

# A Unified Theory of the Term-Structure and Monetary Stabilization

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Bernanke (2014): “QE works in practice but not in theory”

Blanchard (2016): “Solution is to introduce two interest rates, the policy rate set by the central bank in the LM equation and the rate at which people and firms can borrow, which enters the IS equation, and then to discuss how the financial system determines the spread between the two.”

A need for a framework addressing Bernanke (2014)

Need for a deviation from the ‘expectation hypothesis’

⇒ **quantity** matters!

Addressing Blanchard (2016)

Term-structure + private capital market needed

Example: IS equation with 3 maturities (short-term, 10 years, 30 years)

$$\hat{z}_t = E_t \left[ \hat{c}_{t+1} + \beta \left( \hat{r}_{t+1}^S - \hat{r}_{t+1}^L \right) \right]$$

where

$$\hat{r}_{t+1}^S = \hat{r}_{t+1}^L + w_t^{10} (\hat{r}_{t+1}^{10} - \hat{r}_{t+1}^L) + w_t^{30} (\hat{r}_{t+1}^{30} - \hat{r}_{t+1}^L)$$

Policy rate

Up to a first-order, portfolio demand ( $w_t^{10}; w_t^{30}$ ) depend on relative returns:

$$w_t^{10} = w_t^{10} \left( \hat{r}_{t+1}^{10} - \hat{r}_{t+1}^L \right) / \left( \hat{r}_{t+1}^{10} - \hat{r}_{t+1}^L + \left( \hat{r}_{t+1}^{30} - \hat{r}_{t+1}^L \right) \right)$$

Demand elasticity with respect to returns is finite: market segmentation

With  $\hat{r}_{t+1}^L$ , we have  $(w_t^{10}, w_t^{30})$ , leading to  $(\hat{r}_{t+1}^{10}, \hat{r}_{t+1}^{30})$  (i.e., portfolio re-balancing), thereby  $\hat{r}_{t+1}^S$ , but not one-to-one

Then real effects on  $\hat{c}_t$

A [quantitative macroeconomic framework](#) that incorporates

The general equilibrium term-structure of interest rates

Multiple asset classes (government bonds vs. private bond)

Endogenous portfolio shares among different kinds of assets

all of which address [Blanchard \(2016\)](#)

Market segmentation across different maturities (how?: [methodological contribution](#))

that makes LSAPs work in theory (a demand curve for each maturity bond slopes down)  $\Rightarrow$  addressing [Bernanke \(2014\)](#)

Government and central bank's explicit balance sheets

A micro-founded welfare criterion

which are necessary for quantitative policy experiments (ex. [conventional](#) vs. [unconventional](#) monetary policies)

1. **Provide** an efficient way to generate the **market segmentation** across bonds of different maturities based on **Eaton and Kortum (2002)**

Each atomic investor subject to some expectation shock **Fréchet**: these shocks have a structural meaning (e.g., liquidity premium)

9 Downward-sloping demand curve for each bond of different maturities

Estimate the demand elasticity for the Treasury bonds based on macro data

2. **Compare conventional** monetary policy where

Central bank adjusts its **balance sheet holding of the shortest-term bond** to control the shortest-term yield

The shortest-term yield follows the Taylor rule (targeting business cycle)

with the **unconventional** monetary policy where

Central bank adjusts its **entire bond portfolio** along the yield curve to control yields (yields of which maturities to be controlled: chosen by central bank)

Controlled yields follow the Taylor rule (targeting business cycle)

Similar to a complete **yield-curve-control (YCC)** policy

## Big Findings (Conventional vs. Unconventional)

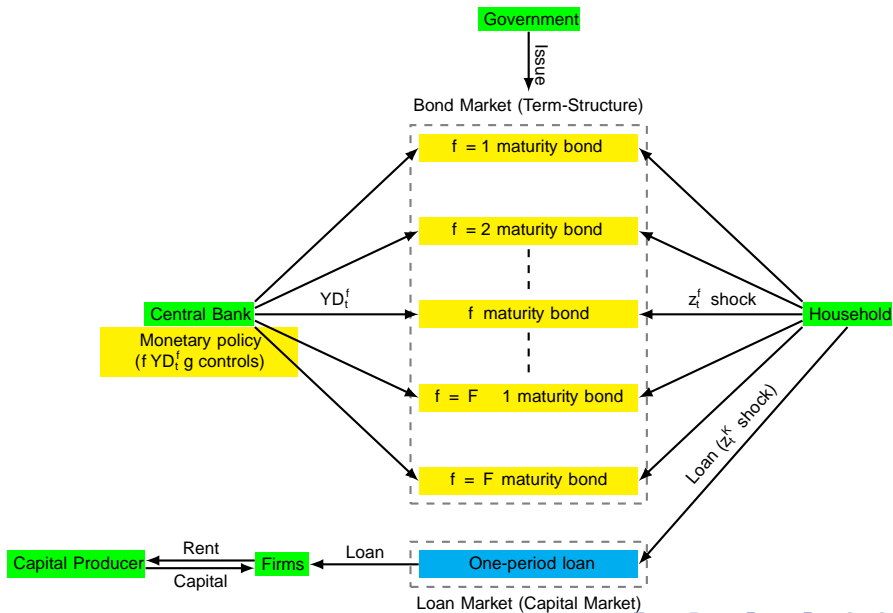
- 1 Quantity matters! (confirm results in **Krishnamurthy and Vissing-Jorgensen (2012)** and **Greenwood and Vayanos (2014)** in theory)
- 2 Unconventional monetary policy is very powerful in terms of stabilization in both normal and ZLB periods
- 3 As a drawback, the economy gets addicted to its power under ZLB regimes

**Why?:** long term yields  $\# \Rightarrow$  downward pressure on short term yields  $\# \Rightarrow$  ZLB duration "  $\Rightarrow$  more reliance on LSAPs  
: from the household's **endogenous portfolio** choices

**'ZLB+LSAPs addicted economy'**

▶ Literature

# The Model





The representative household's problem (given  $B_0$ ):

$$\max_{\{C_{t+j}; N_{t+j}\}} E_t \sum_{j=0}^{\infty} \beta^j \log(C_{t+j})$$

subject to

$$C_t + \frac{L_t}{P_t} + \frac{P_{f=1}^F B_t^{H;f}}{P_t} = \frac{P_{f=0}^{F-1} R_t^f B_t^{H;f+1}}{P_t} + \frac{R_t^K L_{t-1}}{P_t} + \frac{Z_{t-1} W_t(\cdot) N_t(\cdot)}{P_t} d + \frac{t}{P_t}$$

Annotations:

- $L_t$ : Loans
- $B_t^{H;f}$ : Nominal bond purchase (f-maturity)
- $R_t^f$ : f-maturity rate
- $R_t^K$ : Loan rate

where

$\beta$ : intermediate term index such that:

$$\beta = \frac{Z_{t-1} W_t(\cdot) N_t(\cdot)}{P_t} d$$

$Q_t^f$  is the nominal price of f-maturity bond with:

$$(\text{Return}) R_t^f = \frac{Q_t^f}{Q_{t+1}^f}; \quad (\text{Yield}) YD_t^f = \frac{1}{Q_t^f}$$

$$\text{Total savings: } S_t = B_t^H + L_t = \sum_{f=1}^X B_t^{H:f} + L_t$$

Usual bond allocation problem (Ricardian) :

$$\max_{f=1}^X E_t \sum_{t+1}^h Q_{t;t+1} R_{t+1}^f B_t^{H:f} \quad \text{s.t.} \quad \sum_{f=1}^X B_t^{H:f} = B_t^H; \quad B_t^{H:f} \geq 0$$

which gives (in equilibrium):

$$E_t \sum_{t+1}^h Q_{t;t+1} R_{t+1}^f = E_t \sum_{t+1}^h Q_{t;t+1} R_{t+1}^0; \quad \forall f \Rightarrow$$

$$E_t [R_{t+1}^f] = R_{t+1}^0$$

'Expectation hypothesis'  
 $\Rightarrow$  quantity does not matter!

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Usual bond allocation problem (Ricardian) :

$$\max_{f=1}^X E_t \sum_{h=1}^i Q_{t;t+1} R_{t+1}^f B_t^{H:f} \quad \text{s.t.} \quad \sum_{f=1}^X B_t^{H:f} = B_t^H; \quad B_t^{H:f} \geq 0$$

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$$E_t [R_{t+1}^f] = R_{t+1}^0$$

Our approach (Non-Ricardian) :

'Expectation hypothesis'  
 $\Rightarrow$  quantity does not matter!

Split the household into a family  $m \in [0, 1]$ , each of which decides whether to invest in **bonds** or **loan**, subject to expectation shock  $\epsilon_{t+1}$

A **bond** family  $m$  is split into members  $n \in [0, 1]$ , each of whom decides maturity  $f$  to invest in, subject to expectation shock  $\epsilon_{t+1}$

Bond family  $m$ : a member  $n$  has the following expectation shock:

$$E_{m;n;t} Q_{t;t+1} R_{t+1}^f = z_{n;t}^f E_t Q_{t;t+1} R_{t+1}^f ; \quad \sum_{n=1}^F z_{n;t}^f = 1 ; \dots ; F$$

with  $z_{n;t}^f$  follows a **Fechet** distribution with location parameter  $0$ , scale parameter  $z_t^f$ , and shape parameter  $B$

Note:  $z_t^f = 1, \quad B \neq 1$ , then  $E_{m;n;t} = E_t$  (i.e., rational expectations)

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with  $z_{n;t}^f$  follows a **Fechet** distribution with location parameter  $0$ , scale parameter  $z_t^f$ , and shape parameter  $B$

Note:  $z_t^f = 1, B \rightarrow 1$ , then  $E_{m;n;t} \rightarrow E_t$  (i.e., rational expectations)

Aggregation (**Eaton and Kortum (2002)**)

$$P E_{m;n;t} Q_{t;t+1} R_{t+1}^f = \max_j E_{m;n;t} Q_{t;t+1} R_{t+1}^j$$

$$= \frac{z_t^f E_t Q_{t;t+1} R_{t+1}^f}{B}$$

$f$ -maturity share

Deviate from expectation hypothesis  $\Rightarrow$  **downward-sloping demand curve**  
after log-linearization with finite demand elasticity

Shape parameter  $B$ : (inverse of) a degree of bonds market segmentation

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Effective bond market rates

$$R_{t+1}^{HB} = \sum_{f=0}^F \frac{HB;f+1}{t} R_{t+1}^f$$

Loan vs. bond decision: a family  $m$  solves the following problem

$$\max E_t^h Q_{t;t+1} R_{t+1}^{HB} B_{m;t}^H + z_{m;t}^K E_t^h Q_{t;t+1} R_{t+1}^K L_{m;t} \quad \text{s.t.}$$

$$B_{m;t}^H + L_{m;t} = S_t; \quad B_{m;t}^H \geq 0; \quad \text{and} \quad L_{m;t} \geq 0$$

with  $z_{m;t}^K$  follows a Fréchet distribution with location parameter  $0$ , scale parameter  $z_t^K$ , and shape parameter  $s$

Loan vs. bond decision: a family  $i$  solves the following problem

$$\max E_t Q_{t;t+1} R_{t+1}^{HB} B_{m;t}^H + z_{m;t}^K E_t Q_{t;t+1} R_{t+1}^K L_{m;t} \quad \text{s.t.}$$

$$B_{m;t}^H + L_{m;t} = S_t; \quad B_{m;t}^H \geq 0; \quad \text{and } L_{m;t} \geq 0$$

with  $z_{m;t}^K$  follows a Fréchet distribution with location parameter 0, scale parameter  $z_t^K$ , and shape parameter  $s$

Aggregation (Eaton and Kortum (2002))

$$z_t^K = \frac{E_t Q_{t;t+1} R_{t+1}^K}{s}$$

Loan share

9 downward-sloping demand curve after log-linearization (for loan and bonds)  
 Shape parameter  $s$ : (inverse of) a degree of market segmentation between government bonds vs loan



Loan vs. bond decision: a family  $i$  solves the following problem

$$\max E_t^h Q_{t;t+1} R_{t+1}^{HB} B_{m;t}^H + z_{m;t}^K E_t^h Q_{t;t+1} R_{t+1}^K L_{m;t} \quad \text{s.t.}$$

$$B_{m;t}^H + L_{m;t} = S_t; \quad B_{m;t}^H \geq 0; \quad \text{and } L_{m;t} \geq 0$$

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Aggregation (Eaton and Kortum (2002))

Loan share  $\overset{K}{z}_t = \frac{z_t^K E_t^h Q_{t;t+1} R_{t+1}^K}{S_t} \quad ! \quad s$

downward-sloping demand curve after log-linearization (for loan and bonds)

Shape parameter  $s$ : (inverse of) a degree of market segmentation between government bonds vs loan

Effective savings rate : governs intertemporal substitution

$$R_t^S = 1 + \frac{K}{t-1} R_t^{HB} + \frac{K}{t-1} R_t^K$$

$$= 1 + \frac{K}{t-1} \sum_{f=0}^{\infty} \frac{HB;f+1}{t-1} R_t^f + \frac{K}{t-1} R_t^K$$

Bond market equilibrium :

$$B_t^{H;f} + B_t^{G;f} + B_t^{CB;f} = 0; \quad \forall f = 1, \dots, F$$

↑  
Depends on  
monetary policy

↑  
Monetary  
policy

Central bank: balance sheet adjustment ↕ monetary policy

Market clearing :

$$C_t = (1 - \beta^G) Y_t + (1 - \delta) K_t - K_{t+1}$$

Under the **conventional** monetary policy, central banks set Taylor rules  $\Delta YD_t^1$  (i.e., the shortest yield) while not manipulating longer term bonds holdings

Long-term yields uctuate endogenously (in response to shocks + changes in short-term rate)

$$R_{t+1}^0 \quad YD_t^1 = \max^n YD_t^1 ; \quad 1 \quad 0$$

↑  
ZLB

$$YD_t^1 = \overline{YD}^1 \frac{YD_{t-1}^1}{\overline{YD}^1} \frac{YD_{t-2}^1}{\overline{YD}^1} \dots \exp \left\{ \underbrace{\sum_{s=0}^{t-1} \left( \frac{Y_t}{Y} - 1 \right)}_{\text{Targeting}} \right\} \exp \left\{ \sum_{s=0}^{t-1} \epsilon_{t-s}^1 \right\}$$

↑  
MP shock (f = 1)

$$\frac{B_t^{CB;f}}{A_t N_t P_t} = \frac{\overline{B^{CB;f}}}{\overline{ANP}} \quad 8f = 2; \dots; F$$

Normalized holding of f > 1 xed

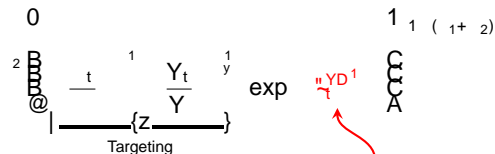
In the **unconventional** monetary policy case, central bank targets all yields along the yield curve, assuming the Taylor-type rule for each maturity yield

Back out the needed purchases of each maturity  $f$ , which are endogenous

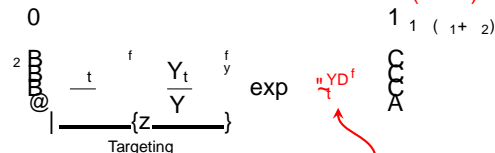
$$R_{t+1}^0 \quad YD_t^1 = \max^n YD_t^1 ; 1$$

↑  
ZLB

$$YD_t^1 = \bar{YD}^1 \frac{YD_{t-1}^1}{\bar{YD}^1} \frac{YD_{t-2}^1}{\bar{YD}^1}$$



$$YD_t^f = \bar{YD}^f \frac{YD_{t-1}^f}{\bar{YD}^f} \frac{YD_{t-2}^f}{\bar{YD}^f}$$



↑ MP shock (f = 1)  
↑ MP shock (8f = 2)

# Steady-state (long-run) analysis

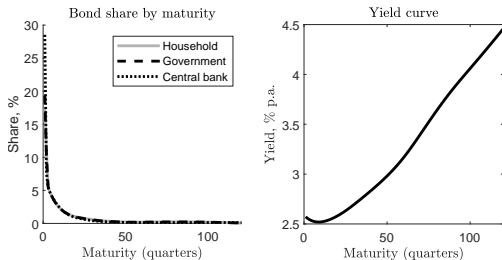


Figure: Steady-state bond portfolios of household, government, and central bank and the resultant yield curve (December 2002 - June 2007)

Estimation:  $B = 10$  from the aggregate bond portfolio data ▶ Estimation

Calibration: given  $B = 10$  and  $s = 6$  (from **Kekre and Lenel (2023)**)

$fz^f g_{f=1}^F$  (i.e., maturity preference for a maturity- $f$ ): matches the yield curve slope;  $z^K$  (i.e., preference for private loan): matches its level

Our private loan rate  $R^K = 8.12\%$  annually ' Moody's seasoned Baa corporate bond average yields

Result:  $z^1 = 1 \gg z^f$  for  $f \geq 2$  (e.g., **safety - liquidity** premium)

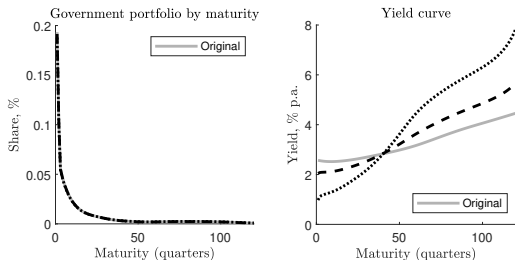


Figure: Government's bond issuance portfolio and yield curve

Government's supply of  $f$ -maturity bond "  $\Rightarrow$  " its yield " (i.e., price effect)

Similar to Krishnamurthy and Vissing-Jorgensen (2012) and Greenwood and Vayanos (2014) in the long run

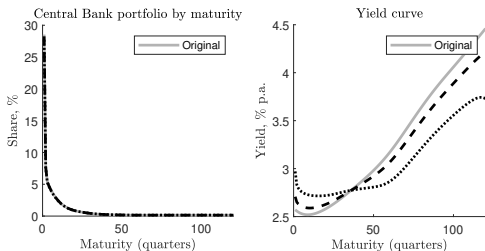


Figure: Central bank's bond demand portfolio and yield curve

Segmented markets  $\Rightarrow$  QE matters in the long run ▶ Deficit ratio



# Short-run analysis (Impulse-responses)

Again...

- 1 Unconventional monetary policy is very powerful in terms of stabilization in both normal and ZLB periods
- 2 As a drawback, the economy gets addicted to its power under ZLB regimes

Why?: long term yields  $\downarrow$   $\Rightarrow$  downward pressure on short term yields  $\Rightarrow$  ZLB  
duration"  $\Rightarrow$  more reliance on LSAPs

Welfare (similar to [Coibion et al. \(2012\)](#)) Trend in inflation term

$$EU_t U^F = \bar{c} + \frac{1}{2} \sigma^2 \text{Var}(\hat{\pi}_t) + \frac{1}{2} \sigma^2 \text{Var}(\hat{r}_t) + \text{t.i.p} + \text{h.o.t}$$

Figure: Impulse response to  $z_t^1$  shock

With **conventional** policy

Short yields  $\downarrow$  other yields, capital return, and wage  $\downarrow$  output  $\downarrow$  (labor supply  $\downarrow$ ) and inflation  $\downarrow$

With **yield-curve-control** (YCC): stabilizing (flattening) gaps in bond demand

Figure: ZLB impulse response to  $z_t^1$  shock

With **yield-curve-control** (YCC): stabilizing ( lling gaps in bond demand)

But duration of ZLB episodes<sup>#</sup>

ZLB  $\Rightarrow$  long-term rates<sup>#</sup>  $\Rightarrow$  ZLB possibility<sup>#</sup>

▶▶ ZLB IRF ( $z_t^K$ )

Figure: ZLB impulse response to  $T_t$  shock

With **conventional** policy: non-Ricardian

Tax"  $\Rightarrow$  bond supply  $\neq$  ZLB ) recessions (Caballero and Farhi (2017))

With **yield-curve-control** (YCC): stabilizing

But **duration** of ZLB episodes<sup>#</sup>

We also consider:

**Mixed policy:** central bank starts controlling long-term rates only when FFR hits ZLB, thus **YCC** only at the ZLB

	Conventional	Yield-Curve-Control	Mixed Policy
Mean ZLB duration	4:5533 quarters	6:2103 quarters	5:5974 quarters
Median ZLB duration	3 quarters	3 quarters	2 quarters
ZLB frequency	15:9596%	13:4242%	17:4141%
Welfare	1:393%	1:2424%	1:3662%

Table: Policy comparisons (ex-ante)

ZLB duration : Conventional < Mixed < YCC

ZLB frequency : YCC < Conventional < Mixed

Welfare : Conventional < Mixed < YCC

Thank you very much!  
(Appendix)

The term-structure and macroeconomy: [Ang and Piazzesi \(2003\)](#), [Rudebusch and Wu \(2008\)](#), [Bekaert et al. \(2010\)](#)

Central bank's endogenous balance sheet size as an another form of monetary policy: [Gertler and Karadi \(2011\)](#), [Curdia and Woodford \(2011\)](#), [Christensen and Krogstrup \(2018, 2019\)](#), [Karadi and Nakov \(2021\)](#), [Sims and Wu \(2021\)](#)

Zero lower bound (ZLB) and issuance of safe bonds: [Swanson and Williams \(2014\)](#), [Caballero and Farhi \(2017\)](#), [Caballero et al. \(2021\)](#)

Welfare criterion with a trend in ation: [Coibion et al. \(2012\)](#)

Preferred-habitat term-structure (and limited risk-bearing): [Greenwood et al. \(2020\)](#), [Vayanos and Vila \(2021\)](#), [Gourinchas et al. \(2021\)](#), [Kekre et al. \(2023\)](#)

Preferred-habitat term-structure and the real economy in New-Keynesian macroeconomics: [Ray \(2019\)](#), [Droste, Gorodnichenko, and Ray \(2021\)](#)

Our paper: general equilibrium term-structure (without relying on factor models)  
+ balance sheet quantities of government and central bank + yield-curve-control  
+ novel way to generate and estimate market segmentation



Capital producer: competitive producer of capital (lend capital to intermediate firms at price  $P_t^K$ )

Firms: standard with Cobb-Douglas production (pricing a la Calvo (1983))

One financial friction: firms need secure loans from the household to operate: for simplicity, borrow a portion of the revenue it generates

$$P_t(z) Y_t(z) = (1 + r^F) P_t(z) Y_t(z); \quad \text{Loan of firm}$$

Government: with the following budget constraint

$$\frac{B_t^G}{P_t} = \frac{R_t^G B_{t-1}^G}{P_t} + \frac{G_t}{Y_t} + \frac{T_t}{Y_t} Y_t; \quad R_t^G = \frac{R_{t-1}^{G,f+1}}{R_t^f}$$

↑  $\frac{G_t}{Y_t}$  (Exogenous)     
 ↑  $\frac{T_t}{Y_t}$  (Exogenous)     
 ↑  $\frac{R_{t-1}^{G,f+1}}{R_t^f}$  (Exogenous)

Government: a natural issuer of the entire bond market

From portfolio equations:

$$P E_{m;n;t}^h Q_{t;t+1} R_{t+1}^{f,1} = \max_j E_{m;n;t}^h Q_{t;t+1} R_{t+1}^{j,1}$$

$$= \frac{z_t^f E_t Q_{t;t+1} R_{t+1}^{f,1}}{B_t}$$

$\nearrow$  HB;<sub>t</sub><sup>f</sup>  
 f-maturity share leading to:

$$\log \frac{H_{t+1}^f}{H_t^f} - \log \frac{H_{t+1}^l}{H_t^l} = \beta E_t^h \left[ r_{t+1}^{f,1} - r_{t+1}^{l,1} \right] + \epsilon_t \quad (1)$$

Jorda local projection :

$$\log \frac{H_{t+h}^f}{H_t^f} - \log \frac{H_{t+h}^l}{H_t^l} = \alpha_h + \beta_{h,h} yd_t^f - yd_t^l + x_t^0 \alpha_h + \epsilon_{t+h}; \quad \alpha_h = 0; \quad (2)$$

Long maturity:  $f = 5$  10 years and short:  $l = 15$  90 days (bunching) for portfolio shares and use  $\epsilon = 7$  years and  $l = 1$  month for yields

Instrument  $yd_t^f - yd_t^l$  with  $yd_{t-1}^f - yd_{t-1}^l$  (? with portfolio demand shocks, i.e.,  $z_t^f, z_t^l$ )

Control other variables (e.g., lagged  $\log \frac{H_{t-1}^f}{H_t^f} - \log \frac{H_{t-1}^l}{H_t^l}$  for serial correlation)



Go back

Figure: Variations in a de cit ratio  $\frac{G}{Y} + \frac{F}{Y} - \frac{T}{Y}$

A higher de cit ratio ) depressed economy (for  $R^G \#$ )

Figure: Variations in a de cit ratio  $\frac{G}{t} + \frac{F}{t} - \frac{T}{t}$

A higher de cit ratio ) depressed economy (for  $R^G$ )  
An entire yield curve#

Figure: Impulse response to  $T_t$  shock

Tax" ) bond supply# ) yields#, loan rates#, and wages# (i.e., real effects)

The **yield-curve-control** (YCC): stabilizing

