in $\hat{G}\left(x\right)$ and $\hat{\pi}\left(x\right)$, we have:

$$\int_0^1 v\left(x;g_z\right)\mathrm{d}\hat{G}\left(x\right) - v\left(1;g_z\right) = \frac{1}{\rho+\theta g_z}\int_0^1 \Gamma\left(\frac{\theta-1}{\theta}, \lambda S^{\gamma^s}B^{\gamma^b-1}x^{\theta}\right)x^{\theta}\,\mathrm{d}x. \tag{41}$$

Intuitively, as search efficiency improves (i.e., higher $\lambda S^{\gamma^s}B^{\gamma^b-1}$), the resulting tougher

[22] Note that x here denotes the inverse relative productivity. Hence, $\hat{\pi}\left(x\right)$ is decreasing in x, implying $-\mathrm{d}\hat{\pi}\left(x\right) > 0$.

competition forces all firms to reduce prices, with price cuts being more pronounced for higher-productivity firms that actively engage in competition. The detrended profit thus becomes less sensitive to productivity changes, weakening incentives to adopt. We refer to this channel as the *partial-equilibrium effect*.

However, by enhancing the allocative efficiency of production labor, higher search efficiency generates a *general-equilibrium effect* that promotes technology adoption, reflected in an increase in the detrended demand shifter $\widetilde{D}$. By comparing (38) and (41), it turns out that the partial- and general-equilibrium effects *exactly* offset each other, rendering the magnitude of *finite* search efficiency irrelevant for aggregate growth. The intuition is that, in the presence of search frictions, labor misallocation arises from firms' static aggregate profitability, which is exactly proportional to the expected detrended adoption value because the dynamic value-loss component is already absorbed into the effective discount rate via θg_z.

In stark contrast, (32) is no longer valid for solving g_z when $\lambda = \infty$. Instead, the market structure collapses to perfect competition where atomistic firms at the technology frontier produce and break even, while all other firms remain inactive. In this limiting case, no firm has an incentive to incur the fixed labor cost $\kappa > 0$ to adopt new technology, leading to $g = 0$.

Given a finite λ, what determines the real GDP growth rate g in the baseline model? The first determinant is the composite term $S\kappa\theta$, the labor required for technology adoption per unit of growth rate g_z. A higher $S\kappa\theta$ diverts labor away from production (i.e., the *labor composition effect*), reduces the detrended demand shifter $\widetilde{D}$, and thereby weakens adoption incentives and aggregate growth. The Pareto shape parameter θ also affects the growth rate by altering the misallocation of labor, together with the labor share $1 - \alpha$. In addition to reducing labor misallocation, a lower labor share also strengthens the amplification effect from roundabout production, as reflected in a higher Leontief inverse $\frac{1}{1-\alpha}$. Finally, as the discount rate ρ rises, the future payoff from technology adoption becomes less attractive, slowing aggregate growth.

Lastly, equation (40) decomposes consumer welfare into two components: the *value of current consumption*, $\frac{\ln C_t}{\rho}$, and the *value of future consumption growth*, $\frac{g}{\rho^2}$. Even though the magnitude of finite search efficiency does not affect GDP growth g in the baseline economy, it remains crucial for determining the level of current consumption C_t by changing the real wage w_t and aggregate real profits Π_t.

6 Extensions

In this section, we present several extensions of the baseline search economy to explore the implications under alternative settings. We show that once additional features are introduced that break the balance between the partial- and general-equilibrium effects, the magnitude of search efficiency begins to affect aggregate growth, either directly or indirectly, with the mechanisms closely tied to the particular feature considered.

6.1 Endogenous entry

We first examine entry decisions of firms by allowing the total measure of firms, S, to be determined endogenously. Following Hopenhayn (1992), we assume that entrants at time t must pay a fixed cost ξ, measured in units of labor, to draw their initial productivity from the current productivity distribution $G_t(\cdot)$. The expected net value of entry is therefore given by $V_{e,t} = \int_{\underline{z}_t}^{\infty} V_{p,t}(z)\,\mathrm{d}G_t(z) - \xi w_t$, and firms continue to enter whenever $V_{e,t} \geq 0$. The following proposition characterizes the equilibrium conditions in the search economy with endogenous entry:

Proposition 5 (Endogenous Entry)**.** *Suppose that $\xi > \kappa$, $\sigma = 1$, and Assumption 1 holds. Along the BGP, the total measure of firms S is the solution to the following fixed-point problem:*

$$\frac{1+\rho S\kappa}{(\xi-\kappa)\left[(1-\alpha)\theta+1\right]\rho S} = \frac{\int_0^1 \Gamma\left(\frac{\theta-1}{\theta}, \lambda S^{\gamma^s} B^{\gamma^b-1} x^{\theta}\right) x^{\theta}\,\mathrm{d}x}{\left(\lambda S^{\gamma^s} B^{\gamma^b-1}\right)^{-\frac{1}{\theta}} \exp\left(-\lambda S^{\gamma^s} B^{\gamma^b-1}\right) - \Gamma\left(\frac{\theta-1}{\theta}, \lambda S^{\gamma^s} B^{\gamma^b-1}\right)}, \tag{42}$$

and the solution is unique.

Proof. See Appendix A.8. □

Discussion The key insight conveyed by Proposition 5 is that, once entry is endogenous, finite search efficiency λ still does not *directly* affect the real GDP growth rate g given by (39). However, it does affect g *indirectly* by endogenously determining the total measure of firms S in (42). In turn, S directly enters (39) through the labor composition effect.

To see why, we first rewrite the free entry condition $V_{e,t} = 0$ in the following form:

$$\widetilde{D}v(1) + \widetilde{D}\left[\int_0^1 v(x)\,\mathrm{d}\hat{G}(x) - v(1)\right] = \xi. \tag{43}$$

Compared with (32), it is clear that the second term on the left-hand side of (43) is governed by adopters' equilibrium behavior and is always equal to the adoption cost κ. A higher κ implies less threat to entrants from technology adoption by competitors, making entry more attractive. Hence, an entrant's decision boils down to comparing the *variable* value of entry, $\widetilde{D}v(1)$, and the effective entry cost, $\xi - \kappa$, where $v(1) = \frac{\hat{\pi}(1)}{\rho}$ as shown in (29).

Now equation (42) becomes easily interpretable. An increase in search efficiency λ is reflected on the right-hand side through two channels: a reduction in aggregate misallocation captured by the numerator (i.e., the general-equilibrium effect), and a decrease in the detrended profit for the least productive firms, $\hat{\pi}(1)$, captured by the denominator.[23] However, unlike the partial-equilibrium effect on cumulative marginal profit gains, the channel operating through $\hat{\pi}(1)$ cannot be fully offset by the general-equilibrium force. Intuitively, the general-equilibrium effect averages out labor reallocation from low- to high-productivity firms, while the least productive firms—where misallocation is most severe—lose labor (and thus profits) disproportionately.

As a result, a higher search efficiency λ lowers the variable value of entry, which reduces the total measure of firms S and in turn spurs technology adoption by increasing the average production labor per firm, $\frac{1-S\kappa\theta g_z}{S}$, and hence the detrended demand shifter $\widetilde{D}$. Other related parameters also affect equilibrium firm entry in the expected way.[24]

6.2 Endogenous search effort

Next, we consider the case where sellers can actively choose their search efforts. In contrast to the baseline economy, each seller in this setting jointly chooses its price and search effort to maximize expected profit. For simplicity, we abstract from endogenous search on the buyer side. One can think of this case as sellers' marketing activities in reality, such

[23] Note from (28) that the denominator is equal to $\hat{\pi}(1)$.

[24] In particular, a higher entry cost ξ, a higher discount rate ρ, and greater misallocation induced by a higher $(1-\alpha)\theta$ all deter entry.

as advertising, with buyers being passive recipients of these advertisements.

Since for each variety firms differ only in productivity, we denote the optimal search effort of a firm with productivity z at time t as $\lambda_t(z)$. With search effort $\lambda_t(z)$, the Poisson arrival rate of encounters between the firm and potential buyers is $\lambda_t(z) S^{\gamma^s} B^{\gamma^b}$. Based on the superposition property of the Poisson process,[25] the arrival rate of a random encounter in the economy remains Poisson and is given by:

$$M_t = \overline{\lambda}_t S^{\gamma^s} B^{\gamma^b}, \tag{44}$$

where $\overline{\lambda}_t = S \int_{\underline{z}_t}^{\infty} \lambda_t(z) \, \mathrm{d}G_t(z)$ represents the aggregate search effort in equilibrium.

Search is costly, where its cost is measured in units of labor. We define the search cost function as a convex power function of the search effort λ:

$$\frac{\chi}{\varphi} \lambda^{\varphi} w_t, \tag{45}$$

with $\chi > 0$ and $\varphi > 1$.

Let $\pi_t(\lambda, p, z)$ denote the expected sales profit of a firm with search effort λ, price p, and productivity z at time t. Given the aggregate search effort $\overline{\lambda}_t$, the firm at time t chooses its optimal search effort $\lambda_t(z)$ and optimal price $\tilde{p}_t\left(z; \overline{\lambda}_t\right)$ to maximize its expected net profit $\pi_t(\lambda, p, z) - \frac{\chi}{\varphi} \lambda^{\varphi} w_t$. As in Assumption 1, we impose a similar regularity assumption that allows for analytical tractability:

Assumption 2 (Regularity Assumption with Endogenous Search Effort)**.** *At each time t, given the aggregate search effort $\overline{\lambda}_t$, a firm at the technology frontier sets its price as:*

$$\lim_{z \to \infty} \tilde{p}\left(z; \overline{\lambda}_t\right) = \frac{\left(\overline{\lambda}_t S^{\gamma^s} B^{\gamma^b - 1}\right)^{-\frac{1}{\theta}}}{\Gamma\left(\frac{\theta - 1}{\theta}\right)} \overline{c}. \tag{46}$$

Let

$$\hat{\pi}\left(\hat{z}; \overline{\lambda}_t\right) = \left(\overline{\lambda}_t S^{\gamma^s} B^{\gamma^b - 1}\right)^{-\frac{1}{\theta}} \exp\left(-\overline{\lambda}_t S^{\gamma^s} B^{\gamma^b - 1} \hat{z}^{\theta}\right) - \Gamma\left(\frac{\theta - 1}{\theta}, \overline{\lambda}_t S^{\gamma^s} B^{\gamma^b - 1} \hat{z}^{\theta}\right) \hat{z} \tag{47}$$

takes the same functional form as $\hat{\pi}(\hat{z})$ in the baseline economy, except that the exogenous

[25] For this property, see, e.g., Crane and Mccullagh (2015).

search efficiency λ is replaced by the endogenous aggregate search effort $\overline{\lambda}_t$. We term it the baseline detrended profit. The following proposition then characterizes the equilibrium conditions in the search economy with endogenous search effort:

Proposition 6 (Endogenous Search Effort)**.** *Suppose that $\sigma = 1$ and Assumption 2 holds. Along the BGP, aggregate search effort remains constant, i.e., $\overline{\lambda}_t = \overline{\lambda}$. Given $\overline{\lambda}$, the optimal search effort as a function of inverse relative productivity $\hat{z}$ is:*

$$\hat{\lambda}\left(\hat{z};\overline{\lambda}\right) = \left[\frac{\widetilde{D}\left(\overline{\lambda}\right)}{\chi}\right]^{\frac{1}{\varphi}} \hat{\pi}\left(\hat{z};\overline{\lambda}\right)^{\frac{1}{\varphi-1}}, \tag{48}$$

where

$$\widetilde{D}\left(\overline{\lambda}\right) = \frac{1 - S\kappa\theta g_z\left(\overline{\lambda}\right)}{S\left[\frac{1}{\varphi}\int_0^1 \hat{\pi}\left(x;\overline{\lambda}\right)^{\frac{\varphi}{\varphi-1}} \mathrm{d}\hat{G}\left(x\right) + \left(1-\alpha\right)\theta\int_0^1 \hat{\pi}\left(x;\overline{\lambda}\right)^{\frac{1}{\varphi-1}} \Gamma\left(\frac{\theta-1}{\theta}, \overline{\lambda}S^{\gamma^s}B^{\gamma^b-1}x^{\theta}\right)x^{\theta}\,\mathrm{d}x\right]} \tag{49}$$

is the detrended demand shifter with $g_z\left(\overline{\lambda}\right)$ given by (54). The aggregate search effort, $\overline{\lambda}$, in turn, is endogenously determined as the solution to the following fixed-point problem:

$$\overline{\lambda} = \left[\frac{\widetilde{D}\left(\overline{\lambda}\right)}{\chi}\right]^{\frac{1}{\varphi}} S\int_0^1 \hat{\pi}\left(x;\overline{\lambda}\right)^{\frac{1}{\varphi-1}} \mathrm{d}\hat{G}\left(x\right). \tag{50}$$

Aggregate real profit is given by:

$$\Pi_t\left(\overline{\lambda}\right) = \left[\widetilde{D}\left(\overline{\lambda}\right) S\int_0^1 \hat{\pi}\left(x;\overline{\lambda}\right)^{\frac{\varphi}{\varphi-1}} \mathrm{d}\hat{G}\left(x\right)\right] w_t. \tag{51}$$

The detrended value function $v\left(\hat{z};\overline{\lambda}\right)$ is the solution to the following first-order linear ODE:

$$v'\left(\hat{z};\overline{\lambda}\right) = \frac{\rho}{g_z\hat{z}} v\left(\hat{z};\overline{\lambda}\right) - \frac{\varphi-1}{\varphi}\frac{\hat{\pi}\left(\hat{z};\overline{\lambda}\right)^{\frac{\varphi}{\varphi-1}}}{g_z\hat{z}}, \tag{52}$$

and the solution reads:

$$v\left(\hat{z};\overline{\lambda}\right) = \frac{\varphi-1}{\rho\varphi}\left[\hat{\pi}\left(\hat{z};\overline{\lambda}\right)^{\frac{\varphi}{\varphi-1}} + \int_{\hat{z}}^1 \left(\frac{\hat{z}}{x}\right)^{\frac{\rho}{g_z}} \mathrm{d}\hat{\pi}\left(x;\overline{\lambda}\right)^{\frac{\varphi}{\varphi-1}}\right]. \tag{53}$$

Suppose that $\left[\Xi\left(\overline{\lambda}\right)+(1-\alpha)\,\theta\right]\rho S\kappa<1$. *The real GDP growth rate for* $\overline{\lambda}\in\mathbb{R}_{++}$ *is given by:*

$$g\left(\overline{\lambda}\right)=\frac{g_z\left(\overline{\lambda}\right)}{1-\alpha}=\frac{1-\left[\Xi\left(\overline{\lambda}\right)+(1-\alpha)\,\theta\right]\rho S\kappa}{(1-\alpha)\left[\Xi\left(\overline{\lambda}\right)+(1-\alpha)\,\theta+1\right]S\kappa\theta},\tag{54}$$

where

$$\Xi\left(\overline{\lambda}\right)=\frac{\frac{1}{\varphi}\int_0^1\hat{\pi}\left(x;\overline{\lambda}\right)^{\frac{\varphi}{\varphi-1}}\,\mathrm{d}\hat{G}\left(x\right)}{\int_0^1\hat{\pi}\left(x;\overline{\lambda}\right)^{\frac{1}{\varphi-1}}\,\Gamma\left(\frac{\theta-1}{\theta},\overline{\lambda}S^{\gamma^s}B^{\gamma^b-1}x^{\theta}\right)x^{\theta}\,\mathrm{d}x}.\tag{55}$$

All other variables retain the same expressions as in the baseline economy, except that endogenous $\overline{\lambda}$ *replaces the exogenous search efficiency parameter* λ.

Proof. See Appendix A.9. □

Discussion Proposition 6 provides three key insights. First, consistent with Arkolakis et al. (2025), the convexity of the search cost function makes it optimal for firms with high productivity to invest disproportionately more in search activities and, in return, capture disproportionately higher profits. However, equation (48) implies that the elasticity of optimal search effort $\hat{\lambda}\left(\hat{z};\overline{\lambda}\right)$ with respect to inverse relative productivity $\hat{z}$ is $\frac{1}{\varphi-1}\frac{\mathrm{d}\ln\hat{\pi}\left(\hat{z};\overline{\lambda}\right)}{\mathrm{d}\ln\hat{z}}$, with the elasticity of the baseline detrended profit $\hat{\pi}\left(\hat{z};\overline{\lambda}\right)$ serving as a sufficient statistic. This *non-isoelastic* feature is driven by within-variety search and matching and the resulting variable markups, which are absent from Arkolakis et al. (2025).

As part of the Poisson arrival rate of buyers, $\hat{\lambda}\left(\hat{z};\overline{\lambda}\right)$ enters the expected profit multiplicatively. However, what determines overall competitive pressure—and hence optimal pricing—is the aggregate search effort $\overline{\lambda}$ in equilibrium, which governs the Poisson arrival rate of sellers per buyer. As a result, the detrended net flow profit becomes $\frac{\varphi-1}{\varphi}\hat{\pi}\left(\hat{z};\overline{\lambda}\right)^{\frac{\varphi}{\varphi-1}}$, which appears in ODE (52). Compared with the baseline economy, heterogeneous search effort across firms makes the flow profit more *elastic* with respect to productivity changes, i.e., $\frac{\mathrm{d}\ln\left(\frac{\varphi-1}{\varphi}\hat{\pi}\left(\hat{z};\overline{\lambda}\right)^{\frac{\varphi}{\varphi-1}}\right)}{\mathrm{d}\ln\hat{z}}=\frac{\varphi}{\varphi-1}\frac{\mathrm{d}\ln\hat{\pi}\left(\hat{z};\overline{\lambda}\right)}{\mathrm{d}\ln\hat{z}}$ with $\frac{\varphi}{\varphi-1}>1$.

Second, the fixed-point problem (50) reveals that aggregate search effort $\overline{\lambda}$ is jointly determined by two *externalities* that each seller fails to internalize when choosing individual search effort: a positive externality whereby higher search effort collectively mitigates labor misallocation and boosts the detrended demand shifter $\widetilde{D}\left(\overline{\lambda}\right)$, and a negative externality whereby more intensive search strengthens competition and depresses detrended profits of all firms, i.e., $\hat{\pi}\left(\hat{z};\overline{\lambda}\right)$ for all $\hat{z}$.[26] In equilibrium, $\overline{\lambda}$ adjusts until the two externalities balance so that the right-hand side of (50) coincides with $\overline{\lambda}$ itself. The scale and elasticity parameters of the search cost function, χ and φ, play crucial roles in this process.

Finally, and most importantly, unlike the baseline economy with exogenous search efficiency, endogenous aggregate search effort $\overline{\lambda}$ affects the real GDP growth rate directly through $\Xi\left(\overline{\lambda}\right)$, as shown in (54). All parameters that pin down $\overline{\lambda}$, such as χ and φ, thus also influence aggregate growth. The reason is that costly search gives rise to a new source of *labor misallocation*: the positive labor demand from low-productivity firms for search activities. As shown in Appendix A.9, this misallocation is given by $\frac{1}{\varphi}\int_0^1 \hat{\pi}\left(x;\overline{\lambda}\right)^{\frac{\varphi}{\varphi-1}} \mathrm{d}\hat{G}\left(x\right)$, which directly depresses the detrended demand in (49). It is exactly this additional inefficiency that prevents the general- and partial-equilibrium effects of $\overline{\lambda}$ from completely offsetting each other. Instead, their net effect is summarized by $\Xi\left(\overline{\lambda}\right)$ in (55),[27] with the additional multiplier $\hat{\pi}\left(x;\overline{\lambda}\right)^{\frac{1}{\varphi-1}}$ in the denominator reflecting the impact of heterogeneous search effort on baseline effects. Aggregate growth $g\left(\overline{\lambda}\right)$ therefore depends on the relative strength of the two forces, with a higher $\Xi\left(\overline{\lambda}\right)$ implying relatively greater misallocation, weaker adoption incentives, and slower growth.[28]

6.3 Creative destruction

The Pareto distribution of productivity in the baseline economy implicitly places the technology frontier at infinity. In this section, we instead impose a bounded technology frontier to allow for both technology adoption and *creative destruction* à la Aghion and Howitt (1992).

[26] Note that from the perspective of consumer welfare, the externality operating through $\hat{\pi}\left(\hat{z};\overline{\lambda}\right)$ is positive. Here we label it "negative" only in the sense that intensified search generates negative spillovers from firms' perspective.

[27] Using integration by parts, we obtain:

$$\frac{1}{\varphi}\int_0^1 \hat{\pi}\left(x;\overline{\lambda}\right)^{\frac{\varphi}{\varphi-1}} \mathrm{d}\hat{G}\left(x\right) = \frac{\hat{\pi}\left(1;\overline{\lambda}\right)^{\frac{\varphi}{\varphi-1}}}{\varphi} + \frac{\int_0^1 \hat{\pi}\left(x;\overline{\lambda}\right)^{\frac{1}{\varphi-1}} \Gamma\left(\frac{\theta-1}{\theta}, \overline{\lambda} S^{\gamma^s} B^{\gamma^b-1} x^{\theta}\right) x^{\theta}\,\mathrm{d}x}{\varphi-1},$$

which implies that:

$$\Xi\left(\overline{\lambda}\right) = \frac{\hat{\pi}\left(1;\overline{\lambda}\right)^{\frac{\varphi}{\varphi-1}}}{\varphi\int_0^1 \hat{\pi}\left(x;\overline{\lambda}\right)^{\frac{1}{\varphi-1}} \Gamma\left(\frac{\theta-1}{\theta}, \overline{\lambda} S^{\gamma^s} B^{\gamma^b-1} x^{\theta}\right) x^{\theta}\,\mathrm{d}x} + \frac{1}{\varphi-1},$$

where the first term on the right-hand side can no longer be canceled out and always depends on $\overline{\lambda}$.

[28] Note from (39) and (54) that the baseline economy corresponds to $\Xi\left(\overline{\lambda}\right) = 0$.

Specifically, we assume that initial productivity follows a Pareto distribution truncated from both above and below:

$$\widetilde{G}_0(z) = \frac{1 - \left(\frac{z}{\underline{z}_0}\right)^{-\theta}}{1 - \left(\frac{\overline{z}_0}{\underline{z}_0}\right)^{-\theta}}, \tag{56}$$

where the upper bound $\overline{z}_0$ corresponds to the technology frontier at time 0. Creative destruction here is modeled as a cutting-edge research project that probabilistically enables a firm's productivity to leap to a level marginally above the current technology frontier, with an exogenous Poisson arrival rate $\delta > 0$. Overall, continuous arrivals of creative destruction drive the growth of the technology frontier $\overline{z}_t$.

Engaging in this research project entails costs denominated in labor. We assume that the required investment increases with inverse relative productivity and is defined as:

$$\varsigma \left(\frac{\underline{z}_t}{z}\right)^{\eta} w_t, \tag{57}$$

with $\varsigma \geq 0$ and $\eta \geq 0$.[29] With creative destruction, the Bellman equation for a firm with productivity z is given by:

$$r_t \widetilde{V}_{p,t}(z) = \tilde{\pi}_t(z) + \max\left\{\delta\left[\widetilde{V}'_{p,t}(\overline{z}_t)\frac{\mathrm{d}\overline{z}_t}{\mathrm{d}t} + \widetilde{V}_{p,t}(\overline{z}_t) - \widetilde{V}_{p,t}(z)\right] - \varsigma\left(\frac{\underline{z}_t}{z}\right)^{\eta} w_t, 0\right\} + \frac{\mathrm{d}\widetilde{V}_{p,t}(z)}{\mathrm{d}t}. \tag{58}$$

The firm trades off the value gain from creative destruction against the research cost, and it invests in the research project only when the former exceeds the latter.

Two challenges arise in this extended model. First, with the doubly truncated productivity distribution, the lower bound of marginal cost shifts from 0 to $\frac{\underline{z}_t}{\overline{z}_t}\overline{c}$, which complicates the optimal pricing problem (17). Second, the potential kink and non-monotonicity introduced by the maximum operator make it difficult to solve equation (58). Hence, we impose the following regularity assumption for tractability:

Assumption 3 (Regularity Assumption with Creative Destruction)**.** *At each time t, given*

[29] Therefore, while a low-productivity firm reaps a greater benefit from the research project, it pays higher investment costs according to (57).

the productivity lower bound $\underline{z}_t$, a firm at the technology frontier $\overline{z}_t$ sets its price as:

$$\tilde{p}\left(\overline{z}_t;\underline{z}_t\right)=\frac{\left[\tilde{\lambda}\left(\frac{\underline{z}_t}{\overline{z}_t}\right)S^{\gamma^s}B^{\gamma^b-1}\right]^{-\frac{1}{\theta}}\exp\left(-\tilde{\lambda}\left(\frac{\underline{z}_t}{\overline{z}_t}\right)S^{\gamma^s}B^{\gamma^b-1}\left(\frac{\underline{z}_t}{\overline{z}_t}\right)^{\theta}\right)}{\Gamma\left(\frac{\theta-1}{\theta},\tilde{\lambda}\left(\frac{\underline{z}_t}{\overline{z}_t}\right)S^{\gamma^s}B^{\gamma^b-1}\left(\frac{\underline{z}_t}{\overline{z}_t}\right)^{\theta}\right)}\overline{c},\tag{59}$$

where $\tilde{\lambda}\left(\frac{\underline{z}_t}{\overline{z}_t}\right)=\frac{\lambda}{1-\left(\frac{\underline{z}_t}{\overline{z}_t}\right)^{\theta}}$. The research cost scale parameter ς is sufficiently small such that:

$$\varsigma\leq\inf_{z\in[\underline{z}_t,\overline{z}_t]}\frac{\delta}{w_t}\left[\widetilde{V}'_{p,t}\left(\overline{z}_t\right)\frac{\mathrm{d}\overline{z}_t}{\mathrm{d}t}+\widetilde{V}_{p,t}\left(\overline{z}_t\right)-\widetilde{V}_{p,t}\left(z\right)\right]\left(\frac{z}{\underline{z}_t}\right)^{\eta},\tag{60}$$

and the elasticity parameter η is sufficiently small such that:

$$\eta\leq\frac{\rho+\delta}{\mathrm{d}\ln\underline{z}_t/\mathrm{d}t}.\tag{61}$$

Similar to Assumption 1, (59) serves as a boundary condition and ensures the monotonicity of the optimal pricing function. Inequality (60) implies that every firm has an incentive to invest in the research project. Inequality (61) is somewhat subtle. As shown in Appendix A.10, it serves as a sufficient condition to ensure that the value function $\widetilde{V}_{p,t}\left(z\right)$ is monotonically increasing in productivity z, implying that only firms at the productivity lower bound $\underline{z}_t$ undertake technology adoption at time t as in the baseline model.

Let $v\left(\hat{z};\lambda,\rho\right)$ denote the baseline model's detrended value function (29) of a firm with inverse relative productivity $\hat{z}$, with effective search efficiency λ and discount rate ρ. The following proposition then characterizes the equilibrium conditions in the search economy with both technology adoption and creative destruction:

Proposition 7 (Creative Destruction)**.** *Suppose that $\sigma=1$ and Assumption 3 holds. Along the BGP, both the lower and upper bounds of productivity grow at the same constant rate g_z, and the productivity distribution remains doubly truncated Pareto with shape parameter θ. The constant inverse productivity dispersion, $\iota=\frac{\underline{z}_t}{\overline{z}_t}$ for all t, is endogenously determined as:*

$$\iota=\left(\frac{\delta}{\theta g_z+\delta}\right)^{\frac{1}{\theta}}.\tag{62}$$

Given ι, the markup of a firm with marginal cost $c \in [\iota\bar{c}, \bar{c}]$ is given by:

$$m(c;\iota) = \frac{p(c;\iota)}{c} = \frac{\left[\tilde{\lambda}(\iota) S^{\gamma^s} B^{\gamma^b-1} H(c)\right]^{-\frac{1}{\theta}} \exp\left(-\tilde{\lambda}(\iota) S^{\gamma^s} B^{\gamma^b-1} H(c)\right)}{\int_{\tilde{\lambda}(\iota) S^{\gamma^s} B^{\gamma^b-1} H(c)}^{\infty} x^{-\frac{1}{\theta}} \exp(-x)\,\mathrm{d}x}. \tag{63}$$

The detrended value function takes the following form:

$$\tilde{v}(\hat{z}) = v_\delta(\hat{z}) + \frac{\delta}{\rho}\left[v_\delta(\iota) - v_\delta'(\iota)\,\iota g_z\right], \tag{64}$$

where

$$v_\delta(\hat{z}) = v\left(\hat{z}; \tilde{\lambda}(\iota), \rho+\delta\right) - \frac{c_\delta(\hat{z})}{\rho+\delta-\eta g_z} \tag{65}$$

represents the detrended variable value and

$$c_\delta(\hat{z}) = \frac{\varsigma \hat{z}^\eta}{\widetilde{D}(\iota)}\left(1 - \frac{\eta g_z}{\rho+\delta}\hat{z}^{\frac{\rho+\delta-\eta g_z}{g_z}}\right) \tag{66}$$

is the effective detrended research cost, with

$$\widetilde{D}(\iota) = \left[(1-\alpha)\,\theta \int_\iota^1 \Gamma\left(\frac{\theta-1}{\theta}, \tilde{\lambda}(\iota) S^{\gamma^s} B^{\gamma^b-1} x^\theta\right) x^\theta\,\mathrm{d}x\right]^{-1}\left[\frac{1-\iota^\theta - S\kappa\theta g_z}{S} - \frac{\varsigma\theta\left(1-\iota^{\eta+\theta}\right)}{\eta+\theta}\right] \tag{67}$$

being the detrended demand shifter. The productivity growth rate g_z, in turn, is given by the solution to:

$$\widetilde{D}(\iota, g_z)\left[\int_\iota^1 \tilde{v}(x;\iota,g_z)\,\mathrm{d}\hat{\tilde{G}}(x;\iota) - \tilde{v}(1;\iota,g_z)\right] = \kappa, \tag{68}$$

where $\hat{\tilde{G}}(x;\iota) = \frac{x^\theta-\iota^\theta}{1-\iota^\theta}$. Aggregate real profit $\Pi_t(\iota)$ and the real wage $w_t(\iota)$ are both functions of ι and are given by equations (117) and (119) in Appendix A.10, respectively.

Proof. See Appendix A.10. □

Discussion Three key insights emerge from Proposition 7. First, (62) shows that once creative destruction is introduced, the relative distance between the least productive firms and the technology frontier along the BGP—which is inversely related to ι—is endogenously determined by the interaction between technology adoption and creative destruction.

To build intuitions, consider a positive shock to δ. Following the shock, firms leapfrog the frontier more frequently through creative destruction, raising the frontier growth rate $g_{\overline{z}}$ and stretching the productivity distribution (i.e., ι drops). On the other hand, a larger gap relative to the frontier strengthens adoption incentives, boosting the growth rate of the productivity lower bound, $g_{\underline{z}}$, and thereby compressing the distribution (i.e., ι raises). The economy will reach a new BGP once the two growth rates equalize again, i.e., $g_{\overline{z}} = g_{\underline{z}} = g_z$, at which point ι stabilizes and is given by (62).

The second insight relates to the static effects of ι, which are twofold. Compared with a distribution compressed into a narrow interval, more dispersed productivity (i.e., lower ι) implies weaker within-variety competition among firms. This impact is reflected in the new markup equation (63): When setting prices, firms treat $\tilde{\lambda}(\iota) = \frac{\lambda}{1-\iota^\theta}$, rather than λ, as buyers' *effective* search efficiency.[30]

Changes in ι also affect aggregation by reshaping the CDF of the inverse relative productivity, $\hat{\tilde{G}}(\cdot\,;\iota)$. As a result, all aggregate variables now become functions of ι. For the detrended demand shifter $\widetilde{D}(\iota)$ in (67), ι additionally appears through its effect on the aggregate labor used for research.

The final insight is about the decomposition of the detrended value of firms $\tilde{v}(\hat{z})$. According to (64), $\tilde{v}(\hat{z})$ consists of two components: the *variable* value $v_\delta(\hat{z})$ and the expected value gain from creative destruction, $\frac{\delta}{\rho}[v_\delta(\iota) - v'_\delta(\iota)\,\iota g_z]$.[31] The variable component in (65) can be written as the value from production minus the discounted value of the effective research cost, where the value from production, $v\left(\hat{z}; \tilde{\lambda}(\iota), \rho+\delta\right)$, is the baseline production value evaluated at the effective search efficiency $\tilde{\lambda}(\iota)$ and the effective discount rate $\rho+\delta$. Intuitively, $\tilde{\lambda}(\iota)$ carries over from the optimal markup (63), and the leapfrogging rate δ makes future profit flows less valuable. The effective research cost $c_\delta(\hat{z})$ is discounted at rate $\rho+\delta-\eta g_z$.[32] A fast-growing productivity lower bound (i.e., higher g_z) implies that firms will bear ever higher research costs over time, an effect that is amplified by a more convex cost function (i.e., higher η).

What roles do search frictions play in determining the productivity growth rate g_z in

30 Clearly, markup (18) in the baseline economy corresponds to $\iota \to 0$, the case with the weakest overall competition.

31 Note that $v'_\delta(\iota) < 0$ and thus $-v'_\delta(\iota)\,\iota g_z$ represents the incremental value from being marginally above the current technology frontier, relative to being exactly at the frontier.

32 Note that the effective research cost given by (66) includes an adjustment term $\frac{\eta g_z}{\rho+\delta}\hat{z}^{\frac{\rho+\delta-\eta g_z}{g_z}}$. This term is introduced to ensure that the smooth-pasting condition $\tilde{v}'(1) = v'_\delta(1) = 0$ holds.

the presence of creative destruction, and how does that differ from the baseline economy? To answer these questions, we turn to the following corollary, which unpacks the value-matching condition (68):

Corollary 1 (Adoption Value with Creative Destruction). *In the value-matching condition (68), the expected detrended value of technology adoption can be expressed as:*

$$\int_{\iota}^{1} \tilde{v}\left(x;\iota,g_z\right) \mathrm{d}\hat{\widetilde{G}}\left(x;\iota\right) - \tilde{v}\left(1;\iota,g_z\right) = \Delta v\left(\iota,g_z\right) - \frac{\Delta c_{\delta}\left(\iota,g_z\right)}{\rho+\delta-\eta g_z}, \tag{69}$$

where

$$\Delta v\left(\iota,g_z\right) = \frac{1}{\rho+\delta+\theta g_z}\int_{\iota}^{1}\Gamma\left(\frac{\theta-1}{\theta},\tilde{\lambda}\left(\iota\right)S^{\gamma^s}B^{\gamma^b-1}x^{\theta}\right)\left[\frac{x^{\theta}-\iota^{\theta}}{1-\iota^{\theta}}-\psi\left(x;\iota,g_z\right)\right]\mathrm{d}x, \tag{70}$$

with $\psi\left(x;\iota,g_z\right) = \frac{\theta g_z \iota^{\theta}}{(\rho+\delta)\left(1-\iota^{\theta}\right)}\left[1-\left(\frac{\iota}{x}\right)^{\frac{\rho+\delta}{g_z}}\right]$, *and*

$$\frac{\Delta c_{\delta}\left(\iota,g_z\right)}{\rho+\delta-\eta g_z} = -\frac{\varsigma\eta}{\left(\rho+\delta-\eta g_z\right)\widetilde{D}\left(\iota\right)}\int_{\iota}^{1}x^{\eta-1}\left(1-x^{\frac{\rho+\delta-\eta g_z}{g_z}}\right)\frac{x^{\theta}-\iota^{\theta}}{1-\iota^{\theta}}\,\mathrm{d}x. \tag{71}$$

Proof. See Appendix A.11. □

Discussion Equation (69) shows that the expected detrended value of adoption is given by the expected gain in production value, $\Delta v\left(\iota,g_z\right)$, plus the expected reduction in discounted effective research cost, $-\frac{\Delta c_{\delta}(\iota,g_z)}{\rho+\delta-\eta g_z}$. After netting out heterogeneous research costs, the gross value of creative destruction, $\frac{\delta}{\rho}\left[v_{\delta}\left(\iota\right)-v_{\delta}'\left(\iota\right)\iota g_z\right]$, affects all firms equally and is therefore irrelevant for technology adoption. Instead, the leapfrogging rate δ influences adoption incentives and aggregate growth via two channels: *directly* by entering the discount rates and *indirectly* by determining the inverse productivity dispersion ι.

The impact of ι is summarized in equations (70) and (71). Comparing (70) with (41), an essential difference is that marginal profit gains are no longer weighted by the productivity CDF. Instead, the weight is reduced by a *truncation factor* $\psi\left(x;\iota,g_z\right)$. This factor arises because the productivity distribution is now truncated at an intermediate point where the future value loss from competitors' technology adoption is typically large.[33] The expected

[33] In contrast, in the baseline economy with unbounded productivity, frontier firms are infinitely far away from the productivity lower bound, implying that the value-loss component is zero.

loss component is thus larger and can no longer be fully captured by θg_z in the effective discount rate.

More importantly, the truncation factor $\psi(x;\iota,g_z)$ drives a wedge between the partial- and general-equilibrium effects of search efficiency, so that search efficiency now has a direct impact on aggregate growth.[34] Unfortunately, the intricate effects of g_z on the truncation factor and the effective research cost make a closed-form solution no longer possible.

7 Quantitative Analysis

In this section, we take U.S. data to the model to quantify BGP welfare changes over three decades in the baseline search economy and its variants.

7.1 Parameterization

In this section, we estimate and calibrate the parameters required for the welfare analysis. We treat Poisson arrival rate of sellers $\frac{M}{B}$, Pareto shape θ, and technology adoption cost κ as having undergone structural changes over the past three decades, and quantify their impact on long-run BGP welfare. In the extension with endogenous search effort, the parameters χ and φ that determine $\frac{M}{B}$ are also treated as time-varying. We focus on $\frac{M}{B}$ and θ because they directly affect within-variety markups in (18) and hence the micro-to-macro linkage in our model. We focus on κ since the surge in barriers to knowledge diffusion is highlighted by Akcigit and Ates (2023) as a crucial explanation for the slowdown in U.S. business dynamism. We summarize the parameterization in Table 1.

GMM estimation We use GMM to estimate the Poisson arrival rate of sellers $\frac{M}{B}$ and the Pareto shape parameter θ based on the parsimonious within-variety markup equation (18). The estimation procedure consists of three steps: estimating markups, constructing moments, and estimating parameters.

First, we estimate markups for Compustat firms based on the production approach of De Loecker et al. (2020), which expresses markups as the elasticity of output with respect

[34] Note that the impact of search efficiency also depends on the expected reduction in research cost given by (71). As the dynamics of research cost have been fully captured by ηg_z in the effective discount rate, the remaining component on the right-hand side of (71) is simply the cumulative marginal reduction in research cost.

Table 1. Parameterization

Description (Notation)	Method	Value
Poisson arrival rate of sellers ($\frac{M}{B}$)	GMM estimation (see text)	0.08 $[0.03, 0.13]$ for 1989–1993 0.12 $[0.05, 0.20]$ for 2019–2023
Pareto shape of productivity distribution (θ)	GMM estimation (see text)	7.96 $[6.40, 9.53]$ for 1989–1993 4.84 $[3.86, 5.83]$ for 2019–2023
Consumer discount rate (ρ)	External calibration from Akcigit and Kerr (2018)	0.02
Cost share of intermediate inputs (α)	External calibration from Edmond et al. (2023)	0.45
Poisson arrival rate of creative destruction (δ)	External calibration from Aghion et al. (2019)	0.01
Research cost elasticity (η)	External calibration from Acemoglu et al. (2018)	0.50
Total measure of firms per variety (S)	Internal calibration (see text)	0.01
Fixed cost of technology adoption (κ)	Internal calibration (see text)	Model-specific
Fixed cost of entry (ξ)	Internal calibration (see text)	2670.78
Search cost elasticity (φ)	Internal calibration (see text)	2.32 for 1989–1993 1.36 for 2019–2023
Research cost scale (ς)	Internal calibration (see text)	0.72
Elasticities in Poisson encounter rate (γ^s and γ^b)	No calibration required given $\frac{M}{B}$	–
Search cost scale (χ)	No calibration required given $\frac{M}{B}$	–
Initial productivity lower bound (z_0)	Normalization	1

to a variable input multiplied by the inverse of that input's expenditure share in revenue. Output elasticity estimates, in turn, are obtained by combining the control function approach of Ackerberg et al. (2015) with the refinements in De Ridder et al. (2025).

Since equation (18) characterizes markups for firms producing homogeneous goods, we rely on time-varying product cosine similarity measures from the Embedding-Based Text Network Industry Classification (ETNIC) data developed by Hoberg and Phillips (2025) to identify each firm's closest competitors. Merging ETNIC with financial information from Compustat yields an estimation sample spanning 1989–2023.

For each benchmark firm j, we follow Cabezon and Hoberg (2026) and classify firms i whose cosine similarity with j lies in the top 1% of the sample as its most direct competitors, and then obtain their markup estimates $\hat{m}^j_{it}$ for the resulting subsample. Because firm i may be classified as a direct competitor of multiple firms, we compute the following product-similarity-weighted average to assign greater weight to estimates indexed by benchmark firms j that are closer to firm i in product space:

$$\hat{m}_{it} = \sum_{j \in \mathcal{J}_{it}} \frac{\text{cosine}_{ijt}}{\sum_{h \in \mathcal{J}_{it}} \text{cosine}_{iht}} \hat{m}^j_{it}, \tag{72}$$

where $\mathcal{J}_{it}$ denotes the set of firms with which firm i is a direct competitor at time t and cosine_{ijt} is the ETNIC similarity score between firms i and j at time t. We treat $\hat{m}_{it}$ as the empirical counterpart of (18). For detailed data description, methodology, and discussion of the markup estimation, see Appendix B.

Proposition 1 and equation (23) jointly imply a strictly positive relationship between markup and gross profit, which is confirmed by the data, using the binned scatter plot in Panel A of Figure 1. We exploit this relationship to construct moment conditions. Specifically, we estimate the following specification with the log of estimated markups as the dependent variable:

$$\log \hat{m}_{it} = \beta_1^{\text{data}} + \sum_{n=2}^{4} \beta_{\Delta n}^{\text{data}} Q_{n,it} + \epsilon_{it}, \tag{73}$$

where $Q_{n,it}$ is a dummy variable that equals one if the gross profit of firm i at time t falls into the nth quartile, and zero otherwise. We stack the resulting coefficients into a vector $\boldsymbol{\beta}^{\text{data}}$.

Given parameters $\left(\frac{M}{B}, \theta\right)$, the same regression can be performed using model-implied markups from (18), yielding a coefficient vector $\boldsymbol{\beta}^{\text{model}}$. Therefore, the GMM estimator is given by:

$$\left(\widehat{\frac{M}{B}}, \hat{\theta}\right) = \arg \min_{\left(\frac{M}{B}, \theta\right)} \left(\boldsymbol{\beta}^{\text{data}} - \boldsymbol{\beta}^{\text{model}}\right)' \boldsymbol{W} \left(\boldsymbol{\beta}^{\text{data}} - \boldsymbol{\beta}^{\text{model}}\right), \tag{74}$$

where $\boldsymbol{W}$ is the optimal weighting matrix. To capture structural changes in search efficiency and the shape of the productivity distribution—and their implications for the BGP welfare—we estimate (74) separately for the first and last five years of the sample period, i.e., 1989–1993 and 2019–2023. Importantly, the isomorphic structure of our markup equation across different model variants implies that the estimated structural changes apply to all model extensions.[35]

Panel B of Figure 1 compares the moments from the data and the model. Given that (74) is overidentified, the model does a good job tracking the empirical relationship, except that it fails to capture the sharp increase in estimated markups from the first to the second quartile of gross profit.

Our estimates indicate that the Poisson arrival rate of sellers rises by 50% over the three decades—from 0.08 to 0.12—which is largely attributable to the revolution in information and communication technology (Eaton et al., 2022a). Using supply-chain data on the Japanese corporate universe from 2008 to 2016, Miyauchi (2024) estimates the corresponding object under monopolistic competition to be 0.14, close to our estimate for the

[35] For $\frac{M}{B}$, it corresponds to $\lambda S^{\gamma^s} B^{\gamma^b - 1}$ in the baseline model and the extension with endogenous entry. At the same time, it maps to $\overline{\lambda} S^{\gamma^s} B^{\gamma^b - 1}$ and $\tilde{\lambda}(\iota) S^{\gamma^s} B^{\gamma^b - 1}$ in the endogenous-search-effort and creative-destruction extensions, respectively.

late sub-period.

The estimated Pareto shape parameter falls from 7.96 to 4.84 (i.e., a 39.20% drop), implying a fatter right tail of the productivity distribution. This structural change is consistent with empirical evidence on the rise of superstar firms (Autor et al., 2020; Chen, 2023; Kwon et al., 2024). Still, our estimates of θ lie within the plausible range $[3.60, 8.28]$ suggested by Eaton et al. (2011).[36]

Panels C and D of Figure 1 plot the within-variety markup (18) and pass-through (21), respectively, based on the estimated $\left(\widehat{\frac{M}{B}}, \hat{\theta}\right)$ for the two sub-periods. The upward shift in the markup curve over time is mainly due to the decrease in θ, consistent with the increase in market power driven by the top firms (De Loecker et al., 2020). In our model, small firms exhibit higher pass-through due to the static escape-competition effect. This pattern aligns with existing theory (e.g., Atkeson and Burstein, 2008) and empirical evidence (e.g., Amiti et al., 2019). However, because our model features atomistic firms and thus intense competition, the implied pass-through is too low relative to the estimates in Amiti et al. (2019). We therefore interpret our pass-through as a lower bound.

External calibration We assign the consumer discount rate ρ to 0.02 following Akcigit and Kerr (2018), which corresponds to an annual discount factor of 0.98. The cost share of intermediate inputs in roundabout production, α, is set to 0.45 to match the material share calibrated in Edmond et al. (2023).

Using indirect inference to match U.S. firm dynamics, Aghion et al. (2019) report a creative-destruction arrival rate of around 0.01. We therefore set $\delta = 0.01$. It can be shown that the elasticity parameter η governing research cost is qualitatively equivalent to the elasticity of R&D efficiency with respect to firm size.[37] The corresponding parameter in Acemoglu et al. (2018) is calibrated to 0.50, implying diminishing returns. We set η to the same value.[38]

36 See footnote 40 in Eaton et al. (2011) for details.

37 Specifically, we can define R&D efficiency as the creative-destruction arrival rate achieved per unit of research labor. From (57), the R&D efficiency of a firm with productivity z is given by $\frac{\delta}{\varsigma}\left(\frac{z}{\underline{z}_t}\right)^{\eta}$, where productivity z is positively related to firm size as shown by (23).

38 Our ex-post verification shows that $\eta = 0.5$ satisfies condition (61) in Assumption 3 given our choice of other parameters.

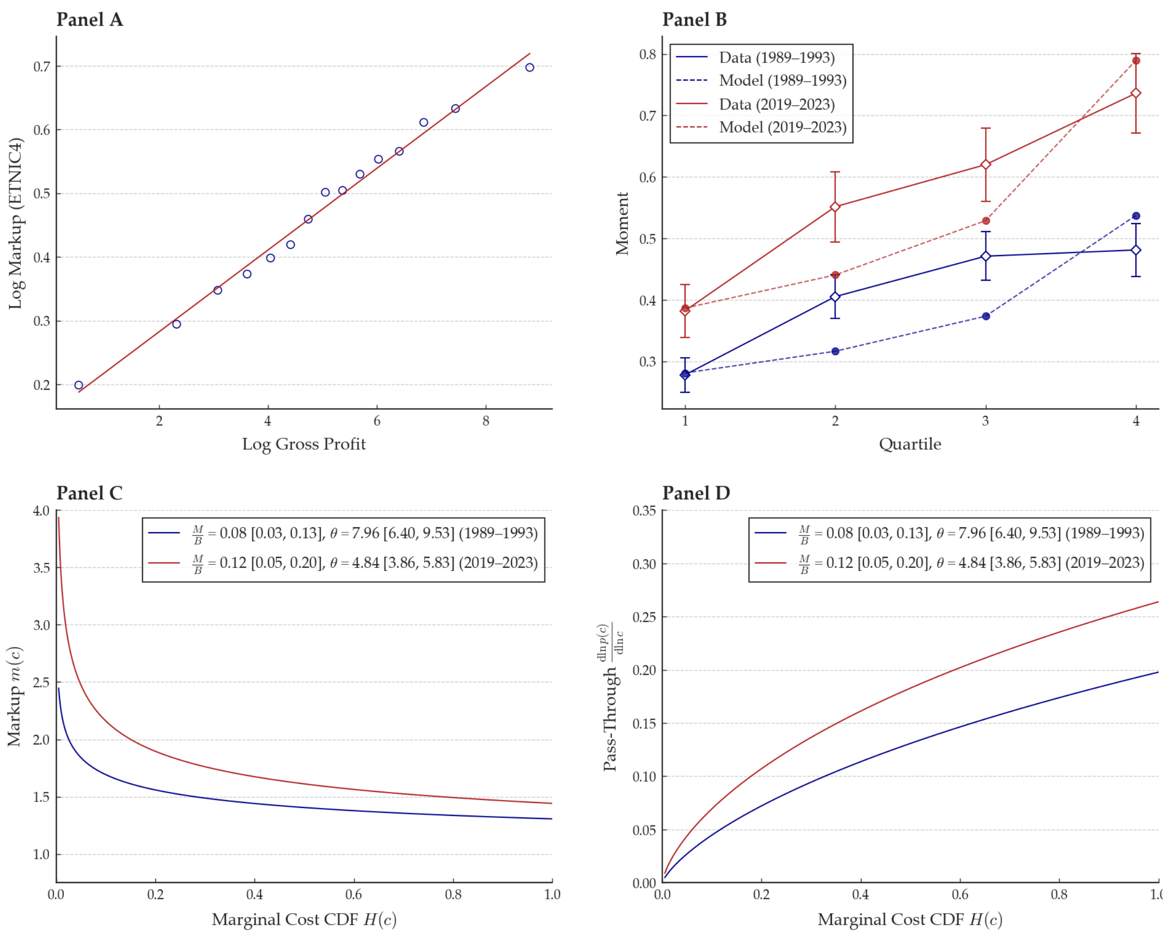

Figure 1. GMM Estimation and Markup Analysis.

This figure summarizes plots related to the GMM estimation and markup analysis. Panel A shows a binned scatter plot of log markup against log gross profit, where markups are estimated using ETNIC data from Hoberg and Phillips (2025) and financial data from Compustat over the sample period from 1989 to 2023. Panel B compares the moments obtained by running the following regression in the data and the model:

$$\log \hat{m}_{it} = \beta_1 + \sum_{n=2}^{4} \beta_{\Delta n} \, Q_{n,it} + \epsilon_{it},$$

where the plotted moments correspond to β_1 and $\beta_1 + \beta_{\Delta n}$ for $n \in \{2, 3, 4\}$. Panels C and D illustrate the model-implied within-variety markup and pass-through, respectively, based on the estimated $\left(\widehat{\frac{M}{B}}, \hat{\theta}\right)$ for the two sub-periods.

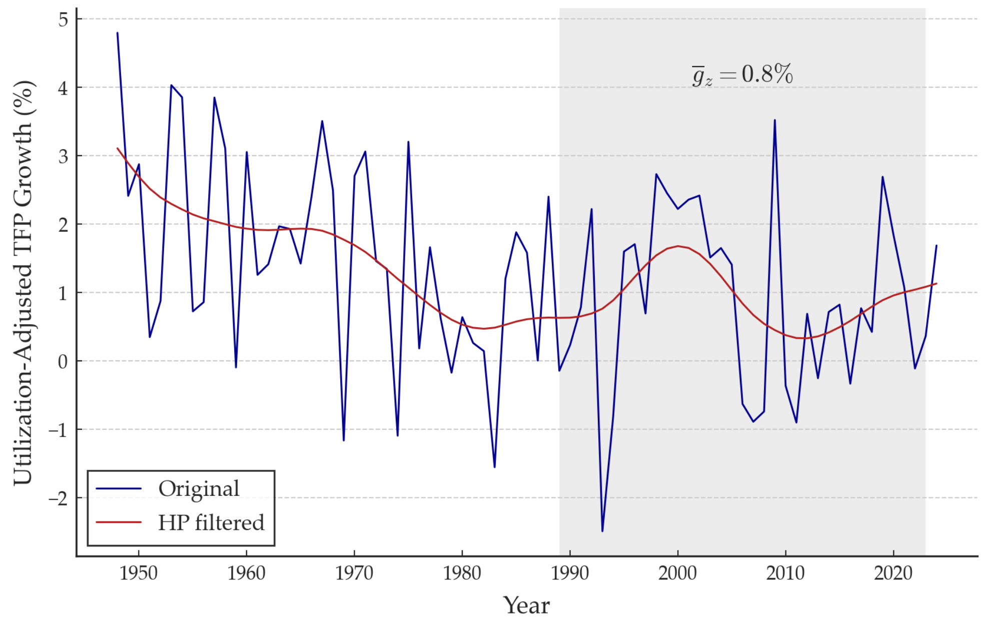

Figure 2. TFP Growth.

This figure plots U.S. utilization-adjusted TFP growth from Fernald (2015) in blue. The red curve corresponds to the Hodrick-Prescott (HP) filtered series using an annual smoothing parameter of 100.

Internal calibration Using FactSet Revere Supply Chain data, Wu et al. (2025) report an average of 7.6 buyers per supplier in the U.S. The model counterpart of this statistic can be derived from (11):

$$\bar{b} = \int_0^{\bar{c}} \bar{b}_t\left(p\left(c\right)\right) \mathrm{d}H\left(c\right) = \frac{B}{S}\left[1 - \exp\left(-\frac{M}{B}\right)\right], \tag{75}$$

where $B = S + 1$. By plugging in the initial estimate of $\frac{M}{B}$, we back out the total measure of firms per variety, $S = 0.01$. When estimating $\frac{M}{B}$, we classify firm pairs with top-1% similarity scores as direct competitors. The value of S therefore coincides with normalizing the entire Compustat sample to have unit mass.

We rely on U.S. utilization-adjusted total factor productivity (TFP) growth data from Fernald (2015) to back out the technology adoption cost κ separately for each model variant using equations (39), (54), and (68). The time series in Figure 2 shows that TFP growth fluctuates around a relatively low level over our sample period, with an average of 0.8%. We take this value as g_z. We then use the κ values calibrated under the two sets of $\left(\widehat{\frac{M}{B}}, \hat{\theta}\right)$ to answer the following question: *How large the change in the technology adoption cost should* *be to bring aggregate growth back to the sample average?*

For the model extension with endogenous entry, we treat $S = 0.01$ as the initial mass of firms and invert the fixed-point problem (42) to pin down the entry cost $\xi = 2670.78$, which is far larger than the technology adoption cost κ. As a benchmark, the ratio of entry cost to initial adoption cost in our baseline model is 11.90, comparable to the corresponding value of 7.88 in Perla et al. (2021).

When search is endogenous, the search cost of a firm with inverse relative productivity $\hat{z}$ can be rewritten as $\frac{1}{\varphi} D_t\left(\overline{\lambda}\right) \hat{\pi}\left(\hat{z}; \overline{\lambda}\right)^{\frac{\varphi}{\varphi-1}}$ by substituting (48) into (45), i.e., a fraction $\frac{1}{\varphi}$ of the gross profit $D_t\left(\overline{\lambda}\right) \hat{\pi}\left(\hat{z}; \overline{\lambda}\right)^{\frac{\varphi}{\varphi-1}}$. Using selling, general, and administrative expenses (SG&A) as a proxy for the search cost, the empirical counterpart of the search-cost elasticity parameter φ is therefore given by $\frac{\text{Gross Profit}}{\text{SG\&A}}$. The binned scatter plot in Panel A of Figure 5 highlights a near-unit elasticity between SG&A and gross profit in the data, providing support for the proportional relationship. We thus set φ to the sample median of $\frac{\text{Gross Profit}}{\text{SG\&A}}$ over the 1989–1993 and 2019–2023 sub-periods, i.e., 2.32 and 1.36, respectively, which implies a declining trend in the elasticity of search cost with respect to search effort over time.[39] For comparison, the corresponding parameter for the 2019 Chilean economy is 4.5 in the goods sector and 2.8 in the service sector (Arkolakis et al., 2025), implying a less efficient search technology than in the United States.

In the extension with creative destruction, we choose the research-cost scale parameter $\varsigma = 0.72$ to match the 0.71% share of research labor in 2000 U.S. total employment obtained from the OECD Main Science and Technology Indicators.[40]

Other parameters Given the estimated Poisson arrival rate of sellers $\frac{M}{B}$, the elasticity parameters γ^s and γ^b need not be calibrated separately, as they affect welfare only through $\frac{M}{B}$. Similarly, in the extension with endogenous search effort, the welfare analysis does not require separate calibration of the search-cost scale parameter χ. Finally, we normalize

[39] We choose the sample median to minimize the influence of outliers. Given that SG&A also includes other costs unrelated to search activities, we view our calibrated values of φ as lower bounds.

[40] Specifically, total research labor in the model is given by:

$$L_\delta = S \int_\iota^1 \varsigma x^\eta \, \mathrm{d}\hat{\tilde{G}}\left(x; \iota\right) = \frac{S \varsigma \theta \left(1 - \iota^{\eta+\theta}\right)}{\left(\eta + \theta\right)\left(1 - \iota^\theta\right)},$$

which is exactly the share of research labor as we normalize total labor supply to one. We verify that $\varsigma = 0.72$ satisfies condition (60) in Assumption 3 given our choice of other parameters.

the initial productivity lower bound $\underline{z}_0$ to one without loss of generality.

7.2 Welfare analysis

Baseline model Figure 3 summarizes the welfare analysis for the baseline model with exogenous search only. Given the initial θ, the pro-competitive effect of the secular reduction in search frictions raises the initial real wage by 8.66% (Panel A) and detrended demand by 3.00% (Panel B), but lowers aggregate profitability by 5.71% (Panel C). In contrast, the increased market power of superstar firms at lower θ reduces the real wage and detrended demand shifter by 20.14% and 8.13%, respectively, while boosting aggregate profitability by 22.20%.

Panel D shows that a fatter tail of the productivity distribution (i.e., lower θ) substantially increases the expected value of adoption and thereby accelerates real GDP growth by 165.89%. Reverting the economy to the original growth rate requires a large 120.61% increase in the technology adoption cost κ. Note that changes in the level of $\frac{M}{B}$ are unrelated to aggregate growth, as we show in Proposition 4.

Taken together, although the static effects on consumption are comparable, the muted dynamic effect makes the overall welfare impact of $\frac{M}{B}$ quantitatively small relative to that of θ: Changes in $\frac{M}{B}$ and θ alone raise BGP welfare by 3.03% and 144.32%, respectively, with a joint effect of 150.58%.

Endogenous entry Figure 4 illustrates the welfare analysis under endogenous firm entry. The intensified competition associated with the three-decade increase in search efficiency depresses the variable value of entry, reducing the total measure of firms per variety by 19.88% (Panel A). Fewer active firms raise the average production labor per firm, $\frac{1-S\kappa\theta g_z}{S}$, and thus increase the detrended demand shifter sharply by 27.43% (Panel B). The impact on firm-level demand is largely offset after aggregation, resulting in a 6.02% decline in aggregate profitability (Panel C), close to 5.71% in the baseline model. The difference is driven by the composition effect between production and adoption labor, $S\kappa\theta g_z$. The drop in θ, in contrast, has opposite effects: Higher aggregate market power elevates profitability by 23.54%, which attracts 68.74% more entry and in turn dilutes demand by 44.21%.

In sharp contrast to the baseline economy with fixed entry, the secular reduction in

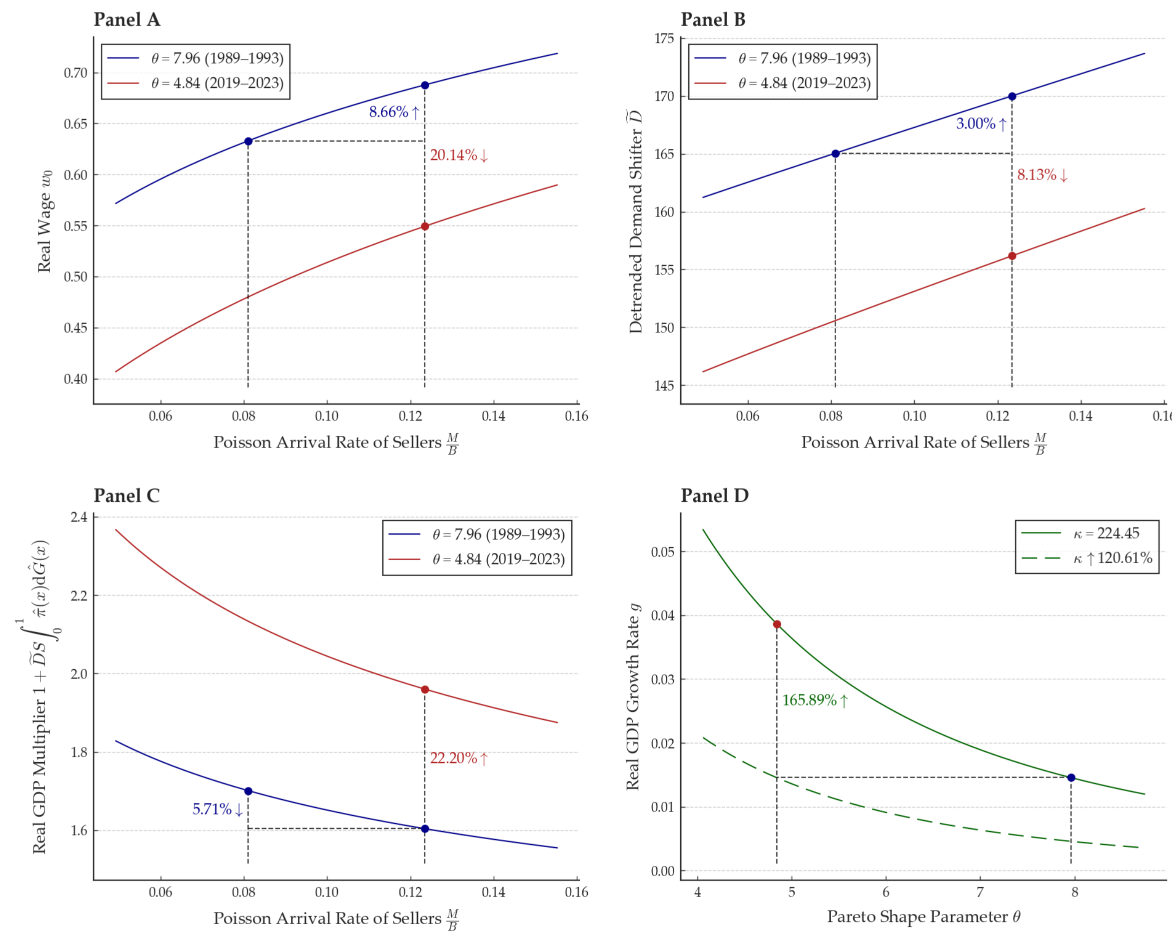

Figure 3. Welfare Analysis: Baseline Model.

This figure plots variables underlying BGP welfare in the baseline model. Panel A illustrates the initial real wage w_0. Panel B shows the detrended demand shifter $\widetilde{D}$. Panel C presents the real GDP multiplier $1 + \widetilde{D}S\int_0^1 \hat{\pi}(x)\,\mathrm{d}\hat{G}(x)$. Panel D visualizes the real GDP growth rate g.

search frictions now indirectly raises real GDP growth by 31.16% through the demand effect, as shown in Panel D. On the other hand, increased entry and the resulting dilution of demand substantially offset the direct effect of a lower θ in reducing within-variety labor misallocation, yielding a 42.51% growth impact much smaller than in the baseline model. Accordingly, a smaller 50.09% increase in the technology adoption cost is sufficient to drag the economy back to the average growth rate.

Altogether, once endogenous entry is taken into account, the separate effects of $\frac{M}{B}$ and θ on BGP welfare become comparable, with nearly identical values of 30.91% and 30.51%, respectively. Their joint effect, by contrast, reaches 79.84%, implying a relatively strong complementarity between the two.

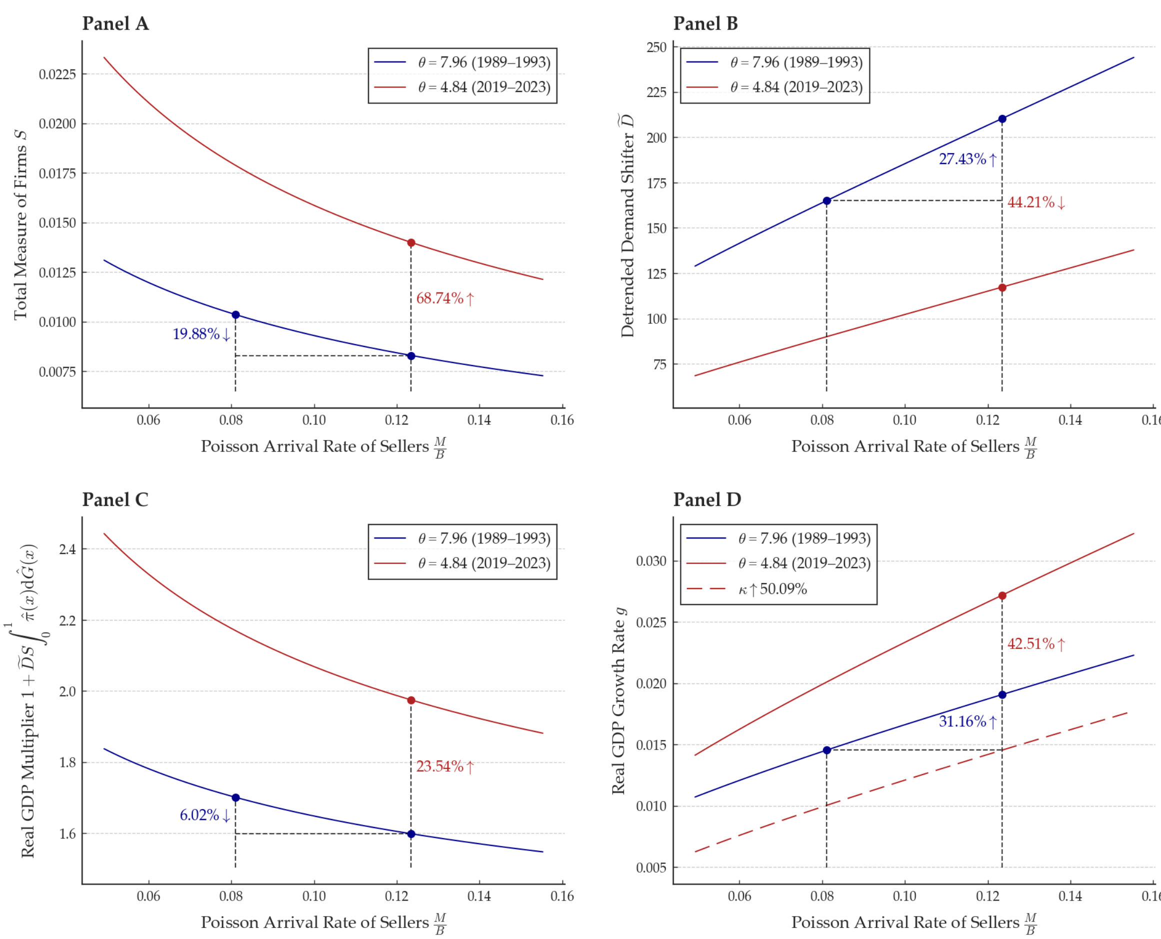

Figure 4. Welfare Analysis: Endogenous Entry.

This figure plots variables underlying BGP welfare in the model extension with endogenous entry. Panel A illustrates the total measure of firms per variety, S. Panel B shows the detrended demand shifter $\widetilde{D}$. Panel C presents the real GDP multiplier $1 + \widetilde{D}S\int_0^1 \hat{\pi}(x)\,\mathrm{d}\hat{G}(x)$. Panel D visualizes the real GDP growth rate g.

Endogenous search effort We now turn to the case in which firms endogenously choose their search effort. In Panel B of Figure 5, we first compare firm value in this scenario to its baseline counterpart. The impact of costly search on detrended firm value is twofold. First, hiring additional labor for search directly reduces the net profit flow of all firms. As high-productivity firms search disproportionately more, however, they crowd out low-productivity firms in buyer matching and capture a larger share of sales profit. As shown in Panel B, the former effect dominates the latter when the search-cost elasticity φ is high. As φ declines over time, the latter effect kicks in, leading to greater polarization in firm value. To illustrate this pattern more clearly, we also plot a hypothetical case in which φ falls to a very low level of 1.10. The value of high-productivity firms explodes, leaving a large mass of firms with extremely low detrended value.

Next, we examine the variables that directly enter BGP welfare. In Panel C of Figure 5, if we attribute all changes in $\frac{M}{B}$ to the search-cost scale χ while holding the elasticity φ fixed, the secular rise in search efficiency implies a 3.52% decline in aggregate profitability. As φ decreases, however, the polarization of sales profit across firms makes aggregate profitability less sensitive to changes in the arrival rate of sellers, resulting in a 1.22% rebound. Incorporating the reduction in θ further raises aggregate profitability by 9.75%. This increase is nonetheless more than halved relative to the 22.20% in the baseline economy, implying a negative net effect of θ on the static consumption level.[41] The difference is driven by the shift of labor from production to search.

Dynamically, attributing all changes in $\frac{M}{B}$ to χ raises aggregate growth by only 3.94% (Panel D), suggesting that the offsetting partial- and general-equilibrium effects still dominate within the empirically relevant range of $\frac{M}{B}$. The drop in φ, on the contrary, induces all firms to allocate more labor to search, including those with low productivity. This additional misallocation diverts labor away from production, contracts demand, and weakens adoption incentives, resulting in a 26.69% decrease in aggregate growth. The contraction in production labor also dampens the within-variety allocative efficiency gains due to a lower θ, leaving a 115.93% effect on real GDP growth, which is smaller than in the baseline economy (165.89%). To bring the growth rate back to the average, the technology adoption cost only needs to rise by 44.73%.

[41] Note that the impact of θ on the initial real wage w_0 is the same as in the baseline model, i.e., a 20.14% decline.

Overall, with costly search, the observed advance in search technology over the past three decades *reduces* BGP welfare by 15.85%. While the rise of superstar firms associated with the decline in θ contributes a 108.35% welfare gain by itself, the effect drops sharply to 55.33% once we jointly account for changes in search technology, as right-tail firms hire disproportionately more search labor.

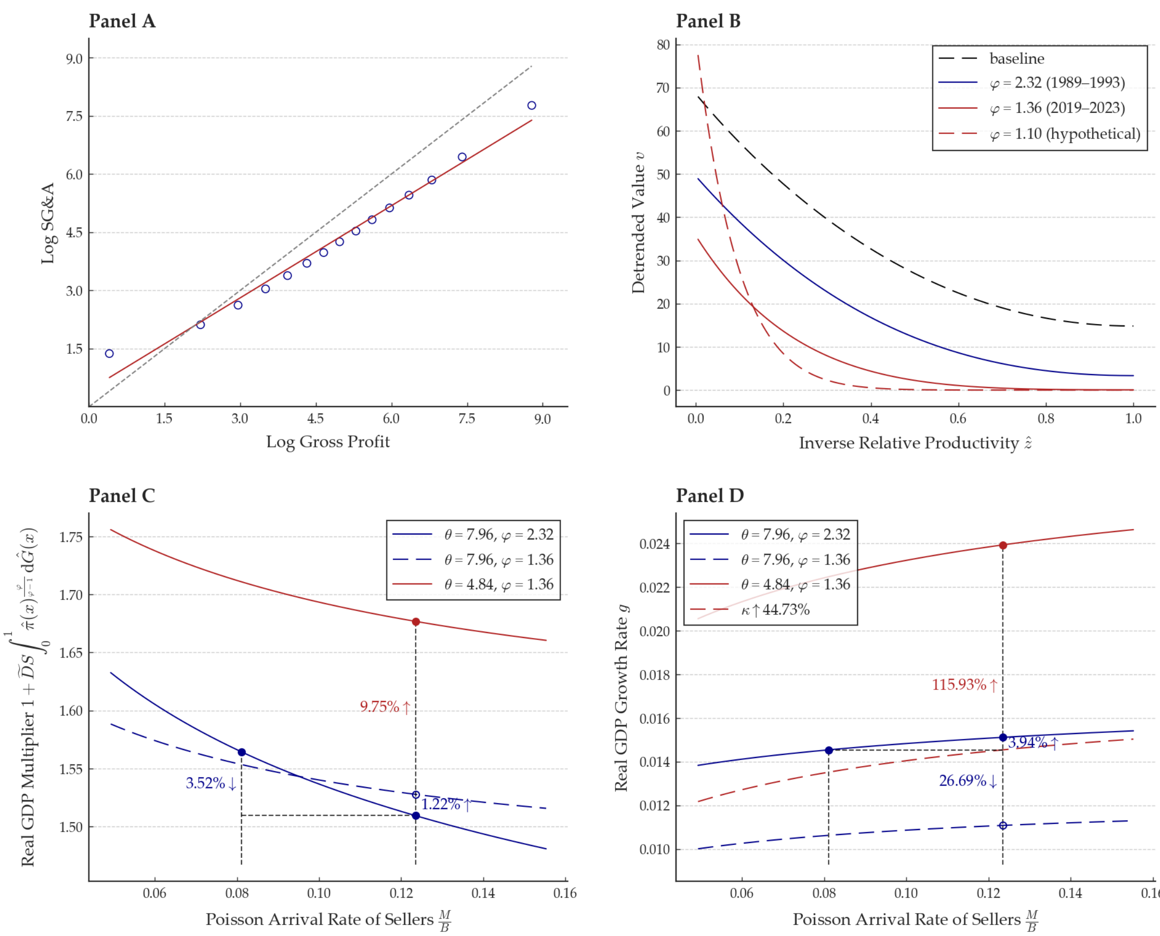

Figure 5. Welfare Analysis: Endogenous Search Effort.

This figure summarizes plots related to the BGP welfare analysis in the model extension with endogenous search effort. Panel A shows a binned scatter plot of log SG&A against log gross profit using Compustat data from 1989 to 2023. Panel B illustrates the detrended value function $v\left(\hat{z};\overline{\lambda}\right)$ for different values of the search-cost elasticity parameter φ. Panel C presents the real GDP multiplier $1 + \widetilde{D}S \int_0^1 \hat{\pi}(x)^{\frac{\varphi}{\varphi-1}} \,\mathrm{d}\hat{G}(x)$. Panel D visualizes the real GDP growth rate g.

Creative destruction Finally, Figure 6 illustrates the welfare analysis for the model extension with creative destruction. For the two sets of $\left(\frac{\widehat{M}}{B}, \hat{\theta}\right)$ in the two sub-periods, the inverse productivity dispersion ι falls from 0.78 to 0.56, implying that the productivity

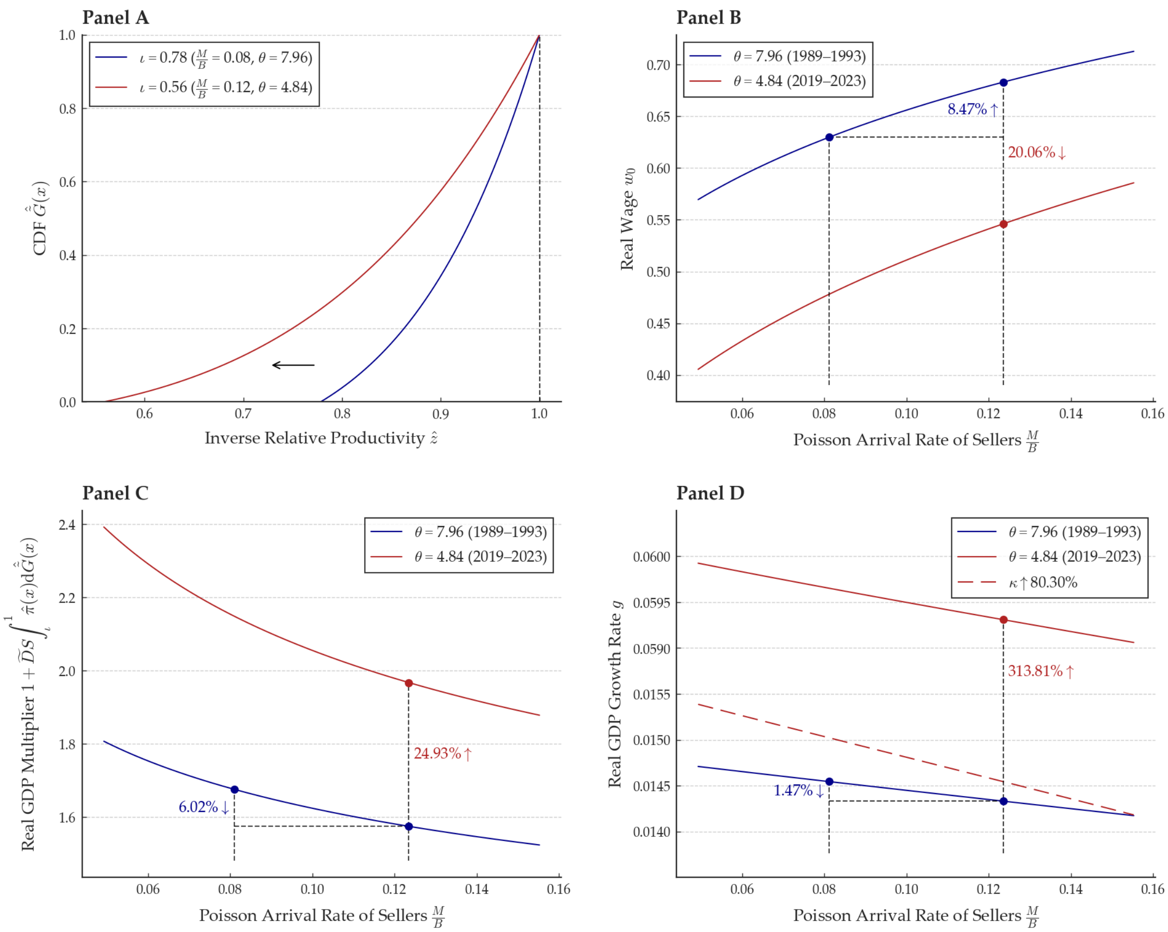

Figure 6. Welfare Analysis: Creative Destruction.

This figure plots variables underlying BGP welfare in the model extension with creative destruction. Panel A illustrates the inverse productivity dispersion ι for two sets of $\left(\widehat{\frac{M}{B}}, \hat{\theta}\right)$. Panel B shows the initial real wage w_0. Panel C presents the real GDP multiplier $1 + \widetilde{D}S \int_\iota^1 \hat{\pi}(x)\, \mathrm{d}\hat{\tilde{G}}(x)$. Panel D visualizes the real GDP growth rate g.

distribution stretches outward.[42] However, Panels B and C show that the effects of $\frac{M}{B}$ and θ on the initial real wage and aggregate profitability remain very close to those in the baseline model, suggesting a limited static impact of the endogenous change in ι. The reason is that, while the outward spread of the productivity distribution raises aggregate productivity, it weakens overall competition and strengthens market power. The two forces largely cancel out.

For the aggregate growth rate, Panel D shows that the estimated change in search frictions is insufficient to induce a large wedge between the partial- and general-equilibrium effects, resulting in a small negative impact of 1.47%. In contrast, by allowing ι to be determined endogenously, the decline in θ now has a twofold growth-enhancing effect. As in the baseline economy, it implies within-variety reallocation of production labor from low- to high-productivity firms. Unlike the baseline model, the lower ι also raises frontier growth driven by creative destruction, which further stimulates technology adoption. As a result, the BGP growth rate increases by 313.81%, almost twice as large as in the baseline model. The impact of adoption cost, by similar logic, is also strengthened: an 80.30% increase is enough to force growth back to the average.

Taken together, this extension generates the largest divergence between the separate BGP welfare effects of $\frac{M}{B}$ and θ, at 1.10% versus 285.26%. Their joint effect is 288.88%, indicating very limited complementarity.

8 Conclusion

Monopoly rents within a product market can drive technological laggards to advance the technology frontier through innovation, but often fall short in explaining their incentives to catch up by adopting existing technologies at a cost. To address this, we propose a search-economy model where incentives for adoption within a variety emerge naturally. Search frictions grant all firms in the same product market strictly positive market power and profits, irrespective of their productivity levels. Meanwhile, buyer-seller matching based on the lowest price encountered maintains the importance of technological advantage in shaping firm profits. This mechanism motivates less productive firms to embrace

42 Note that when plotting the two values of ι in Panel A, we vary only $\frac{M}{B}$ and θ while holding κ fixed at its initial value.

advanced technologies to climb the profit ladder. The adoption of new technologies, in turn, is tightly interconnected with firm dynamics, economic growth, and welfare.

While exogenous search frictions are essential for generating within-variety adoption incentives, conditional on being strictly positive, the magnitude of those frictions has surprisingly little impact on technology adoption and aggregate growth. This is because the partial-equilibrium effect on variable markups is perfectly offset by the general-equilibrium effect on demand. Incorporating additional elements like endogenous entry, endogenous search effort, or creative destruction creates a wedge between these two effects and enables search frictions to have dynamic implications through distinct mechanisms.

We quantify the BGP welfare effects of the improving search efficiency and the thickening right tail of the productivity distribution in the U.S. over the past three decades. While the effect of the former depends strongly on the model variant in question, the effect of the latter is always positive and large. In all cases, a sizable increase in technology adoption barriers is required to reconcile the model with the current slowdown in U.S. productivity growth.

While we explored three model extensions that highlight the dynamic implications of search frictions for economic growth, we view these merely as a starting point. Future research may delve into additional mechanisms, such as persistence and path dependence in buyer-seller matching, input-output linkages and production networks, and product innovation through the creation of new varieties. Finally, given the growing attention to search and matching in the spatial and international economics literature (e.g., Eaton et al., 2022a,b; Miyauchi, 2024; Arkolakis et al., 2025), incorporating search frictions and endogenous technology adoption decisions into an open-economy framework offers a promising direction for future research.

Appendix

A Proofs

A.1 Proof of Lemma 1

For a buyer of any given variety, the Poisson arrival rate of a random seller (i.e., a firm) is:

$$\frac{M}{B} = \lambda S^{\gamma^s} B^{\gamma^b - 1},$$

which implies that at each time t, the probability that the buyer encounters n sellers is:

$$\frac{\exp\left(-\lambda S^{\gamma^s} B^{\gamma^b-1}\right)\left(\lambda S^{\gamma^s} B^{\gamma^b-1}\right)^n}{n!}.$$

For a given seller, the expected number of encounters with buyers who have n other encounters is thus given by:

$$\frac{B\frac{\exp\left(-\lambda S^{\gamma^s} B^{\gamma^b-1}\right)\left(\lambda S^{\gamma^s} B^{\gamma^b-1}\right)^{n+1}}{(n+1)!}(n+1)}{S} = \lambda S^{\gamma^s-1} B^{\gamma^b} \frac{\exp\left(-\lambda S^{\gamma^s} B^{\gamma^b-1}\right)\left(\lambda S^{\gamma^s} B^{\gamma^b-1}\right)^n}{n!}.$$

The term $B\frac{\exp\left(-\lambda S^{\gamma^s} B^{\gamma^b-1}\right)\left(\lambda S^{\gamma^s} B^{\gamma^b-1}\right)^{n+1}}{(n+1)!}$ represents the expected measure of buyers who have $n+1$ total encounters with sellers of the same variety. Multiplying it by $n+1$ yields the expected total measure of encounters for these buyers. Because all sellers of measure S are subject to the same search friction, dividing the expression by S yields the expected number of encounters for a given seller with buyers with n other encounters.[43]

Using the matching probability $(1-F_t(p))^n$ of a seller charging price p, and summing over all types n of matched buyers, we obtain the following expression for the expected number of buyers matched with the seller:

$$\begin{aligned}
\bar{b}_t(p) &= \sum_{n=0}^{\infty} \lambda S^{\gamma^s-1} B^{\gamma^b} \frac{\exp\left(-\lambda S^{\gamma^s} B^{\gamma^b-1}\right)\left(\lambda S^{\gamma^s} B^{\gamma^b-1}\right)^n}{n!}(1-F_t(p))^n \\
&= \lambda S^{\gamma^s-1} B^{\gamma^b} \exp\left(-\lambda S^{\gamma^s} B^{\gamma^b-1} F_t(p)\right) \sum_{n=0}^{\infty} \frac{\exp\left(-\lambda S^{\gamma^s} B^{\gamma^b-1}(1-F_t(p))\right)\left[\lambda S^{\gamma^s} B^{\gamma^b-1}(1-F_t(p))\right]^n}{n!}
\end{aligned}$$

43 Note that these encounters arrive at sellers of measure S with equal probability.

$$= \lambda S^{\gamma^s - 1} B^{\gamma^b} \exp\left(-\lambda S^{\gamma^s} B^{\gamma^b - 1} F_t(p)\right),$$

where the last equality relies on the fact that $\frac{\exp\left(-\lambda S^{\gamma^s} B^{\gamma^b-1}(1-F_t(p))\right)\left[\lambda S^{\gamma^s} B^{\gamma^b-1}(1-F_t(p))\right]^n}{n!}$ is the probability mass function of a Poisson distribution with parameter $\lambda S^{\gamma^s} B^{\gamma^b-1}(1 - F_t(p))$, thereby proving equation (11).[44]

We define $\overline{Q}_t$ as the expected real expenditure of a random buyer—either a consumer or a firm—on the final good. Since both consumers and firms share the same elasticity of substitution σ across varieties, a random buyer's demand function takes the following CES form:[45]

$$\overline{q}_t(p) = \overline{Q}_t p^{-\sigma}.$$

Finally,

$$\overline{\pi}_t(p; z) = p - c_t(z) = p - \frac{w_t^{1-\alpha}}{z}$$

is the firm's profit margin per unit of goods sold. □

A.2 Proof of Proposition 1

To prove Proposition 1, we first guess that the optimal price $p_t(c)$ is strictly increasing in marginal cost c *if and only if* Assumption 1 holds. We derive equations (17) and (18) under this conjecture. Finally, we verify the conjecture to complete the proof.

Optimal pricing and markups Using $z_t(c) = \frac{w_t^{1-\alpha}}{c}$, we can rewrite the optimal price as a function of marginal cost c, i.e., $\tilde{p}_t(z_t(c)) = p_t(c)$. Since $p_t(c)$ is strictly increasing in c under our conjecture, the price distribution $F_t(p_t(c))$ coincides with the time-invariant cost distribution $H(c)$. Hence:

$$\frac{\mathrm{d}F_t(p_t(c))}{\mathrm{d}c} = F_t'(p_t(c))\, p_t'(c) = H'(c).$$

44 Note that $\overline{b}_t(p)$ is similarly derived in Menzio (2024b).

45 Note that the final good price index P_t at each time t is normalized to be 1, and $\overline{Q}_t$ will be determined in equilibrium. Since the demand of a random buyer, regardless of whether it is a consumer or a firm, is proportional to $p^{-\sigma}$, $\overline{q}_t(p) = \overline{Q}_t p^{-\sigma}$ becomes the effective demand of a random buyer as a function of p.

Substituting $c_t(z) = c$, $\tilde{p}_t(z_t(c)) = p_t(c)$, and $F_t'(p_t(c)) = \frac{H'(c)}{p_t'(c)}$, we rewrite equation (14) as:

$$p_t'(c) = \frac{\lambda S^{\gamma^s} B^{\gamma^b - 1} H'(c)\, p_t(c)\left[p_t(c) - c\right]}{(1-\sigma)\, p_t(c) + \sigma c}. \tag{76}$$

It is clear from equation (76) that the solution $p_t(c)$ does not depend on time t, so we can drop the subscript t. For $\sigma = 1$, (76) can be further simplified to the Riccati equation (17) in Proposition 1.

Using $H(c) = \left(\frac{c}{\bar{c}}\right)^{\theta}$ along the BGP,[46] we can plug in $H'(c)$ and obtain:

$$p'(c) = \Lambda\theta c^{\theta-2} p(c)\left[p(c) - c\right], \tag{77}$$

where $\Lambda = \lambda S^{\gamma^s} B^{\gamma^b - 1} \bar{c}^{-\theta}$ is constant over time. Equation (77) can be transformed into the following first-order linear ODE by a change of variables $u(c) = \frac{1}{p(c)}$:

$$u'(c) = -\frac{p'(c)}{p(c)^2} = \Lambda\theta c^{\theta-1} u(c) - \Lambda\theta c^{\theta-2}. \tag{78}$$

Based on equation (78) and the integrating factor $\exp\left(-\int \Lambda\theta c^{\theta-1}\, \mathrm{d}c\right) = \exp\left(-\Lambda c^{\theta}\right)$, we have:

$$\begin{aligned}\frac{\mathrm{d}\left(u(c)\exp\left(-\Lambda c^{\theta}\right)\right)}{\mathrm{d}c} &= \exp\left(-\Lambda c^{\theta}\right)\left[u'(c) - \Lambda\theta c^{\theta-1} u(c)\right] \\ &= -\exp\left(-\Lambda c^{\theta}\right)\Lambda\theta c^{\theta-2}.\end{aligned}$$

Integrating both sides from 0 to c, we obtain:

$$u(c)\exp\left(-\Lambda c^{\theta}\right) - u(0) = -\int_0^c \exp\left(-\Lambda \hat{c}^{\theta}\right)\Lambda\theta \hat{c}^{\theta-2}\, \mathrm{d}\hat{c}.$$

The right-hand side can be further simplified by a change of variables $x = \Lambda\hat{c}^{\theta}$:

$$-\int_0^c \exp\left(-\Lambda \hat{c}^{\theta}\right)\Lambda\theta \hat{c}^{\theta-2}\, \mathrm{d}\hat{c} = -\Lambda^{\frac{1}{\theta}} \int_0^{\Lambda c^{\theta}} \exp(-x)\, x^{-\frac{1}{\theta}}\, \mathrm{d}x = -\Lambda^{\frac{1}{\theta}}\gamma\left(\frac{\theta-1}{\theta}, \Lambda c^{\theta}\right).$$

[46] See Definition 1.

Substituting $u(c) = \frac{1}{p(c)}$ back, we obtain:

$$p(c) = \frac{\exp\left(-\Lambda c^{\theta}\right) p(0)}{1 - \Lambda^{\frac{1}{\theta}} \gamma\left(\frac{\theta-1}{\theta}, \Lambda c^{\theta}\right) p(0)}, \tag{79}$$

where we denote $\lim_{c\to 0} p(c)$ as $p(0)$.

Assumption 1 implies

$$p(0) = \lim_{z\to\infty} \tilde{p}(z) = \frac{\left(\lambda S^{\gamma^{s}} B^{\gamma^{b}-1}\right)^{-\frac{1}{\theta}}}{\Gamma\left(\frac{\theta-1}{\theta}\right)} \bar{c}. \tag{80}$$

Substituting (80) into (79) yields equation (18).

Monotonicity of pricing function We next verify the conjecture that $p(c)$ is strictly increasing in c if and only if Assumption 1 holds. Equation (17) implies that $p'(c) > 0$ for all $c \in (0, \bar{c}]$ if and only if $p(c) > c$ for all $c \in (0, \bar{c}]$. From equation (79), it must be:

$$1 - \Lambda^{\frac{1}{\theta}} \gamma\left(\frac{\theta-1}{\theta}, \Lambda c^{\theta}\right) p(0) > 0 \text{ for all } c \tag{81}$$

and

$$\frac{\exp\left(-\Lambda c^{\theta}\right) p(0)}{1 - \Lambda^{\frac{1}{\theta}} \gamma\left(\frac{\theta-1}{\theta}, \Lambda c^{\theta}\right) p(0)} > c \text{ for all } c. \tag{82}$$

Equation (81) indicates that:

$$p(0) < \min_{c\in(0,\bar{c}]} \frac{1}{\Lambda^{\frac{1}{\theta}} \gamma\left(\frac{\theta-1}{\theta}, \Lambda c^{\theta}\right)} = \frac{1}{\Lambda^{\frac{1}{\theta}} \gamma\left(\frac{\theta-1}{\theta}, \Lambda \bar{c}^{\theta}\right)}. \tag{83}$$

Rearranging equation (82), we obtain:

$$p(0) > \max_{c\in(0,\bar{c}]} \frac{c}{\exp\left(-\Lambda c^{\theta}\right) + \Lambda^{\frac{1}{\theta}} \gamma\left(\frac{\theta-1}{\theta}, \Lambda c^{\theta}\right) c}. \tag{84}$$

Taking the derivative of the right-hand side of equation (84) with respect to c:

$$\frac{\mathrm{d}\left(\frac{c}{\exp\left(-\Lambda c^{\theta}\right)+\Lambda^{\frac{1}{\theta}}\gamma\left(\frac{\theta-1}{\theta},\Lambda c^{\theta}\right)c}\right)}{\mathrm{d}c} = \frac{\exp\left(-\Lambda c^{\theta}\right) + \Lambda^{\frac{1}{\theta}} \gamma\left(\frac{\theta-1}{\theta}, \Lambda c^{\theta}\right) c - \frac{\mathrm{d}\left(\exp\left(-\Lambda c^{\theta}\right)+\Lambda^{\frac{1}{\theta}}\gamma\left(\frac{\theta-1}{\theta},\Lambda c^{\theta}\right)c\right)}{\mathrm{d}c} c}{\left[\exp\left(-\Lambda c^{\theta}\right) + \Lambda^{\frac{1}{\theta}} \gamma\left(\frac{\theta-1}{\theta}, \Lambda c^{\theta}\right) c\right]^{2}}.$$

By the Leibniz rule, we have:

$$\frac{\mathrm{d}\gamma\left(\frac{\theta-1}{\theta},\Lambda c^{\theta}\right)}{\mathrm{d}c}=\theta\Lambda^{\frac{\theta-1}{\theta}}c^{\theta-2}\exp\left(-\Lambda c^{\theta}\right).$$

Plugging in and rearranging, we obtain:

$$\frac{\mathrm{d}\left(\frac{c}{\exp\left(-\Lambda c^{\theta}\right)+\Lambda^{\frac{1}{\theta}}\gamma\left(\frac{\theta-1}{\theta},\Lambda c^{\theta}\right)c}\right)}{\mathrm{d}c}=\frac{\exp\left(-\Lambda c^{\theta}\right)}{\left[\exp\left(-\Lambda c^{\theta}\right)+\Lambda^{\frac{1}{\theta}}\gamma\left(\frac{\theta-1}{\theta},\Lambda c^{\theta}\right)c\right]^{2}}>0.$$

Therefore, $\max_{c\in(0,\bar{c}]}\frac{c}{\exp\left(-\Lambda c^{\theta}\right)+\Lambda^{\frac{1}{\theta}}\gamma\left(\frac{\theta-1}{\theta},\Lambda c^{\theta}\right)c}=\frac{1}{\frac{\exp\left(-\Lambda\bar{c}^{\theta}\right)}{\bar{c}}+\Lambda^{\frac{1}{\theta}}\gamma\left(\frac{\theta-1}{\theta},\Lambda\bar{c}^{\theta}\right)}$. Combining equations (83) and (84), we have:

$$\frac{1}{\frac{\exp\left(-\Lambda\bar{c}^{\theta}\right)}{\bar{c}}+\Lambda^{\frac{1}{\theta}}\gamma\left(\frac{\theta-1}{\theta},\Lambda\bar{c}^{\theta}\right)}<p\left(0\right)<\frac{1}{\Lambda^{\frac{1}{\theta}}\gamma\left(\frac{\theta-1}{\theta},\Lambda\bar{c}^{\theta}\right)}\tag{85}$$

Given Λ, $p\left(0\right)$ must satisfy (85) for all possible $\bar{c}\in(0,\infty)$. Since

$$\max_{\bar{c}}\frac{1}{\frac{\exp\left(-\Lambda\bar{c}^{\theta}\right)}{\bar{c}}+\Lambda^{\frac{1}{\theta}}\gamma\left(\frac{\theta-1}{\theta},\Lambda\bar{c}^{\theta}\right)}=\lim_{\bar{c}\to\infty}\frac{1}{\frac{\exp\left(-\Lambda\bar{c}^{\theta}\right)}{\bar{c}}+\Lambda^{\frac{1}{\theta}}\gamma\left(\frac{\theta-1}{\theta},\Lambda\bar{c}^{\theta}\right)}=\frac{\Lambda^{-\frac{1}{\theta}}}{\Gamma\left(\frac{\theta-1}{\theta}\right)}$$

and

$$\min_{\bar{c}}\frac{1}{\Lambda^{\frac{1}{\theta}}\gamma\left(\frac{\theta-1}{\theta},\Lambda\bar{c}^{\theta}\right)}=\lim_{\bar{c}\to\infty}\frac{1}{\Lambda^{\frac{1}{\theta}}\gamma\left(\frac{\theta-1}{\theta},\Lambda\bar{c}^{\theta}\right)}=\frac{\Lambda^{-\frac{1}{\theta}}}{\Gamma\left(\frac{\theta-1}{\theta}\right)},$$

the squeeze theorem implies that $p'\left(c\right)>0$ if and only if $p\left(0\right)=\frac{\Lambda^{-\frac{1}{\theta}}}{\Gamma\left(\frac{\theta-1}{\theta}\right)}=\frac{\left(\lambda S^{\gamma^{s}}B^{\gamma^{b}-1}\right)^{-\frac{1}{\theta}}}{\Gamma\left(\frac{\theta-1}{\theta}\right)}\bar{c}$, i.e., when Assumption 1 holds.

Monotonicity of markup function We now prove that the markup function $m\left(c\right)$ in (18) is monotonically decreasing in c. Let $f\left(x\right)=x^{-\frac{1}{\theta}}\exp\left(-x\right)$ and $g\left(x\right)=\int_{x}^{\infty}u^{-\frac{1}{\theta}}\exp\left(-u\right)\mathrm{d}u$, we can rewrite the markup function (18) as the following gamma hazard rate:

$$\tilde{m}\left(x\right)=\frac{f\left(x\right)}{g\left(x\right)},\tag{86}$$

where $x=x\left(c\right)=\lambda S^{\gamma^{s}}B^{\gamma^{b}-1}H\left(c\right)$.

Using integration by parts, we obtain:

$$g(x) = f(x) - \frac{1}{\theta} h(x),$$

where $h(x) = \int_x^{\infty} u^{-\frac{\theta+1}{\theta}} \exp(-u) \, \mathrm{d}u$. Substituting into (86) to replace $f(x)$, we have:

$$\tilde{m}(x) = 1 + \frac{h(x)}{\theta g(x)}.$$

Differentiating yields:

$$\tilde{m}'(x) = \frac{x^{-\frac{\theta+1}{\theta}} \exp(-x) \left[x h(x) - g(x) \right]}{\theta g(x)^2}.$$

Since

$$x h(x) - g(x) = \int_x^{\infty} (x - u) \, u^{-\frac{\theta+1}{\theta}} \exp(-u) \, \mathrm{d}u < 0$$

and $x'(c) = \lambda S^{\gamma^s} B^{\gamma^b - 1} H'(c) > 0$, we have $m'(c) = \tilde{m}'(x) \, x'(c) < 0$. □

A.3 Proof of Lemma 2

We first prove that only firms at the lower bound $\underline{z}_t$ of the current productivity distribution $G_t(\cdot)$ choose to adopt new technology. Suppose that firms with productivity $z' > \underline{z}_t$ choose to adopt, then we have $V_{a,t} \geq V_{p,t}(z')$. As the flow profit $\pi_t(z)$ is strictly increasing in productivity z as shown in (23), it follows that $V_{p,t}(z') > V_{p,t}(\underline{z}_t)$. Therefore, all firms with productivity $z \in [\underline{z}_t, z')$ would continue redrawing productivity until they have no incentive to do so, which contradicts the premise that $\underline{z}_t$ is the lower bound of the productivity distribution.

As a result, $\underline{z}_t$ must be the unique productivity level at which firms have the incentive to adopt at time t, and it must satisfy the value-matching condition (24). This is equivalent to an optimal stopping problem with $\underline{z}_t$ as the cutoff. For the optimal stopping problem, the smooth-pasting condition (25) is required to ensure that the derivative of the value function equals that of the payoff function at the cutoff (Stokey, 2009).

Given the decision rule for technology adoption, the fraction of firms that choose to redraw their productivity at time t is:

$$\frac{G_t\left(\underline{z}_{t+\mathrm{d}t}\right)-G_t\left(\underline{z}_t\right)}{\mathrm{d}t}=G_t'\left(\underline{z}_t\right)\underline{z}_t\frac{\mathrm{d}\ln\underline{z}_t}{\mathrm{d}t}.$$

Since these firms have probability $1-G_t(z)$ of drawing a productivity level above z, we obtain the following Kolmogorov Forward Equation (KFE):

$$\frac{\mathrm{d}G_t(z)}{\mathrm{d}t}=-\left[1-G_t(z)\right]G_t'\left(\underline{z}_t\right)\underline{z}_t\frac{\mathrm{d}\ln\underline{z}_t}{\mathrm{d}t}. \tag{87}$$

By Definition 1, the distribution $\hat{G}(\cdot)$ of inverse relative productivity $\hat{z}=\frac{\underline{z}_t}{z}\in(0,1]$ is stationary along the BGP. Given that $\hat{G}(x)=1-G_t\left(\frac{\underline{z}_t}{x}\right)$ for all t, we have $\hat{G}'(1)=G_t'\left(\underline{z}_t\right)\underline{z}_t$ for all t. Plugging $G_t'\left(\underline{z}_t\right)\underline{z}_t=G_0'\left(\underline{z}_0\right)\underline{z}_0=\theta$ into (87), rearranging, and integrating both sides from 0 to $\mathrm{d}t$, we have:

$$\int_0^{\mathrm{d}t}\mathrm{d}\ln\left[1-G_s(z)\right]=\theta\int_0^{\mathrm{d}t}\mathrm{d}\ln\underline{z}_s. \tag{88}$$

Solving (88), we get:

$$G_{\mathrm{d}t}(z)=1-\left[1-G_0(z)\right]\left(\frac{\underline{z}_{\mathrm{d}t}}{\underline{z}_0}\right)^{\theta}=1-\left(\frac{z}{\underline{z}_{\mathrm{d}t}}\right)^{-\theta}.$$

Thus, the productivity distribution $G_t(\cdot)$ becomes always Pareto with shape parameter θ, while its lower bound $\underline{z}_t$ grows over time. As we show in Appendix A.6, the growth rate of $\underline{z}_t$ along the BGP must be a constant, denoted by g_z, to ensure constant aggregate growth. □

A.4 Proof of Proposition 2

Real interest rate First, consumers' utility function (1) and budget constraint (3) jointly yield the following current-value Hamiltonian:

$$\mathcal{H}_t\left(C_t,A_t\right)=\ln C_t+\mu_t\left(w_t+\Pi_t+r_tA_t-C_t\right),$$

with first-order conditions:

$$\frac{\partial \mathcal{H}_t\left(C_t, A_t\right)}{\partial C_t}=\frac{1}{C_t}-\mu_t=0$$

and

$$\frac{\partial \mathcal{H}_t\left(C_t, A_t\right)}{\partial A_t}=\mu_t r_t=\rho \mu_t-\dot{\mu}_t .$$

Solving the system of equations, we obtain the usual Euler equation for consumers:

$$\frac{\dot{C}_t}{C_t}=r_t-\rho .$$

With the real output growing at a constant rate g along the BGP, implying $\frac{\dot{C}_t}{C_t}=g$, the real interest rate $r=\rho+g$ remains constant over time.

Detrended value function Next, given that $V_{p,t}(z)=D_t v\left(\frac{z_t}{z}\right)$, we have:

$$\frac{\mathrm{d} V_{p,t}(z)}{\mathrm{d} t}=\frac{\mathrm{d}\left(D_t v\left(\frac{z_t}{z}\right)\right)}{\mathrm{d} t}=D_t \frac{\mathrm{d} \ln D_t}{\mathrm{d} t} v(\hat{z})+D_t v^{\prime}(\hat{z}) \hat{z} \frac{\mathrm{d} \ln z_t}{\mathrm{d} t} . \tag{89}$$

Along the BGP, the growth rate of demand shifter D_t coincides with that of real output, i.e., $\frac{\mathrm{d} \ln D_t}{\mathrm{d} t}=g$, and Lemma 2 indicates that $\frac{\mathrm{d} \ln z_t}{\mathrm{d} t}=g_z$. Substituting them into (89) and then (89) into (26) and dividing both sides by D_t, we obtain the ODE (27).

To solve (27), we first use the integrating factor $\exp\left(-\int \frac{\rho}{g_z \hat{z}} \mathrm{d} \hat{z}\right)=\hat{z}^{-\frac{\rho}{g_z}}$ to get:

$$\frac{\mathrm{d}\left(v(\hat{z}) \hat{z}^{-\frac{\rho}{g_z}}\right)}{\mathrm{d} \hat{z}}=\hat{z}^{-\frac{\rho}{g_z}}\left[v^{\prime}(\hat{z})-\frac{\rho}{g_z \hat{z}} v(\hat{z})\right]=-\frac{\hat{z}^{-\frac{\rho}{g_z}-1}}{g_z} \hat{\pi}(\hat{z})$$

Integrating both sides from $\hat{z}$ to 1 yields:

$$\begin{aligned} v(1)-v(\hat{z}) \hat{z}^{-\frac{\rho}{g_z}} & =\frac{1}{\rho} \int_{\hat{z}}^1 \hat{\pi}(x) \mathrm{d} x^{-\frac{\rho}{g_z}} \\ & =\frac{\hat{\pi}(1)-\hat{\pi}(\hat{z}) \hat{z}^{-\frac{\rho}{g_z}}}{\rho}-\frac{1}{\rho} \int_{\hat{z}}^1 x^{-\frac{\rho}{g_z}} \mathrm{d} \hat{\pi}(x), \end{aligned} \tag{90}$$

which implies that:

$$v\left(\hat{z}\right)=\frac{1}{\rho}\left[\hat{\pi}\left(\hat{z}\right)-\hat{z}^{\frac{\rho}{gz}}\int_{0}^{\hat{z}}x^{-\frac{\rho}{gz}}\,\mathrm{d}\hat{\pi}\left(x\right)\right]+k\hat{z}^{\frac{\rho}{gz}},\tag{91}$$

where k is a constant.

To pin down k, we require the smooth-pasting condition (25). Replacing $V_{p,t}\left(z\right)$ with $D_t v\left(\frac{\underline{z}_t}{z}\right)$ yields:

$$\left.\frac{\partial V_{p,t}\left(z\right)}{\partial z}\right|_{z=\underline{z}_t}=\left.\frac{\partial\left(D_t v\left(\frac{\underline{z}_t}{z}\right)\right)}{\partial z}\right|_{z=\underline{z}_t}=-\frac{D_t}{\underline{z}_t}v'\left(1\right)=0,$$

which implies $v'\left(1\right)=0$. Differentiating (91) with respect to $\hat{z}$ and substituting $\hat{z}=1$, we obtain:

$$k=\frac{1}{\rho}\int_{0}^{1}x^{-\frac{\rho}{gz}}\,\mathrm{d}\hat{\pi}\left(x\right).$$

Substituting k into (91) and rearranging yields (29).

From equation (22), we have:

$$\hat{\pi}\left(x\right)=\frac{\pi_t\left(\frac{\underline{z}_t}{x}\right)}{D_t}=\left(\lambda S^{\gamma^s}B^{\gamma^b-1}\right)^{-\frac{1}{\theta}}\exp\left(-\lambda S^{\gamma^s}B^{\gamma^b-1}x^{\theta}\right)-\Gamma\left(\frac{\theta-1}{\theta},\lambda S^{\gamma^s}B^{\gamma^b-1}x^{\theta}\right)x.\tag{92}$$

Taking the differential yields:

$$\mathrm{d}\hat{\pi}\left(x\right)=-\Gamma\left(\frac{\theta-1}{\theta},\lambda S^{\gamma^s}B^{\gamma^b-1}x^{\theta}\right)\mathrm{d}x,\tag{93}$$

where $\Gamma\left(\frac{\theta-1}{\theta},\lambda S^{\gamma^s}B^{\gamma^b-1}x^{\theta}\right)$ can be made tractable using Taylor expansion:

$$\begin{aligned}\Gamma\left(\frac{\theta-1}{\theta},\lambda S^{\gamma^s}B^{\gamma^b-1}x^{\theta}\right)&=\Gamma\left(\frac{\theta-1}{\theta}\right)-\int_{0}^{\lambda S^{\gamma^s}B^{\gamma^b-1}x^{\theta}}\exp\left(-u\right)u^{-\frac{1}{\theta}}\,\mathrm{d}u\\&=\Gamma\left(\frac{\theta-1}{\theta}\right)-\int_{0}^{\lambda S^{\gamma^s}B^{\gamma^b-1}x^{\theta}}\sum_{n=0}^{\infty}\frac{\left(-1\right)^{n}u^{n-\frac{1}{\theta}}}{n!}\,\mathrm{d}u\\&=\Gamma\left(\frac{\theta-1}{\theta}\right)-\sum_{n=0}^{\infty}\frac{\left(-1\right)^{n}\left(\lambda S^{\gamma^s}B^{\gamma^b-1}\right)^{n+\frac{\theta-1}{\theta}}x^{n\theta+\theta-1}}{n!\left(n+\frac{\theta-1}{\theta}\right)}.\end{aligned}$$

Plugging into $\mathrm{d}\hat{\pi}(x)$, we have:

$$\int_{\hat{z}}^{1} x^{-\frac{\rho}{g_z}} \,\mathrm{d}\hat{\pi}(x) = -\Gamma\left(\frac{\theta-1}{\theta}\right)\int_{\hat{z}}^{1} x^{-\frac{\rho}{g_z}} \,\mathrm{d}x + \sum_{n=0}^{\infty}\left[\frac{(-1)^n \left(\lambda S^{\gamma^s} B^{\gamma^b-1}\right)^{n+\frac{\theta-1}{\theta}}}{n!\left(n+\frac{\theta-1}{\theta}\right)}\int_{\hat{z}}^{1} x^{n\theta+\theta-\frac{\rho}{g_z}-1}\,\mathrm{d}x\right]$$

$$= -\Gamma\left(\frac{\theta-1}{\theta}\right)\frac{g_z}{g_z-\rho}\left(1-\hat{z}^{1-\frac{\rho}{g_z}}\right) + \sum_{n=0}^{\infty}\frac{(-1)^n\left(\lambda S^{\gamma^s}B^{\gamma^b-1}\right)^{n+\frac{\theta-1}{\theta}}}{n!\left(n+\frac{\theta-1}{\theta}\right)\left(n\theta+\theta-\frac{\rho}{g_z}\right)}\left(1-\hat{z}^{n\theta+\theta-\frac{\rho}{g_z}}\right).$$

Substituting into (29), we obtain (30). □

A.5 Proof of Lemma 3

For a given variety and a buyer who encounters n sellers of that variety, the probability that the lowest marginal cost among these sellers is no greater than c is:

$$1-[1-H(c)]^n = 1-\left[1-\left(\frac{c}{\bar{c}}\right)^{\theta}\right]^n.$$

For any given buyer, the probability that the buyer has access to a variety *and* that the lowest marginal cost among the sellers they encounter does not exceed c is therefore:

$$\Omega H_{min}(c) = \sum_{n=1}^{\infty}\frac{\exp\left(-\lambda S^{\gamma^s}B^{\gamma^b-1}\right)\left(\lambda S^{\gamma^s}B^{\gamma^b-1}\right)^n}{n!}\left\{1-\left[1-\left(\frac{c}{\bar{c}}\right)^{\theta}\right]^n\right\}$$

$$= 1-\exp\left(-\lambda S^{\gamma^s}B^{\gamma^b-1}\right)-\exp\left(-\lambda S^{\gamma^s}B^{\gamma^b-1}\right)\sum_{n=1}^{\infty}\frac{\left\{\lambda S^{\gamma^s}B^{\gamma^b-1}\left[1-\left(\frac{c}{\bar{c}}\right)^{\theta}\right]\right\}^n}{n!}$$

$$= 1-\exp\left(-\lambda S^{\gamma^s}B^{\gamma^b-1}\bar{c}^{-\theta}c^{\theta}\right),$$

where the last equality applies the Taylor expansion:

$$\exp\left(\lambda S^{\gamma^s}B^{\gamma^b-1}\left(1-\left(\frac{c}{\bar{c}}\right)^{\theta}\right)\right) = 1+\sum_{n=1}^{\infty}\frac{\left\{\lambda S^{\gamma^s}B^{\gamma^b-1}\left[1-\left(\frac{c}{\bar{c}}\right)^{\theta}\right]\right\}^n}{n!}.$$

□

A.6 Proof of Proposition 3

Real consumption Since risk-free bond market clearing implies that $A_t = 0$ and $\dot{A}_t = 0$ in equilibrium, the budget constraint (3) becomes the final-good market-clearing condition (35):

$$C_t = w_t + \Pi_t.$$

Real wage To solve for the real wage w_t, we rely on the expression for the final good price index. For $\sigma = 1$, the final good price index takes the following Cobb-Douglas form:

$$\begin{aligned} P &= \exp\left(\Omega \int_0^{\overline{c}} \ln p\left(c\right) \mathrm{d}H_{min}\left(c\right)\right) \\ &= \exp\left(\Omega \int_0^{\overline{c}} \left[-\frac{1}{\theta}\ln\Lambda - \Lambda c^{\theta} - \ln\Gamma\left(\frac{\theta-1}{\theta}, \Lambda c^{\theta}\right)\right] \mathrm{d}H_{min}\left(c\right)\right) \\ &= 1, \end{aligned} \tag{94}$$

where the second equality is obtained by substituting $p\left(c\right)$ from equation (18).

To simplify (94), we first have:

$$\int_0^{\overline{c}} -\frac{1}{\theta}\ln\Lambda \,\mathrm{d}H_{min}\left(c\right) = -\frac{1}{\theta}\ln\Lambda = -\frac{1}{\theta}\ln\left(\lambda S^{\gamma^s} B^{\gamma^b - 1}\overline{c}^{-\theta}\right).$$

Next, using integration by parts, we obtain:

$$\begin{aligned} \int_0^{\overline{c}} -\Lambda c^{\theta} \,\mathrm{d}H_{min}\left(c\right) &= -\int_0^{\overline{c}} \Lambda c^{\theta} \,\mathrm{d}\frac{1-\exp\left(-\Lambda c^{\theta}\right)}{\Omega} \\ &= -\frac{1}{\Omega}\left[\Lambda c^{\theta}\left[1-\exp\left(-\Lambda c^{\theta}\right)\right]\right]_0^{\overline{c}} + \frac{1}{\Omega}\int_0^{\overline{c}} \left[1-\exp\left(-\Lambda c^{\theta}\right)\right] \mathrm{d}\Lambda c^{\theta} \\ &= \frac{\left(\Lambda\overline{c}^{\theta}+1\right)\exp\left(-\Lambda\overline{c}^{\theta}\right)-1}{\Omega} \\ &= \frac{\lambda S^{\gamma^s} B^{\gamma^b-1}\exp\left(-\lambda S^{\gamma^s} B^{\gamma^b-1}\right)}{\Omega} - 1. \end{aligned}$$

Finally, applying a change of variables $x = \Lambda c^{\theta}$ yields:

$$\int_0^{\overline{c}} -\ln\Gamma\left(\frac{\theta-1}{\theta}, \Lambda c^{\theta}\right) \mathrm{d}H_{min}\left(c\right) = -\int_0^{\overline{c}} \ln\Gamma\left(\frac{\theta-1}{\theta}, \Lambda c^{\theta}\right) \mathrm{d}\frac{1-\exp\left(-\Lambda c^{\theta}\right)}{\Omega}$$

$$
\begin{aligned}
&= -\frac{1}{\Omega}\int_0^{\lambda S^{\gamma^s} B^{\gamma^b-1}} \frac{\ln\Gamma\left(\frac{\theta-1}{\theta}, x\right)}{\exp(x)}\, \mathrm{d}x \\
&= -\frac{\mu\left(\lambda S^{\gamma^s} B^{\gamma^b-1}\right)}{\Omega},
\end{aligned}
$$

where $\mu\left(\lambda S^{\gamma^s} B^{\gamma^b-1}\right) = \int_0^{\lambda S^{\gamma^s} B^{\gamma^b-1}} \frac{\ln\Gamma\left(\frac{\theta-1}{\theta}, x\right)}{\exp(x)}\, \mathrm{d}x$ is a constant. Substituting the above three components into (94) gives:

$$
P = \exp\left(\lambda S^{\gamma^s} B^{\gamma^b-1} \exp\left(-\lambda S^{\gamma^s} B^{\gamma^b-1}\right) - \mu\left(\lambda S^{\gamma^s} B^{\gamma^b-1}\right) - \Omega\right)\left(\lambda S^{\gamma^s} B^{\gamma^b-1}\right)^{-\frac{\Omega}{\theta}} \bar{c}^{\Omega} = 1.
$$

Plugging in $\bar{c} = \frac{w_t^{1-\alpha}}{\underline{z}_t}$ and rearranging, we obtain (36):

$$
w_t = \left[\exp\left(1 - \frac{\lambda S^{\gamma^s} B^{\gamma^b-1}\exp\left(-\lambda S^{\gamma^s} B^{\gamma^b-1}\right) - \mu\left(\lambda S^{\gamma^s} B^{\gamma^b-1}\right)}{\Omega}\right)\left(\lambda S^{\gamma^s} B^{\gamma^b-1}\right)^{\frac{1}{\theta}} \underline{z}_t\right]^{\frac{1}{1-\alpha}}.
$$

This equation also confirms that the marginal cost upper bound $\bar{c} = \frac{w_t^{1-\alpha}}{\underline{z}_t}$ does not vary over time, consistent with Definition 1.

Aggregate real profits We next solve for aggregate real profits Π_t. Using the law of large numbers, we have (37):

$$
\Pi_t = S\int_{\underline{z}_t}^{\infty} \pi_t(z)\, \mathrm{d}G_t(z) = \left[\widetilde{D} S \int_0^1 \hat{\pi}(x)\, \mathrm{d}\hat{G}(x)\right] w_t,
$$

where the second equality uses $\pi_t(z) = D_t \hat{\pi}(\hat{z})$ and $D_t = \widetilde{D} w_t$.

Detrended demand shifter Finally, to find the detrended demand shifter $\widetilde{D}$, we need the labor market clearing condition $L_p + L_a = 1$, where

$$
L_p = S\int_0^{\bar{c}} \tilde{l}_t(c)\, \mathrm{d}H(c) = \frac{(1-\alpha) S \int_0^{\bar{c}} c\tilde{y}_t(c)\, \mathrm{d}H(c)}{w_t}
$$

represents the labor used in production and

$$
L_a = \frac{G_t\left(\underline{z}_{t+\mathrm{d}t}\right) - G_t\left(\underline{z}_t\right)}{\mathrm{d}t} S\kappa = S\kappa\theta g_z
$$

measures the labor used in technology adoption.

In equilibrium, the output of a firm with marginal cost c must equal the total demand from its buyers, i.e., $\tilde{y}_t(c) = \overline{b}_t(p(c))\,\overline{q}_t(p(c))$. Using Lemma 1 and Proposition 1 to replace $\overline{b}_t(p(c))$ and $\overline{q}_t(p(c))$ for $\sigma = 1$, we obtain:

$$\tilde{y}_t(c) = \lambda^{\frac{\theta+1}{\theta}} S^{\frac{\theta+1}{\theta}\gamma^s - 1} B^{\frac{\theta+1}{\theta}\gamma^b - \frac{1}{\theta}} \overline{Q}_t \overline{c}^{-1} \Gamma\left(\frac{\theta-1}{\theta}, \lambda S^{\gamma^s} B^{\gamma^b - 1}\left(\frac{c}{\overline{c}}\right)^{\theta}\right).$$

Plugging $\tilde{y}_t(c)$ into the labor market clearing condition and using a change of variables $x = \frac{c}{\overline{c}}$, we have:

$$(1-\alpha)\,\theta\lambda^{\frac{\theta+1}{\theta}} S^{\frac{\theta+1}{\theta}\gamma^s} B^{\frac{\theta+1}{\theta}\gamma^b - \frac{1}{\theta}} \frac{\overline{Q}_t}{w_t} \int_0^1 \Gamma\left(\frac{\theta-1}{\theta}, \lambda S^{\gamma^s} B^{\gamma^b-1} x^{\theta}\right) x^{\theta}\,\mathrm{d}x + S\kappa\theta g_z = 1. \tag{95}$$

We can rearrange equation (95) to solve for a firm's expected demand from a random buyer, $\overline{Q}_t$:

$$\overline{Q}_t = \frac{1 - S\kappa\theta g_z}{(1-\alpha)\,\theta\lambda^{\frac{\theta+1}{\theta}} S^{\frac{\theta+1}{\theta}\gamma^s} B^{\frac{\theta+1}{\theta}\gamma^b - \frac{1}{\theta}} \int_0^1 \Gamma\left(\frac{\theta-1}{\theta}, \lambda S^{\gamma^s} B^{\gamma^b-1} x^{\theta}\right) x^{\theta}\,\mathrm{d}x} w_t.$$

Equation (38) is thus derived by substituting $\overline{Q}_t$:

$$\widetilde{D} = \frac{D_t}{w_t} = \lambda^{\frac{\theta+1}{\theta}} S^{\frac{\theta+1}{\theta}\gamma^s - 1} B^{\frac{\theta+1}{\theta}\gamma^b - \frac{1}{\theta}} \frac{\overline{Q}_t}{w_t} = \frac{1 - S\kappa\theta g_z}{(1-\alpha)\,\theta S \int_0^1 \Gamma\left(\frac{\theta-1}{\theta}, \lambda S^{\gamma^s} B^{\gamma^b-1} x^{\theta}\right) x^{\theta}\,\mathrm{d}x}.$$

□

A.7 Proof of Proposition 4

Aggregate growth Using integration by parts, we obtain:

$$\int_0^1 v(x)\,\mathrm{d}\hat{G}(x) - v(1) = -\int_0^1 \hat{G}(x)\,v'(x)\,\mathrm{d}x.$$

Plugging in $\hat{G}(x) = x^{\theta}$ and $v'(x) = \frac{1}{g_z} x^{\frac{\rho}{g_z}-1} \int_x^1 u^{-\frac{\rho}{g_z}} \mathrm{d}\hat{\pi}(u)$ and rearranging yields:

$$\int_0^1 v(x)\,\mathrm{d}\hat{G}(x) - v(1) = -\frac{1}{g_z}\int_0^1 \left[\int_x^1 u^{-\frac{\rho}{g_z}} \mathrm{d}\hat{\pi}(u)\right] x^{\frac{\rho}{g_z}+\theta-1}\,\mathrm{d}x.$$

Applying the Fubini–Tonelli theorem, we have:

$$\begin{aligned}\int_0^1 \left[\int_x^1 u^{-\frac{\rho}{g_z}} \mathrm{d}\hat{\pi}(u)\right] x^{\frac{\rho}{g_z}+\theta-1}\,\mathrm{d}x &= \int_0^1 \left[\int_0^u x^{\frac{\rho}{g_z}+\theta-1}\,\mathrm{d}x\right] u^{-\frac{\rho}{g_z}} \mathrm{d}\hat{\pi}(u) \\ &= \frac{g_z}{\rho+\theta g_z}\int_0^1 u^{\theta}\mathrm{d}\hat{\pi}(u).\end{aligned}$$

Hence, substituting $\mathrm{d}\hat{\pi}(x)$ given by (93) yields:

$$\int_0^1 v(x)\,\mathrm{d}\hat{G}(x) - v(1) = \frac{1}{\rho+\theta g_z}\int_0^1 \Gamma\left(\frac{\theta-1}{\theta}, \lambda S^{\gamma^s} B^{\gamma^b-1} x^{\theta}\right) x^{\theta}\,\mathrm{d}x. \tag{96}$$

For $\lambda \in \mathbb{R}_{++}$, plugging (38) and (96) into (32) and rearranging, we obtain:

$$g_z = \frac{1-(1-\alpha)\rho S\kappa\theta}{[(1-\alpha)\theta+1]\,S\kappa\theta}. \tag{97}$$

Equations (35), (36), and (37) jointly imply that the real GDP growth rate satisfies $g = \frac{g_z}{1-\alpha}$, which corresponds to (39).

For $\lambda = \infty$, equation (32) is no longer valid for solving g_z. Instead, the market structure collapses to perfect competition where atomistic firms at the technology frontier produce and break even, while all other firms remain inactive. In this limiting case, no firm has an incentive to incur the fixed labor cost $\kappa > 0$ to adopt new technology, which implies $g = g_z = 0$.

Welfare Along the BGP with real GDP growth rate g, we have $C_s = \exp(g(s-t))\,C_t$. Substituting into the utility function (1) yields (40):

$$\begin{aligned}U_t &= \ln C_t \int_t^{\infty} \exp(-\rho(s-t))\,\mathrm{d}s + g\int_t^{\infty} \exp(-\rho(s-t))(s-t)\,\mathrm{d}s \\ &= \frac{1}{\rho}\left(\ln C_t + \frac{g}{\rho}\right),\end{aligned}$$

where the second equality applies integration by parts. □

A.8 Proof of Proposition 5

Fixed-point problem First, the free entry condition $V_{e,t} = 0$ implies:

$$\widetilde{D} \int_0^1 v(x)\, \mathrm{d}\hat{G}(x) = \xi. \tag{98}$$

By subtracting (32) from (98), we have:

$$\widetilde{D} v(1) = \xi - \kappa. \tag{99}$$

It follows from (29) that:

$$v(1) = \frac{\hat{\pi}(1)}{\rho} = \frac{1}{\rho}\left[\left(\lambda S^{\gamma^s} B^{\gamma^b - 1}\right)^{-\frac{1}{\theta}} \exp\left(-\lambda S^{\gamma^s} B^{\gamma^b - 1}\right) - \Gamma\left(\frac{\theta - 1}{\theta}, \lambda S^{\gamma^s} B^{\gamma^b - 1}\right)\right].$$

Finally, plugging (38) and (97) into (99) to replace $\widetilde{D}$, and then rearranging, we obtain the fixed-point problem (42).

Uniqueness To prove that the fixed-point problem (42) has a unique solution, we can rewrite it as $f(S) = \frac{g(\Lambda)}{h(\Lambda)}$, where $\Lambda = \Lambda(S) = \lambda S^{\gamma^s} B^{\gamma^b - 1}$. It is clear that $f(S)$ is strictly decreasing in S, with $f(S) \to \infty$ as $S \to 0$ and $f(S) \to \frac{\kappa}{(\xi - \kappa)[(1-\alpha)\theta + 1]}$ as $S \to \infty$.

Next, it is straightforward that $\frac{g(\Lambda)}{h(\Lambda)} \to 0$ as $\Lambda \to 0$. Using a change of variables $u = \Lambda x^\theta$ and integration by parts, we have:

$$\begin{aligned}
g(\Lambda) &= \frac{\Lambda^{-\frac{\theta+1}{\theta}}}{\theta} \int_0^\Lambda \Gamma\left(\frac{\theta - 1}{\theta}, u\right) u^{\frac{1}{\theta}}\, \mathrm{d}u \\
&= \frac{\Lambda^{-\frac{\theta+1}{\theta}}}{\theta + 1} \left\{\left[\Gamma\left(\frac{\theta - 1}{\theta}, u\right) u^{\frac{\theta+1}{\theta}}\right]_0^\Lambda + \int_0^\Lambda \exp(-u)\, u\, \mathrm{d}u\right\} \\
&= \frac{1}{\theta + 1}\left\{\Gamma\left(\frac{\theta - 1}{\theta}, \Lambda\right) + \Lambda^{-\frac{\theta+1}{\theta}}\left[1 - (\Lambda + 1)\exp(-\Lambda)\right]\right\}.
\end{aligned}$$

It follows that:

$$g'(\Lambda) = -\frac{\Lambda^{-\frac{2\theta+1}{\theta}}}{\theta}\left[1 - (\Lambda + 1)\exp(-\Lambda)\right] < 0 \ \text{ for } \Lambda > 0.$$

Together with $h'(\Lambda) = -\frac{\Lambda^{-\frac{\theta+1}{\theta}}}{\theta}\exp(-\Lambda) < 0$, L'Hôpital's rule implies that:

$$\lim_{\Lambda\to\infty}\frac{g(\Lambda)}{h(\Lambda)} = \lim_{\Lambda\to\infty}\frac{g'(\Lambda)}{h'(\Lambda)} = \lim_{\Lambda\to\infty}\frac{\exp(\Lambda)-\Lambda-1}{\Lambda} = \infty.$$

It remains to prove that $\frac{g(\Lambda)}{h(\Lambda)}$ is strictly increasing in Λ. Since Λ is strictly increasing in S, it then follows that the solution for S is unique. Differentiating with respect to Λ yields:

$$\frac{\mathrm{d}\left(\frac{g(\Lambda)}{h(\Lambda)}\right)}{\mathrm{d}\Lambda} = \frac{h'(\Lambda)\left[\frac{g'(\Lambda)}{h'(\Lambda)} - \frac{g(\Lambda)}{h(\Lambda)}\right]}{h(\Lambda)}.$$

By the fundamental theorem of calculus, we obtain:

$$\frac{g(\Lambda)}{h(\Lambda)} = \frac{\int_\Lambda^\infty g'(x)\,\mathrm{d}x}{\int_\Lambda^\infty h'(x)\,\mathrm{d}x} = \int_\Lambda^\infty \frac{g'(x)}{h'(x)}\cdot\frac{h'(x)}{\int_\Lambda^\infty h'(x')\,\mathrm{d}x'}\,\mathrm{d}x,$$

which implies that $\frac{g(\Lambda)}{h(\Lambda)}$ is a weighted average of $\frac{g'(x)}{h'(x)}$ over $[\Lambda,\infty)$, with weights given by $\frac{h'(x)}{\int_\Lambda^\infty h'(x')\mathrm{d}x'}$. For the function $\frac{g'(x)}{h'(x)}$, we have:

$$\frac{\mathrm{d}\left(\frac{g'(x)}{h'(x)}\right)}{\mathrm{d}x} = \frac{\mathrm{d}\left(\frac{\exp(x)-x-1}{x}\right)}{\mathrm{d}x} = \frac{(x-1)\exp(x)+1}{x^2} > 0 \;\text{ for } x > 0.$$

Therefore, $\frac{g(\Lambda)}{h(\Lambda)} > \frac{g'(\Lambda)}{h'(\Lambda)}$. Together with $h(\Lambda) > 0$ and $h'(\Lambda) < 0$, it follows that $\frac{\mathrm{d}\left(\frac{g(\Lambda)}{h(\Lambda)}\right)}{\mathrm{d}\Lambda} > 0$. □

A.9 Proof of Proposition 6

Optimal search effort, optimal pricing, and profits For a buyer of any given variety, the Poisson arrival rate of a random seller (i.e., a firm) is:

$$\frac{M_t}{B} = \overline{\lambda}_t S^{\gamma^s} B^{\gamma^b - 1},$$

which implies that at each time t, the probability that the buyer encounters n sellers is:

$$\frac{\exp\left(-\overline{\lambda}_t S^{\gamma^s} B^{\gamma^b-1}\right)\left(\overline{\lambda}_t S^{\gamma^s} B^{\gamma^b-1}\right)^n}{n!}.$$

For a given seller with search effort λ, the expected number of encounters with buyers who have n other encounters is thus given by:

$$B\frac{\exp\left(-\overline{\lambda}_t S^{\gamma^s}B^{\gamma^b-1}\right)\left(\overline{\lambda}_t S^{\gamma^s}B^{\gamma^b-1}\right)^{n+1}}{(n+1)!}(n+1)\frac{\lambda}{\overline{\lambda}_t}=\lambda S^{\gamma^s}B^{\gamma^b}\frac{\exp\left(-\overline{\lambda}_t S^{\gamma^s}B^{\gamma^b-1}\right)\left(\overline{\lambda}_t S^{\gamma^s}B^{\gamma^b-1}\right)^{n}}{n!}.$$

As in Appendix A.1, the term $B\frac{\exp\left(-\overline{\lambda}_t S^{\gamma^s}B^{\gamma^b-1}\right)\left(\overline{\lambda}_t S^{\gamma^s}B^{\gamma^b-1}\right)^{n+1}}{(n+1)!}(n+1)$ denotes the expected total measure of encounters for buyers with n other encounters. However, in contrast to the case with exogenous search efficiency, the probability that these encounters are paired with the given seller is $\frac{\lambda}{\overline{\lambda}_t}$ rather than $\frac{1}{S}$.[47]

Similar to Lemma 1, the expected number of buyers matched with a seller with search effort λ and price p is given by:

$$\begin{aligned}\overline{b}_t(\lambda,p)&=\sum_{n=0}^{\infty}\lambda S^{\gamma^s}B^{\gamma^b}\frac{\exp\left(-\overline{\lambda}_t S^{\gamma^s}B^{\gamma^b-1}\right)\left(\overline{\lambda}_t S^{\gamma^s}B^{\gamma^b-1}\right)^{n}}{n!}(1-F_t(p))^n\\&=\lambda S^{\gamma^s}B^{\gamma^b}\exp\left(-\overline{\lambda}_t S^{\gamma^s}B^{\gamma^b-1}F_t(p)\right).\end{aligned}$$

With $\overline{b}_t(\lambda,p)$, the expected sales profit of a firm with search effort λ, price p, and productivity z at time t is given by:

$$\begin{aligned}\pi_t(\lambda,p,z)&=\overline{b}_t(\lambda,p)\,\overline{q}_t(p)\,\overline{\pi}_t(p,z)\\&=\lambda S^{\gamma^s}B^{\gamma^b}\exp\left(-\overline{\lambda}_t S^{\gamma^s}B^{\gamma^b-1}F_t(p)\right)\overline{Q}_t p^{-\sigma}\left[p-c_t(z)\right].\end{aligned}$$

The first-order condition for the optimal search effort λ is given by:

$$\frac{\partial\left(\pi_t(\lambda,p,z)-\frac{\chi}{\varphi}\lambda^{\varphi}w_t\right)}{\partial\lambda}=S^{\gamma^s}B^{\gamma^b}\exp\left(-\overline{\lambda}_t S^{\gamma^s}B^{\gamma^b-1}F_t(p)\right)\overline{Q}_t p^{-\sigma}\left[p-c_t(z)\right]-\chi\lambda^{\varphi-1}w_t=0,$$

leading to

$$\lambda_t(p,z)=\left\{\frac{S^{\gamma^s}B^{\gamma^b}\overline{Q}_t}{\chi w_t}\exp\left(-\overline{\lambda}_t S^{\gamma^s}B^{\gamma^b-1}F_t(p)\right)p^{-\sigma}\left[p-c_t(z)\right]\right\}^{\frac{1}{\varphi-1}}. \tag{100}$$

The first-order condition for the optimal price, $\frac{\partial\left(\pi_t(\lambda,p,z)-\frac{\chi}{\varphi}\lambda^{\varphi}w_t\right)}{\partial p}=0$, yields an ODE

47 The encounter share $\frac{\lambda}{\overline{\lambda}_t}$ is derived from the thinning property of the Poisson process.

similar to (14), except that the exogenous search efficiency is replaced by the endogenous aggregate search effort $\overline{\lambda}_t$:

$$F_t'\left(\tilde{p}_t\left(z;\overline{\lambda}_t\right)\right)=\frac{(1-\sigma)\,\tilde{p}_t\left(z;\overline{\lambda}_t\right)+\sigma c_t\left(z\right)}{\overline{\lambda}_t S^{\gamma^s}B^{\gamma^b-1}\tilde{p}_t\left(z;\overline{\lambda}_t\right)\left[\tilde{p}_t\left(z;\overline{\lambda}_t\right)-c_t\left(z\right)\right]}.$$

Similar to Assumption 1, we assume Assumption 2 and focus on the case where $\sigma=1$. Using the same procedure as in Appendix A.2, we obtain the following optimal price as a function of marginal cost c, given aggregate search effort $\overline{\lambda}_t$:

$$p\left(c;\overline{\lambda}_t\right)=\frac{\left[\overline{\lambda}_t S^{\gamma^s}B^{\gamma^b-1}H\left(c\right)\right]^{-\frac{1}{\theta}}\exp\left(-\overline{\lambda}_t S^{\gamma^s}B^{\gamma^b-1}H\left(c\right)\right)}{\Gamma\left(\frac{\theta-1}{\theta},\overline{\lambda}_t S^{\gamma^s}B^{\gamma^b-1}H\left(c\right)\right)}c.$$

Using $c_t\left(z\right)=\frac{w_t^{1-\alpha}}{z}$ to recover $\tilde{p}_t\left(z;\overline{\lambda}_t\right)$, substituting $\tilde{p}_t\left(z;\overline{\lambda}_t\right)$ back into $\lambda_t\left(p,z\right)$ in equation (100), and applying the change of variables $\hat{z}=\frac{z_t}{z}$, we obtain the following expression for the optimal search effort as a function of inverse relative productivity $\hat{z}$, given aggregate search effort $\overline{\lambda}_t$:

$$\hat{\lambda}_t\left(\hat{z};\overline{\lambda}_t\right)=\left[\frac{\overline{\lambda}_t^{\frac{1}{\theta}}S^{\frac{\theta+1}{\theta}\gamma^s}B^{\frac{\theta+1}{\theta}\gamma^b-\frac{1}{\theta}}\overline{Q}_t}{\chi w_t}\hat{\pi}\left(\hat{z};\overline{\lambda}_t\right)\right]^{\frac{1}{\varphi-1}},\tag{101}$$

where

$$\hat{\pi}\left(\hat{z};\overline{\lambda}_t\right)=\left(\overline{\lambda}_t S^{\gamma^s}B^{\gamma^b-1}\right)^{-\frac{1}{\theta}}\exp\left(-\overline{\lambda}_t S^{\gamma^s}B^{\gamma^b-1}\hat{z}^{\theta}\right)-\Gamma\left(\frac{\theta-1}{\theta},\overline{\lambda}_t S^{\gamma^s}B^{\gamma^b-1}\hat{z}^{\theta}\right)\hat{z}$$

takes the same form as $\hat{\pi}\left(\hat{z}\right)=\frac{\pi_t(z)}{D_t}$ in equation (92) of the baseline economy, except that the exogenous search efficiency λ is replaced by the endogenous aggregate search effort $\overline{\lambda}_t$.

Likewise, the expected sales profit as a function of $\hat{z}$ is given by:

$$\pi_t\left(\hat{\lambda}_t\left(\frac{z_t}{z};\overline{\lambda}_t\right),\tilde{p}_t\left(z;\overline{\lambda}_t\right),z\right)=\chi w_t\hat{\lambda}_t\left(\hat{z};\overline{\lambda}_t\right)^{\varphi}=D_t\left(\overline{\lambda}_t\right)\hat{\pi}\left(\hat{z};\overline{\lambda}_t\right)^{\frac{\varphi}{\varphi-1}},\tag{102}$$

where $D_t\left(\overline{\lambda}_t\right)=\left(\chi w_t\right)^{\frac{1}{1-\varphi}}\left(\overline{\lambda}_t^{\frac{1}{\theta}}S^{\frac{\theta+1}{\theta}\gamma^s}B^{\frac{\theta+1}{\theta}\gamma^b-\frac{1}{\theta}}\overline{Q}_t\right)^{\frac{\varphi}{\varphi-1}}$ represents the demand shifter given aggregate search effort $\overline{\lambda}_t$.

Aggregation To close the model, we next solve for the expected demand of a firm from a random buyer $\overline{Q}_t$. First, aggregate real profits are given by:

$$\Pi_t\left(\overline{\lambda}_t\right) = S\int_{\underline{z}_t}^{\infty} \pi_t\left(\hat{\lambda}_t\left(\frac{\underline{z}_t}{z};\overline{\lambda}_t\right), \tilde{p}_t\left(z;\overline{\lambda}_t\right), z\right) \mathrm{d}G_t\left(z\right) = SD_t\left(\overline{\lambda}_t\right)\int_0^1 \hat{\pi}\left(x;\overline{\lambda}_t\right)^{\frac{\varphi}{\varphi-1}} \mathrm{d}\hat{G}\left(x\right). \tag{103}$$

As additional labor is required to cover search costs, the labor market clearing condition becomes $L_p + L_a + L_s = 1$, where the labor allocated to production, technology adoption, and search is given by:

$$\begin{aligned}
L_p &= \frac{(1-\alpha)\, S\int_0^{\overline{c}} c\tilde{y}_t\left(c;\overline{\lambda}_t\right) \mathrm{d}H\left(c\right)}{w_t},\\
L_a &= S\kappa\theta g_z,\\
L_s &= \frac{\chi S}{\varphi}\int_0^1 \hat{\lambda}_t\left(x;\overline{\lambda}_t\right)^{\varphi} \mathrm{d}\hat{G}\left(x\right) = \frac{\Pi_t}{\varphi w_t} = \frac{SD_t\left(\overline{\lambda}_t\right)\int_0^1 \hat{\pi}\left(x;\overline{\lambda}_t\right)^{\frac{\varphi}{\varphi-1}} \mathrm{d}\hat{G}\left(x\right)}{\varphi w_t}.
\end{aligned}$$

In equilibrium, the output of a firm with marginal cost c must equal the total demand from its buyers, that is:

$$\begin{aligned}
\tilde{y}_t\left(c;\overline{\lambda}_t\right) &= \overline{b}_t\left(\hat{\lambda}_t\left(\frac{c}{\overline{c}};\overline{\lambda}_t\right), p\left(c;\overline{\lambda}_t\right)\right)\overline{q}_t\left(p\left(c;\overline{\lambda}_t\right)\right)\\
&= \Gamma\left(\frac{\theta-1}{\theta}, \overline{\lambda}_t S^{\gamma^s} B^{\gamma^b-1}\left(\frac{c}{\overline{c}}\right)^{\theta}\right)\hat{\lambda}_t\left(\frac{c}{\overline{c}};\overline{\lambda}_t\right)\overline{\lambda}_t^{\frac{1}{\theta}} S^{\frac{\theta+1}{\theta}\gamma^s} B^{\frac{\theta+1}{\theta}\gamma^b-\frac{1}{\theta}}\overline{Q}_t\overline{c}^{-1}\\
&= \Gamma\left(\frac{\theta-1}{\theta}, \overline{\lambda}_t S^{\gamma^s} B^{\gamma^b-1}\left(\frac{c}{\overline{c}}\right)^{\theta}\right) D_t\left(\overline{\lambda}_t\right)\hat{\pi}\left(\frac{c}{\overline{c}};\overline{\lambda}_t\right)^{\frac{1}{\varphi-1}}\overline{c}^{-1},
\end{aligned}$$

which implies:

$$\int_0^{\overline{c}} c\tilde{y}_t\left(c;\overline{\lambda}_t\right) \mathrm{d}H\left(c\right) = D_t\left(\overline{\lambda}_t\right)\theta\int_0^1 \Gamma\left(\frac{\theta-1}{\theta}, \overline{\lambda}_t S^{\gamma^s} B^{\gamma^b-1} x^{\theta}\right)\hat{\pi}\left(x;\overline{\lambda}_t\right)^{\frac{1}{\varphi-1}} x^{\theta}\, \mathrm{d}x.$$

Rewriting the labor market clearing condition, we obtain:

$$\frac{SD_t\left(\overline{\lambda}_t\right)\nu\left(\overline{\lambda}_t\right)}{w_t} + S\kappa\theta g_z = 1,$$

where

$$\nu\left(\overline{\lambda}_t\right) = \frac{1}{\varphi}\int_0^1 \hat{\pi}\left(x;\overline{\lambda}_t\right)^{\frac{\varphi}{\varphi-1}} \mathrm{d}\hat{G}\left(x\right) + (1-\alpha)\,\theta\int_0^1 \hat{\pi}\left(x;\overline{\lambda}_t\right)^{\frac{1}{\varphi-1}}\Gamma\left(\frac{\theta-1}{\theta}, \overline{\lambda}_t S^{\gamma^s} B^{\gamma^b-1} x^{\theta}\right) x^{\theta}\, \mathrm{d}x.$$

We therefore obtain:

$$D_t\left(\overline{\lambda}_t\right) = \widetilde{D}\left(\overline{\lambda}_t\right) w_t = \frac{\left(1 - S\kappa\theta g_z\right) w_t}{S\nu\left(\overline{\lambda}_t\right)}, \tag{104}$$

which in turn pins down $\overline{Q}_t$:

$$\overline{Q}_t = \left[\frac{1 - S\kappa\theta g_z}{\nu\left(\overline{\lambda}_t\right)}\right]^{\frac{\varphi-1}{\varphi}} \frac{\chi^{\frac{1}{\varphi}} w_t}{\overline{\lambda}_t^{\frac{1}{\theta}} S^{\frac{\theta+1}{\theta}\gamma^s + \frac{\varphi-1}{\varphi}} B^{\frac{\theta+1}{\theta}\gamma^b - \frac{1}{\theta}}}.$$

Substituting $\overline{Q}_t$ back into the optimal search effort $\hat{\lambda}_t\left(\hat{z}; \overline{\lambda}_t\right)$ in equation (101), we have:

$$\hat{\lambda}_t\left(\hat{z}; \overline{\lambda}_t\right) = \left[\frac{1 - S\kappa\theta g_z}{\chi S\nu\left(\overline{\lambda}_t\right)}\right]^{\frac{1}{\varphi}} \hat{\pi}\left(\hat{z}; \overline{\lambda}_t\right)^{\frac{1}{\varphi-1}} = \left[\frac{\widetilde{D}\left(\overline{\lambda}_t\right)}{\chi}\right]^{\frac{1}{\varphi}} \hat{\pi}\left(\hat{z}; \overline{\lambda}_t\right)^{\frac{1}{\varphi-1}}. \tag{105}$$

The fixed-point problem (50) for $\overline{\lambda}_t$ is obtained by integrating $\hat{\lambda}_t\left(\hat{z}; \overline{\lambda}_t\right)$ over $\hat{z} \in (0, 1]$:

$$\overline{\lambda}_t = S\int_0^1 \hat{\lambda}_t\left(\hat{z}; \overline{\lambda}_t\right) \mathrm{d}\hat{G}\left(\hat{z}\right) = \left[\frac{\widetilde{D}\left(\overline{\lambda}_t\right)}{\chi}\right]^{\frac{1}{\varphi}} S\int_0^1 \hat{\pi}\left(\hat{z}; \overline{\lambda}_t\right)^{\frac{1}{\varphi-1}} \mathrm{d}\hat{G}\left(\hat{z}\right).$$

It is clear that the solution $\overline{\lambda}_t$ does not depend on time t, so we can drop its subscript t from all the relevant equations above. Hence, equations (105), (104), and (103) correspond to (48), (49), and (51), respectively.

Detrended value function and aggregate growth Given aggregate search effort $\overline{\lambda}$ and the net flow profit $\frac{\varphi-1}{\varphi} D_t\left(\overline{\lambda}\right) \hat{\pi}\left(\hat{z}; \overline{\lambda}\right)^{\frac{\varphi}{\varphi-1}}$, we use the same procedure as in Appendix A.4 to obtain the ODE (52) for the detrended value function $v\left(\hat{z}; \overline{\lambda}\right) = \frac{V_{p,t}\left(z; \overline{\lambda}\right)}{D_t\left(\overline{\lambda}\right)}$. Clearly, (52) takes the same form as (27), except that the detrended flow profit $\hat{\pi}\left(\hat{z}\right)$ is replaced by $\frac{\varphi-1}{\varphi}\hat{\pi}\left(\hat{z}; \overline{\lambda}\right)^{\frac{\varphi}{\varphi-1}}$. It follows immediately that the solution is given by (53), which corresponds to (29).

Finally, the value-matching condition gives:

$$\widetilde{D}\left(\overline{\lambda}, g_z\right)\left[\int_0^1 v\left(x; \overline{\lambda}, g_z\right) \mathrm{d}\hat{G}\left(x\right) - v\left(1; \overline{\lambda}, g_z\right)\right] = \kappa.$$

Using the same procedure as in Appendix A.7, we obtain:

$$\int_0^1 v\left(x;\overline{\lambda},g_z\right)\mathrm{d}\hat{G}\left(x\right)-v\left(1;\overline{\lambda},g_z\right)=\frac{1}{\rho+\theta g_z}\int_0^1\hat{\pi}\left(x;\overline{\lambda}\right)^{\frac{1}{\varphi-1}}\Gamma\left(\frac{\theta-1}{\theta},\overline{\lambda}S^{\gamma^s}B^{\gamma^b-1}x^{\theta}\right)x^{\theta}\,\mathrm{d}x.$$

Substituting $\int_0^1 v\left(x;\overline{\lambda},g_z\right)\mathrm{d}\hat{G}\left(x\right)-v\left(1;\overline{\lambda},g_z\right)$ and $\widetilde{D}\left(\overline{\lambda},g_z\right)$ into the value-matching condition yields:

$$g_z\left(\overline{\lambda}\right)=\frac{1-\left[\Xi\left(\overline{\lambda}\right)+\left(1-\alpha\right)\theta\right]\rho S\kappa}{\left[\Xi\left(\overline{\lambda}\right)+\left(1-\alpha\right)\theta+1\right]S\kappa\theta}.$$

The real GDP growth rate (54) is therefore given by $g\left(\overline{\lambda}\right)=\frac{g_z\left(\overline{\lambda}\right)}{1-\alpha}$. □

A.10 Proof of Proposition 7

Productivity distribution As we will verify later, under Assumption 3, the value function with creative destruction, $\widetilde{V}_{p,t}\left(z\right)$, remains increasing in productivity z, so that only firms at the productivity lower bound $\underline{z}_t$ choose to adopt new technology at time t.

The KFE incorporating creative destruction is given by:

$$\frac{\mathrm{d}\widetilde{G}_t\left(z\right)}{\mathrm{d}t}=-\left[1-\widetilde{G}_t\left(z\right)\right]\widetilde{G}_t'\left(\underline{z}_t\right)\underline{z}_t\frac{\mathrm{d}\ln\underline{z}_t}{\mathrm{d}t}-\delta\widetilde{G}_t\left(z\right). \tag{106}$$

Compared with equation (87) in Appendix A.3, an additional term $\delta\widetilde{G}_t\left(z\right)$ appears in (106) to account for the fraction of firms jumping marginally above the technology frontier $\overline{z}_t$. Differentiating (106), we have:

$$\frac{\mathrm{d}\widetilde{G}_t'\left(z\right)}{\mathrm{d}t}=\widetilde{G}_t'\left(z\right)\left[\widetilde{G}_t'\left(\underline{z}_t\right)\underline{z}_t\frac{\mathrm{d}\ln\underline{z}_t}{\mathrm{d}t}-\delta\right]. \tag{107}$$

By Definition 1, the distribution $\hat{\widetilde{G}}\left(\cdot\right)$ of inverse relative productivity $\hat{z}=\frac{\underline{z}_t}{z}\in\left[\frac{\underline{z}_t}{\overline{z}_t},1\right]$ is stationary along the BGP. Therefore, as in Appendix A.3, we obtain:

$$\widetilde{G}_t'\left(\underline{z}_t\right)\underline{z}_t=\widetilde{G}_0'\left(\underline{z}_0\right)\underline{z}_0=\frac{\theta}{1-\left(\frac{\overline{z}_0}{\underline{z}_0}\right)^{-\theta}}\ \text{for all}\ t. \tag{108}$$

Plugging (108) into (107), rearranging, and integrating both sides from 0 to $\mathrm{d}t$, we have:

$$\int_0^{\mathrm{d}t} \mathrm{d}\ln \widetilde{G}'_s(z) = \frac{\theta}{1-\left(\frac{\overline{z}_0}{\underline{z}_0}\right)^{-\theta}} \int_0^{\mathrm{d}t} \mathrm{d}\ln \underline{z}_s - \int_0^{\mathrm{d}t} \delta\, \mathrm{d}s,$$

which implies that:

$$\widetilde{G}'_{\mathrm{d}t}(z) = \left(\frac{\underline{z}_{\mathrm{d}t}}{\underline{z}_0}\right)^{\frac{\theta}{1-\left(\frac{\overline{z}_0}{\underline{z}_0}\right)^{-\theta}}} \exp(-\delta \mathrm{d}t)\, \widetilde{G}'_0(z) = \frac{\theta \underline{z}_0^{\theta} z^{-\theta-1}}{1-\left(\frac{\overline{z}_0}{\underline{z}_0}\right)^{-\theta}} \left(\frac{\underline{z}_{\mathrm{d}t}}{\underline{z}_0}\right)^{\frac{\theta}{1-\left(\frac{\overline{z}_0}{\underline{z}_0}\right)^{-\theta}}} \exp(-\delta \mathrm{d}t). \tag{109}$$

Substituting $z = \underline{z}_{\mathrm{d}t}$ into (109) and applying (108) yields:

$$\underline{z}_{\mathrm{d}t} = \exp\left(\left(\frac{\delta}{\theta}\left(\frac{\overline{z}_0}{\underline{z}_0}\right)^{\theta} - \frac{\delta}{\theta}\right)\mathrm{d}t\right)\underline{z}_0.$$

Hence, along the BGP, the lower bound of the productivity distribution grows at constant rate $g_z = \frac{\delta}{\theta}\left[\left(\frac{\overline{z}_0}{\underline{z}_0}\right)^{\theta} - 1\right]$. Using g_z, (109) simplifies to:

$$\widetilde{G}'_{\mathrm{d}t}(z) = \frac{\theta \underline{z}_{\mathrm{d}t}^{\theta} z^{-\theta-1}}{1-\left(\frac{\overline{z}_0}{\underline{z}_0}\right)^{-\theta}}.$$

Finally, the boundary condition $\int_{\underline{z}_{\mathrm{d}t}}^{\overline{z}_{\mathrm{d}t}} \widetilde{G}'_{\mathrm{d}t}(z)\, \mathrm{d}z = 1$ implies that:

$$\frac{1-\left(\frac{\overline{z}_{\mathrm{d}t}}{\underline{z}_{\mathrm{d}t}}\right)^{-\theta}}{1-\left(\frac{\overline{z}_0}{\underline{z}_0}\right)^{-\theta}} = 1. \tag{110}$$

Since $\underline{z}_t$ grows at rate g_z, for (110) to hold, the technology frontier $\overline{z}_t$ must grow at the same rate g_z along the BGP. We have thus proved that the ratio $\frac{\underline{z}_t}{\overline{z}_t} = \iota$ remains constant over time. Substituting ι into the equation for g_z, we obtain (62).

Optimal pricing, markups, and profits Given the doubly truncated Pareto distribution, the marginal cost distribution becomes $\widetilde{H}(c;\iota) = \frac{\left(\frac{c}{\overline{c}}\right)^{\theta} - \iota^{\theta}}{1-\iota^{\theta}}$ with support $[\iota\overline{c}, \overline{c}]$. Since the lower bound is no longer zero, the optimal pricing function corresponding to (79) is given by:

$$p\left(c\right)=\frac{\exp\left(-\tilde{\Lambda}c^{\theta}\right)p\left(\iota\bar{c}\right)}{\exp\left(-\tilde{\Lambda}\left(\iota\bar{c}\right)^{\theta}\right)-\tilde{\Lambda}^{\frac{1}{\theta}}\left[\gamma\left(\frac{\theta-1}{\theta},\tilde{\Lambda}c^{\theta}\right)-\gamma\left(\frac{\theta-1}{\theta},\tilde{\Lambda}\left(\iota\bar{c}\right)^{\theta}\right)\right]p\left(\iota\bar{c}\right)},\tag{111}$$

where $\tilde{\Lambda}=\frac{\lambda S^{\gamma^{s}}B^{\gamma^{b}-1}\bar{c}^{-\theta}}{1-\iota^{\theta}}$. Using the same logic as in Appendix A.2, we obtain the following constraint on $p\left(\iota\bar{c}\right)$ such that $p'\left(c\right)>0$ for all c:

$$\frac{\exp\left(-\tilde{\Lambda}\left(\iota\bar{c}\right)^{\theta}\right)}{\frac{\exp\left(-\tilde{\Lambda}\bar{c}^{\theta}\right)}{\bar{c}}+\tilde{\Lambda}^{\frac{1}{\theta}}\left[\gamma\left(\frac{\theta-1}{\theta},\tilde{\Lambda}\bar{c}^{\theta}\right)-\gamma\left(\frac{\theta-1}{\theta},\tilde{\Lambda}\left(\iota\bar{c}\right)^{\theta}\right)\right]}<p\left(\iota\bar{c}\right)<\frac{\exp\left(-\tilde{\Lambda}\left(\iota\bar{c}\right)^{\theta}\right)}{\tilde{\Lambda}^{\frac{1}{\theta}}\left[\gamma\left(\frac{\theta-1}{\theta},\tilde{\Lambda}\bar{c}^{\theta}\right)-\gamma\left(\frac{\theta-1}{\theta},\tilde{\Lambda}\left(\iota\bar{c}\right)^{\theta}\right)\right]}.$$

With a non-zero lower bound $\iota\bar{c}$, the above constraint is no longer monotonic in $\bar{c}$ on either side. We therefore obtain $p\left(\iota\bar{c}\right)$ through a guess-and-verify approach. Conjecture that

$$p\left(\iota\bar{c}\right)=\frac{\exp\left(-\tilde{\Lambda}\left(\iota\bar{c}\right)^{\theta}\right)}{\tilde{\Lambda}^{\frac{1}{\theta}}\left[\Gamma\left(\frac{\theta-1}{\theta}\right)-\gamma\left(\frac{\theta-1}{\theta},\tilde{\Lambda}\left(\iota\bar{c}\right)^{\theta}\right)\right]},\tag{112}$$

which corresponds to (59) in Assumption 3. Since $\Gamma\left(\frac{\theta-1}{\theta}\right)>\gamma\left(\frac{\theta-1}{\theta},\tilde{\Lambda}\bar{c}^{\theta}\right)$, the right-hand side of the constraint is clearly satisfied. It remains to verify that

$$\frac{\tilde{\Lambda}^{-\frac{1}{\theta}}\exp\left(-\tilde{\Lambda}\bar{c}^{\theta}\right)+\gamma\left(\frac{\theta-1}{\theta},\tilde{\Lambda}\bar{c}^{\theta}\right)\bar{c}}{\bar{c}}>\Gamma\left(\frac{\theta-1}{\theta}\right)$$

holds for all $\bar{c}\in(0,\infty)$ such that the left-hand side of the constraint is also satisfied. It is easy to show that the derivative of the left-hand side with respect to $\bar{c}$ is $-\frac{\tilde{\Lambda}^{-\frac{1}{\theta}}\exp\left(-\tilde{\Lambda}\bar{c}^{\theta}\right)}{\bar{c}^{2}}<0$ and thus:

$$\min_{\bar{c}}\frac{\tilde{\Lambda}^{-\frac{1}{\theta}}\exp\left(-\tilde{\Lambda}\bar{c}^{\theta}\right)+\gamma\left(\frac{\theta-1}{\theta},\tilde{\Lambda}\bar{c}^{\theta}\right)\bar{c}}{\bar{c}}=\lim_{\bar{c}\to\infty}\frac{\tilde{\Lambda}^{-\frac{1}{\theta}}\exp\left(-\tilde{\Lambda}\bar{c}^{\theta}\right)+\gamma\left(\frac{\theta-1}{\theta},\tilde{\Lambda}\bar{c}^{\theta}\right)\bar{c}}{\bar{c}}=\Gamma\left(\frac{\theta-1}{\theta}\right).$$

Therefore, (112) constitutes an admissible boundary pricing. Plugging (112) into (111) yields:

$$p\left(c;\iota\right)=\frac{\tilde{\Lambda}^{-\frac{1}{\theta}}\exp\left(-\tilde{\Lambda}c^{\theta}\right)}{\Gamma\left(\frac{\theta-1}{\theta},\tilde{\Lambda}c^{\theta}\right)}=\frac{\left(\frac{\lambda S^{\gamma^{s}}B^{\gamma^{b}-1}}{1-\iota^{\theta}}\left(\frac{c}{\bar{c}}\right)^{\theta}\right)^{-\frac{1}{\theta}}\exp\left(-\frac{\lambda S^{\gamma^{s}}B^{\gamma^{b}-1}}{1-\iota^{\theta}}\left(\frac{c}{\bar{c}}\right)^{\theta}\right)}{\Gamma\left(\frac{\theta-1}{\theta},\frac{\lambda S^{\gamma^{s}}B^{\gamma^{b}-1}}{1-\iota^{\theta}}\left(\frac{c}{\bar{c}}\right)^{\theta}\right)}c,$$

which corresponds to equation (63).

Substituting $p(c;\iota)$ into the profit function and applying the change of variables $\hat{z} = \frac{\underline{z}_t}{z} = \frac{c}{\bar{c}}$, we have:

$$\tilde{\pi}_t(z;\iota) = D_t(\iota)\,\hat{\pi}\left(\hat{z};\tilde{\lambda}(\iota)\right),$$

where

$$D_t(\iota) = \left(1-\iota^{\theta}\right)\tilde{\lambda}(\iota)^{\frac{\theta+1}{\theta}} S^{\frac{\theta+1}{\theta}\gamma^s-1} B^{\frac{\theta+1}{\theta}\gamma^b-\frac{1}{\theta}} \exp\left(\tilde{\lambda}(\iota) S^{\gamma^s} B^{\gamma^b-1}\iota^{\theta}\right)\overline{Q}_t \tag{113}$$

denotes the demand shifter with creative destruction and $\hat{\pi}\left(\hat{z};\tilde{\lambda}(\iota)\right)$ takes the same form as $\hat{\pi}(\hat{z}) = \frac{\pi_t(z)}{D_t}$ in the baseline economy except that λ is replaced by $\tilde{\lambda}(\iota)$.

Detrended value function Next, we solve for the detrended value function. Using $\tilde{v}(\hat{z}) = \frac{\widetilde{V}_{p,t}(z)}{D_t(\iota)} = \frac{\widetilde{V}_{p,t}(z)}{\widetilde{D}(\iota)w_t}$, we have:

$$\widetilde{V}'_{p,t}(\overline{z}_t)\frac{\mathrm{d}\overline{z}_t}{\mathrm{d}t} = \left.\frac{\mathrm{d}\left(D_t(\iota)\,\tilde{v}\left(\frac{\underline{z}_t}{z}\right)\right)}{\mathrm{d}z}\right|_{z=\overline{z}_t}\overline{z}_t\frac{\mathrm{d}\ln\overline{z}_t}{\mathrm{d}t} = -D_t(\iota)\,\tilde{v}'(\iota)\,\iota g_z$$

and $\frac{\mathrm{d}\widetilde{V}_{p,t}(z)}{\mathrm{d}t}$, which takes the same form as (89). Substituting into (58), dividing both sides by $D_t(\iota)$, and rearranging, we obtain the following ODE under Assumption 3:

$$\tilde{v}'(\hat{z}) = \frac{\rho+\delta}{g_z\hat{z}}\tilde{v}(\hat{z}) - \frac{1}{g_z\hat{z}}\left\{\hat{\pi}\left(\hat{z};\tilde{\lambda}(\iota)\right) + \delta\left[\tilde{v}(\iota) - \tilde{v}'(\iota)\,\iota g_z\right] - \frac{\varsigma\hat{z}^{\eta}}{\widetilde{D}(\iota)}\right\}.$$

The integrating factor, $\exp\left(-\int\frac{\rho+\delta}{g_z\hat{z}}\,\mathrm{d}\hat{z}\right) = \hat{z}^{-\frac{\rho+\delta}{g_z}}$, implies:

$$\begin{aligned}
\frac{\mathrm{d}\left(\tilde{v}(\hat{z})\,\hat{z}^{-\frac{\rho+\delta}{g_z}}\right)}{\mathrm{d}z} &= \hat{z}^{-\frac{\rho+\delta}{g_z}}\left[\tilde{v}'(\hat{z}) - \frac{\rho+\delta}{g_z\hat{z}}\tilde{v}(\hat{z})\right] \\
&= -\frac{\hat{z}^{-\frac{\rho+\delta}{g_z}-1}}{g_z}\left\{\hat{\pi}\left(\hat{z};\tilde{\lambda}(\iota)\right) + \delta\left[\tilde{v}(\iota) - \tilde{v}'(\iota)\,\iota g_z\right] - \frac{\varsigma\hat{z}^{\eta}}{\widetilde{D}(\iota)}\right\}.
\end{aligned}$$

Integrating both sides and applying the results from Appendix A.4, together with the smooth-pasting condition $\tilde{v}'(1) = 0$, we obtain:

$$\tilde{v}(\hat{z}) = v\left(\hat{z};\tilde{\lambda}(\iota),\rho+\delta\right) + \frac{\delta}{\rho+\delta}\left[\tilde{v}(\iota) - \tilde{v}'(\iota)\,\iota g_z\right] - \frac{c_{\delta}(\hat{z})}{\rho+\delta-\eta g_z}, \tag{114}$$

where $v\left(\hat{z};\tilde{\lambda}(\iota),\rho+\delta\right)$ takes the same form as the detrended value function in the baseline model, given by equation (29), with the search efficiency parameter λ replaced by $\tilde{\lambda}(\iota)$ and the discount rate ρ replaced by $\rho+\delta$. $c_\delta(\hat{z})$ is given by (66).

Substituting $\hat{z}=\iota$ into (114) and rearranging, we have:

$$\tilde{v}(\iota)=\frac{1}{\rho}\left[(\rho+\delta)\,v\left(\iota;\tilde{\lambda}(\iota),\rho+\delta\right)-\delta\tilde{v}'(\iota)\,\iota g_z-\frac{(\rho+\delta)\,c_\delta(\iota)}{\rho+\delta-\eta g_z}\right].$$

Plugging $\tilde{v}(\iota)$ into (114) yields:

$$\tilde{v}(\hat{z})=v\left(\hat{z};\tilde{\lambda}(\iota),\rho+\delta\right)+\frac{\delta}{\rho}\left[v\left(\iota;\tilde{\lambda}(\iota),\rho+\delta\right)-\tilde{v}'(\iota)\,\iota g_z-\frac{c_\delta(\iota)}{\rho+\delta-\eta g_z}\right]-\frac{c_\delta(\hat{z})}{\rho+\delta-\eta g_z}. \tag{115}$$

Finally, differentiating (114) and substituting $\hat{z}=\iota$ yields:

$$\tilde{v}'(\iota)=v'\left(\iota;\tilde{\lambda}(\iota),\rho+\delta\right)-\frac{c_\delta'(\iota)}{\rho+\delta-\eta g_z},$$

where $c_\delta'(\hat{z})=\frac{\varsigma\eta\hat{z}^{\eta-1}}{\widetilde{D}(\iota)}\left(1-\hat{z}^{\frac{\rho+\delta-\eta g_z}{g_z}}\right)$. Plugging $\tilde{v}'(\iota)$ into (115), we obtain:

$$\begin{aligned}\tilde{v}(\hat{z})=\;&v\left(\hat{z};\tilde{\lambda}(\iota),\rho+\delta\right)-\frac{c_\delta(\hat{z})}{\rho+\delta-\eta g_z}\\&+\frac{\delta}{\rho}\left\{v\left(\iota;\tilde{\lambda}(\iota),\rho+\delta\right)-\frac{c_\delta(\iota)}{\rho+\delta-\eta g_z}-\left[v'(\iota;\rho+\delta)-\frac{c_\delta'(\iota)}{\rho+\delta-\eta g_z}\right]\iota g_z\right\},\end{aligned} \tag{116}$$

which corresponds to equation (64), with $v_\delta(\hat{z})$ given by (65).[48]

Since $\hat{z}\in[\iota,1]$, with $\eta\leq\frac{\rho+\delta}{g_z}$ under Assumption 3, we have $c_\delta'(\hat{z})\geq 0$. Together with $v'\left(\hat{z};\tilde{\lambda}(\iota),\rho+\delta\right)\leq 0$, we have verified that $\tilde{v}'(\hat{z})\leq 0$ and thus only firms at the productivity lower bound $\underline{z}_t$ (i.e., $\hat{z}=1$) choose to adopt new technology at time t.

Aggregation We complete the proof by deriving the aggregate variables. The labor market clearing condition with creative destruction is given by $L_p+L_a+L_\delta=1$, where the labor allocated to production, technology adoption, and the research project for creative

[48] Note that the second line in (116) is constant for any $\hat{z}\in[\iota,1]$.

destruction is given by:

$$L_p = S \int_{\iota\bar{c}}^{\bar{c}} \tilde{l}_t (c;\iota)\, \mathrm{d}\widetilde{H}(c;\iota) = \frac{(1-\alpha) S \int_{\iota\bar{c}}^{\bar{c}} c\tilde{y}_t (c;\iota)\, \mathrm{d}\widetilde{H}(c;\iota)}{w_t},$$
$$L_a = \frac{S\kappa\theta g_z}{1-\iota^\theta},$$
$$L_\delta = S \int_\iota^1 \varsigma x^\eta \, \mathrm{d}\hat{\widetilde{G}}(x;\iota) = \frac{S\varsigma\theta\left(1-\iota^{\eta+\theta}\right)}{(\eta+\theta)\left(1-\iota^\theta\right)}.$$

In equilibrium, the output of a firm with marginal cost c must equal the total demand from its buyers. Deriving the total demand and using (113), we obtain:

$$\tilde{y}_t(c;\iota) = D_t(\iota)\,\Gamma\left(\frac{\theta-1}{\theta}, \tilde{\lambda}(\iota) S^{\gamma^s} B^{\gamma^b-1}\left(\frac{c}{\bar{c}}\right)^\theta\right)\bar{c}^{-1}.$$

Plugging $\tilde{y}_t(c;\iota)$ into L_p and rearranging the labor market clearing condition yields (67):

$$\widetilde{D}(\iota) = \left[(1-\alpha)\theta \int_\iota^1 \Gamma\left(\frac{\theta-1}{\theta}, \tilde{\lambda}(\iota) S^{\gamma^s} B^{\gamma^b-1} x^\theta\right) x^\theta \,\mathrm{d}x\right]^{-1}\left[\frac{1-\iota^\theta - S\kappa\theta g_z}{S} - \frac{\varsigma\theta\left(1-\iota^{\eta+\theta}\right)}{\eta+\theta}\right].$$

Finally, aggregate real profit is simply given by:

$$\Pi_t(\iota) = S\int_{\underline{z}_t}^{\bar{z}_t} \tilde{\pi}_t(z;\iota)\,\mathrm{d}\widetilde{G}_t(z) = \left[\widetilde{D}(\iota) S \int_\iota^1 \hat{\pi}\left(x;\tilde{\lambda}(\iota)\right)\mathrm{d}\hat{\widetilde{G}}(x)\right] w_t(\iota). \tag{117}$$

To derive the real wage w_t, we first follow the same procedure as in Appendix A.5 to recover the distribution of the minimum marginal cost accessible to a buyer:

$$\widetilde{H}_{min}(c;\iota) = \frac{1-\exp\left(-\tilde{\lambda}(\iota) S^{\gamma^s} B^{\gamma^B-1}\left(\bar{c}^{-\theta}c^\theta - \iota^\theta\right)\right)}{\Omega}. \tag{118}$$

The final good price index takes the form:

$$P = \exp\left(\Omega \int_{\iota\bar{c}}^{\bar{c}} \ln p(c;\iota)\,\mathrm{d}\widetilde{H}_{min}(c;\iota)\right) = 1.$$

Plugging in $p(c;\iota)$ from (63) and $\widetilde{H}_{min}(c;\iota)$ from (118), and proceeding as in Appendix A.6, we have:

$$1 = \exp\left(\tilde{\lambda}(\iota) S^{\gamma^s} B^{\gamma^b-1}\left(\exp\left(-\lambda S^{\gamma^s} B^{\gamma^b-1}\right) - \iota^\theta\right) - \exp\left(\tilde{\lambda}(\iota) S^{\gamma^s} B^{\gamma^b-1}\iota^\theta\right)\tilde{\mu}\left(\tilde{\lambda}(\iota) S^{\gamma^s} B^{\gamma^b-1}\right) - \Omega\right)$$

$$\left[\tilde{\lambda}\left(\iota\right)S^{\gamma^{s}}B^{\gamma^{b}-1}\right]^{-\frac{\Omega}{\theta}}\bar{c}^{\Omega},$$

where $\tilde{\mu}\left(\tilde{\lambda}\left(\iota\right)S^{\gamma^{s}}B^{\gamma^{b}-1}\right)=\int_{\tilde{\lambda}(\iota)S^{\gamma^{s}}B^{\gamma^{b}-1}\iota^{\theta}}^{\tilde{\lambda}(\iota)S^{\gamma^{s}}B^{\gamma^{b}-1}}\frac{\ln\Gamma\left(\frac{\theta-1}{\theta},x\right)}{\exp(x)}\,\mathrm{d}x$. Plugging in $\bar{c}=\frac{w_t^{1-\alpha}}{\underline{z}_t}$ and rearranging, we obtain:

$$w_t=\left\{\exp\left(1-\frac{\tilde{\lambda}\left(\iota\right)S^{\gamma^{s}}B^{\gamma^{b}-1}\left(\exp\left(-\lambda S^{\gamma^{s}}B^{\gamma^{b}-1}\right)-\iota^{\theta}\right)-\exp\left(\tilde{\lambda}\left(\iota\right)S^{\gamma^{s}}B^{\gamma^{b}-1}\iota^{\theta}\right)\tilde{\mu}\left(\tilde{\lambda}\left(\iota\right)S^{\gamma^{s}}B^{\gamma^{b}-1}\right)}{\Omega}\right)\right.$$
$$\left.\left[\tilde{\lambda}\left(\iota\right)S^{\gamma^{s}}B^{\gamma^{b}-1}\right]^{\frac{1}{\theta}}\underline{z}_t\right\}^{\frac{1}{1-\alpha}}. \tag{119}$$

□

A.11 Proof of Corollary 1

Substituting (116) into the value-matching condition and rearranging, we obtain:

$$\begin{aligned}\kappa&=\widetilde{D}\left(\iota,g_z\right)\left\{\left[\int_{\iota}^{1}v\left(x;\tilde{\lambda}\left(\iota\right),\rho+\delta,g_z\right)\mathrm{d}\hat{\widetilde{G}}\left(x;\iota\right)-v\left(1;\tilde{\lambda}\left(\iota\right),\rho+\delta,g_z\right)\right]\right.\\&\quad\left.-\frac{1}{\rho+\delta-\eta g_z}\left[\int_{\iota}^{1}c_{\delta}\left(x;\iota,g_z\right)\mathrm{d}\hat{\widetilde{G}}\left(x;\iota\right)-c_{\delta}\left(1;\iota,g_z\right)\right]\right\}\\&=\widetilde{D}\left(\iota,g_z\right)\left[\Delta v\left(\iota,g_z\right)-\frac{\Delta c_{\delta}\left(\iota,g_z\right)}{\rho+\delta-\eta g_z}\right].\end{aligned}\tag{120}$$

Using the same procedure as in Appendix A.7, together with $\hat{\widetilde{G}}\left(x;\iota\right)=\frac{x^{\theta}-\iota^{\theta}}{1-\iota^{\theta}}$, we have:

$$\begin{aligned}\Delta v\left(\iota,g_z\right)&=-\int_{\iota}^{1}\hat{\widetilde{G}}\left(x;\iota\right)v'\left(x;\tilde{\lambda}\left(\iota\right),\rho+\delta,g_z\right)\mathrm{d}x\\&=-\frac{1}{\rho+\delta+\theta g_z}\int_{\iota}^{1}\left\{\frac{x^{\theta}-\iota^{\theta}}{1-\iota^{\theta}}-\frac{\theta g_z}{\rho+\delta}\frac{\iota^{\theta}}{1-\iota^{\theta}}\left[1-\left(\frac{\iota}{x}\right)^{\frac{\rho+\delta}{g_z}}\right]\right\}\mathrm{d}\hat{\pi}\left(x;\tilde{\lambda}\left(\iota\right)\right)\\&=\frac{1}{\rho+\delta+\theta g_z}\int_{\iota}^{1}\Gamma\left(\frac{\theta-1}{\theta},\tilde{\lambda}\left(\iota\right)S^{\gamma^{s}}B^{\gamma^{b}-1}x^{\theta}\right)\left[\frac{x^{\theta}-\iota^{\theta}}{1-\iota^{\theta}}-\psi\left(x;\iota,g_z\right)\right]\mathrm{d}x,\end{aligned}\tag{121}$$

where $\psi\left(x;\iota,g_z\right)=\frac{\theta g_z\iota^{\theta}}{(\rho+\delta)\left(1-\iota^{\theta}\right)}\left[1-\left(\frac{\iota}{x}\right)^{\frac{\rho+\delta}{g_z}}\right]$. Next, plugging in $c_{\delta}\left(\hat{z}\right)$ from (66) yields:

$$\begin{aligned}\Delta c_{\delta}\left(\iota,g_z\right)&=-\int_{\iota}^{1}\hat{\widetilde{G}}\left(x;\iota\right)c_{\delta}'\left(x;\iota,g_z\right)\mathrm{d}x\\&=-\frac{\varsigma\eta}{\widetilde{D}\left(\iota\right)}\int_{\iota}^{1}x^{\eta-1}\left(1-x^{\frac{\rho+\delta-\eta g_z}{g_z}}\right)\frac{x^{\theta}-\iota^{\theta}}{1-\iota^{\theta}}\,\mathrm{d}x.\end{aligned}\tag{122}$$

Equations (120), (121), and (122) therefore correspond to (69), (70), and (71), respectively. $\square$

B Markup Estimation

B.1 Data

Industry classification To estimate within-variety markups given by (18), we need an industry classification that accurately identifies a firm's most direct product-market competitors based on product similarity. We therefore use the newly developed ETNIC data from Hoberg and Phillips (2025).

ETNIC applies a doc2vec embedding model, e.g., Le and Mikolov (2014), to the 10-K business descriptions of all Compustat firms from 1988 to 2023 to compute the text-based cosine similarity for each pair of firms. Compared with the original TNIC data (Hoberg and Phillips, 2016), ETNIC not only has broader coverage—both cross-sectionally and over time—but also better captures information from synonyms and context.

Following Cabezon and Hoberg (2026), we treat firm pairs whose cosine similarity lies in the top 1% of the sample as the most direct competitors. The high granularity—roughly equivalent to 4-digit SIC codes—ensures that firms grouped together produce highly substitutable varieties, so that within-group price dispersion is more likely to be driven by search frictions rather than product differentiation, in line with equation (18).

Compared with traditional industry classification schemes such as NAICS and SIC, ETNIC offers two additional advantages in our setting. First, our model abstracts from firms' decisions to introduce new products. The time-varying nature of ETNIC allows firms to be promptly regrouped after substantial product updates, thereby reducing potential bias in within-variety markup estimation stemming from variety shocks. Second, Chen et al. (2016) find that traditional industry classifications are subject to managerial manipulation. By contrast, Regulation S-K mandates that firms disclose detailed and accurate product-related information, which helps ensure the objectivity of ETNIC and its strong link to product markets.

Financial information We merge ETNIC with firms' financial information from Compustat. For markup estimation, we extract gross sales (sale), cost of goods sold (cogs), and gross and net property, plant, and equipment (ppegt and ppent, respectively). We drop observations with missing values for any of these variables. Because lagged inputs are required as instruments, our estimation sample covers the period 1989–2023.

Deflators To express financial variables in real terms, we obtain GDP and investment deflators (Gross domestic product (implicit price deflator) and Gross private domestic investment: Fixed investment: Nonresidential (implicit price deflator), respectively) from the Federal Reserve Economic Data (FRED).

B.2 Methodology

In this section, we explain in detail how to obtain the markup estimates corresponding to equation (18). All notation introduced in this section is independent of that used in our main text.

Production approach Following De Loecker et al. (2020), we estimate markups using the production approach. Specifically, the cost-minimization problem of firm i in industry j at time t yields the following Lagrangian:

$$\mathcal{L}^j \left(V_{it}, K_{it}, c_{it}\right) = W_t^j V_{it} + R_t^j K_{it} - c_{it} \left[Y^j \left(V_{it}, K_{it}, \Omega_{it}\right) - \overline{Y}_{it}\right], \tag{123}$$

where V_{it} denotes the quantity of variable inputs with price W_t^j, K_{it} denotes capital stock with user cost R_t^j, $Y^j \left(V_{it}, K_{it}, \Omega_{it}\right)$ is the production function in industry j with Ω_{it} representing firm i's productivity, and the Lagrange multiplier c_{it} equals firm i's marginal cost according to the envelope theorem.

The first-order condition with respect to V_{it} in (123) delivers the markup expression:

$$m_{it} = \frac{P_{it}}{c_{it}} = \alpha_{it}^j \frac{P_{it} Y_{it}}{W_t^j V_{it}}, \tag{124}$$

where P_{it} is the output price charged by firm i and $\alpha_{it}^j = \frac{\partial \ln Y^j(V_{it}, K_{it}, \Omega_{it})}{\partial \ln V_{it}}$ denotes the output elasticity in variable input for firm i in industry j. We proxy $W_t^j V_{it}$ with cost of goods sold and $P_{it}Y_{it}$ with gross sales, so that markup estimation reduces to estimating α^j_{it}.

Production function estimation We combine the control function approach of Ackerberg et al. (2015) with the refinements in De Ridder et al. (2025) to obtain output elasticity estimates $\hat{\alpha}^j_{it}$. Using lowercase letters to denote log-transformed variables, we estimate the following translog production function:

$$y_{it} = \boldsymbol{x}'_{it}\boldsymbol{\beta}^j + \omega_{it} + \varepsilon_{it}, \tag{125}$$

where

$$\boldsymbol{x}'_{it}\boldsymbol{\beta}^j = \beta^j_0 + \beta^j_v v_{it} + \beta^j_k k_{it} + \beta^j_{vv} v^2_{it} + \beta^j_{kk} k^2_{it} + \beta^j_{vk} v_{it} k_{it},$$

ω_{it} denotes productivity observed by firms but unobserved by the econometrician, and ε_{it} represents i.i.d. noise unobserved by both, including measurement errors in output y_{it}. The translog production function is a second-order approximation to any production function and yields more general output elasticities that vary with a firm's inputs.

Because input choices are usually made based on productivity ω_{it}, a simple OLS estimation of (125) suffers from omitted-variable bias. However, under the assumptions of Ackerberg et al. (2015), ω_{it} can be recovered as a function of inputs, that is:

$$\omega_{it} = f^j\left(v_{it}, k_{it}\right), \tag{126}$$

where $f^j(\cdot)$ is called the control function. Substituting (126) into (125) implies that we can purge the noise term ε_{it} by regressing y_{it} on a high-order polynomial in (v_{it}, k_{it}), yielding fitted output $\hat{y}_{it}$.

We assume that productivity evolves according to the following nonlinear process:

$$\omega_{it} = \rho^j_1 \omega_{it-1} + \rho^j_2 \omega^2_{it-1} + \xi_{it}, \tag{127}$$

where ξ_{it} denotes i.i.d. innovations to productivity at time t and is therefore orthogonal to input choices at time $t-1$. Given that $\omega_{it} = \hat{y}_{it} - \boldsymbol{x}'_{it}\boldsymbol{\beta}^j$, substituting into (127) yields:

$$\xi_{it}\left(\rho^j_1, \rho^j_2, \boldsymbol{\beta}^j\right) = \hat{y}_{it} - \boldsymbol{x}'_{it}\boldsymbol{\beta}^j - \rho^j_1\left(\hat{y}_{it-1} - \boldsymbol{x}'_{it-1}\boldsymbol{\beta}^j\right) - \rho^j_2\left(\hat{y}_{it-1} - \boldsymbol{x}'_{it-1}\boldsymbol{\beta}^j\right)^2.$$

The GMM estimator $\left(\rho_1^j, \rho_2^j, \boldsymbol{\beta}^j\right)$ is thus defined by the following moment condition:

$$\mathbb{E}\left[\xi_{it}\left(\hat{\rho}_1^j, \hat{\rho}_2^j, \hat{\boldsymbol{\beta}}^j\right) \otimes \begin{pmatrix} \boldsymbol{x}_{it-1} \\ \hat{y}_{it-1} - \boldsymbol{x}'_{it-1}\hat{\boldsymbol{\beta}}^j \\ \left(\hat{y}_{it-1} - \boldsymbol{x}'_{it-1}\hat{\boldsymbol{\beta}}^j\right)^2 \end{pmatrix}\right] = \mathbf{0}, \tag{128}$$

which gives the following output elasticity estimates:

$$\hat{\alpha}_{it}^j = \hat{\beta}_v^j + 2\hat{\beta}_{vv}^j v_{it} + \hat{\beta}_{vk}^j k_{it}.$$

In estimating (128), we implement the following procedure. First, because we group firms based on ETNIC, industry j here refers to a benchmark firm. Hence, for each firm j, we estimate (128) using the subsample of its closest product-market competitors defined by the top-1% cosine-similarity cutoff. We impose a minimum sample size of 50 for the central limit theorem to be applied. This filter also effectively excludes niche markets with few direct competitors, where markups may deviate from equation (18). We construct the capital stock using the perpetual inventory method, proxying investment by changes in net property, plant, and equipment and deflating it with the investment deflator. For other nominal variables, we deflate them based on the GDP deflator. Finally, when observations are missing for some intermediate years, we interpolate using the average of adjacent-year observations whenever available.

Product-similarity-weighted markup estimates Substituting the estimated output elasticity $\hat{\alpha}_{it}^j$, we obtain the markup estimate $\hat{m}_{it}^j$ for firm i at time t in the competitor subsample defined by benchmark firm j. As ETNIC provides a continuous similarity measure at the firm-pair level, it is possible that firm i is classified as a direct competitor of multiple benchmark firms, yielding multiple markup estimates. To capture the within-variety markup driven by search frictions, we assign higher weight to estimates indexed by benchmark firms j that are closer to firm i in product space.

Specifically, let $\mathcal{J}_{it}$ denote the set of firms for which firm i is a direct competitor at time t, and let cosine_{ijt} denote the ETNIC similarity score between firms i and j at time t. The product-similarity-weighted markup estimate for firm i at time t is given by:

$$\hat{m}_{it} = \sum_{j \in \mathcal{J}_{it}} \frac{\text{cosine}_{ijt}}{\sum_{h \in \mathcal{J}_{it}} \text{cosine}_{iht}} \hat{m}_{it}^{j},$$

which we treat as the empirical counterpart of (18). Finally, we truncate the estimates at the 1st and 99th percentiles to mitigate the influence of outliers.

B.3 Discussion

A key caveat in the estimation procedure of markups for Compustat firms is that we only observe gross sales, rather than quantities of goods sold. As a result, the left-hand side of (125) becomes $p_{it} + y_{it}$. In this case, the GMM estimator suffers from omitted-variable bias whenever the current log price is correlated with lagged log inputs, i.e., $\mathbb{E}\left[p_{it}\boldsymbol{x}_{it-1}\right] \neq \mathbf{0}$. Under imperfect competition and a persistent productivity process, this bias generically exists.

However, as shown by De Ridder et al. (2025), the bias becomes a constant common to all firms as long as the demand system is invertible so that the log price can be written as:

$$p_{it} = -\sum_{k} d_{ikt} y_{kt},$$

where d_{ikt} denotes the heterogeneous cross-elasticity of firm i's price with respect to firm k's output. In our search economy, (14) implies that a firm's price depends on the prices (and hence output) of all other firms, so the argument in De Ridder et al. (2025) applies.

With the constant bias, regressions based on log markups can still precisely capture markup variation (Li et al., 2025), which is the approach we employ in the GMM estimation of model parameters.

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