Higher-Order Forward Guidance

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Motivation

> “The Federal Reserve, . . . affirmed today its **readiness to serve** as a source of liquidity **to support the economic and financial system**”
>
> - Alan Greenspan, 1987 (Black Monday)

> “Within our mandate, the ECB is **ready to do whatever it takes** to preserve the euro. And believe me, **it will be enough**.”
>
> - Mario Draghi, 2012 (European Crisis)

Observation

Modern central banks: expanding their roles beyond the conventional policy rate setting

Unconventional policy interventions (e.g. forward guidance): more prevalent

Motivation

Forward guidance:

- **Odyssean guidance:** Communication about future policy commitment

  "The Central Bank **commits to maintaining** *the current interest rate at its present low level* **until the unemployment rate falls** *below 5 percent, irrespective of fluctuations in inflation."*

- **Delphic guidance:** Communication about forecasts of future macroeconomic performance

  "The Central Bank **anticipates that the prevailing economic conditions** *will necessitate an upward adjustment in interest rates in the foreseeable future."*

How does it work, exactly?

- First-order effects (levels): "Interest rates will stay low" ⟶ intertemporal substitution channel: many works in the literature (Eggertsson et al. (2003), Campbell et al. (2012, 2019), Del Negro et al. (2013), McKay et al. (2016), Caballero and Farhi (2017))
- **Second-order effects (volatility):** Reduce uncertainty, avoid worst-case scenarios, "whatever it takes" ⟶ precautionary savings channel

Motivation

Big Question (Uncertainty Management)

How does the central bank manage economic uncertainty at the zero lower bound (ZLB)? Is it possible? Desirable?

Uncertainty (i.e., volatility): an important source of business cycle fluctuations

- The literature: Bloom (2009), Ludvigson et al. (2015)
- Finance: risk-premium $\propto$ volatility^2 (e.g., Merton (1971))
- **VAR analysis**: financial and real volatility ⏩ VAR analysis

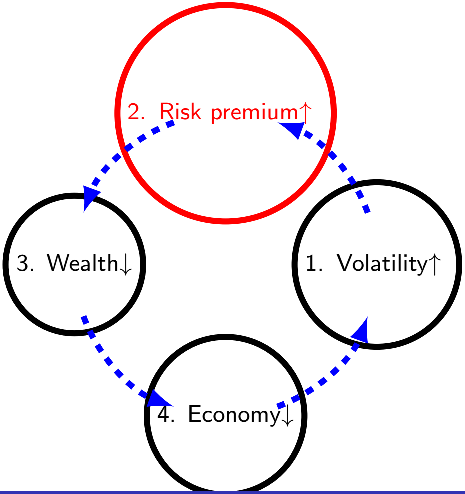
This paper: forward guidance with a focus on strategic uncertainty management and coordination. We show it is possible for central banks to pick an equilibrium where:

- During the ZLB (now): reduce aggregate volatility (and risk premium) for aggregate demand↑
- But after the ZLB (future): less stabilization
- Welfare-enhancing overall

Theoretical framework

Non-linear Two-Agent New-Keynesian (TANK) model with a stock market + portfolio choice

- **Build** a parsimonious New-Keynesian framework where: ⏩ Explain



Model

Model structure

Identical **capitalists** and **hand-to-mouth workers** (Two types of agents)

- **Capitalists**: consumption - portfolio decision (between stock and bond)
- **Workers**: supply labor to firms (hand-to-mouth)

1. **Technology**

Fundamental risk (Exogenous)

$$\frac{dA_t}{A_t} = \underbrace{g}_{\text{Growth}} \cdot dt + \sigma \cdot \underbrace{dZ_t}_{\text{Aggregate shock}}$$

2. **Hand-to-mouth workers**: supply labor + solves the following problem:

$$\max_{C_t^w, N_t^w} \frac{\left(\frac{C_t^w}{A_t}\right)^{1-\varphi}}{1-\varphi} - \frac{(N_t^w)^{1+\chi_0}}{1+\chi_0} \quad \text{s.t.} \quad \bar{p} C_t^w = w_t N_t^w$$

- Hand-to-mouth assumption can be relaxed, without changing implications

3. **Firms**: Dixit-Stiglitz production using labor + perfectly rigid prices ($\pi_t = 0$)

4. **Financial market**: zero net-supplied risk-free bond + stock (index) market

Capitalists

Capitalists: standard portfolio and consumption decisions (very simple)

1. Total financial wealth $a_t = \bar{p} A_t Q_t$, where (real) stock price Q_t follows:

$$\frac{dQ_t}{Q_t} = \mu_t^q \cdot dt + \sigma_t^q \cdot dZ_t$$

Financial risk (Endogenous)

- μ_t^q and σ_t^q are both endogenous (to be determined)

2. Each solves the following optimization (standard)

$$\max_{C_t, \theta_t} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \log C_t dt \quad \text{s.t.}$$

$$da_t = (a_t(i_t + \theta_t(i_t^m - i_t)) - \bar{p} C_t) dt + \theta_t a_t (\sigma + \sigma_t^q) dZ_t$$

- Aggregate consumption of capitalists ∝ aggregate financial wealth

$$C_t = \rho A_t Q_t$$

- Equilibrium **risk-premium** is determined by the total risk

$$i_t^m - i_t \equiv \text{rp}_t = \boxed{(\sigma + \sigma_t^q)^2}$$

Other equilibrium conditions

Dividend yield: dividend yield= ρ, as in Caballero and Simsek (2020)

- A positive feedback loop between asset price $\Longleftrightarrow$ dividend (output)

Determination of nominal stock return $d\mathbf{I}_t^m$

$$d\mathbf{I}_t^m = \underbrace{[\; \underbrace{\rho}_{\text{Dividend yield}} + \underbrace{g + \mu_t^q + \overbrace{\sigma\sigma_t^q}^{\text{Covariance}}}_{\text{Capital gain}} \;]}_{= \underbrace{i_t^m}_{\text{Drift}} = \underbrace{i_t}_{\text{Monetary policy}} + \underbrace{(\sigma+\sigma_t^q)^2}_{\text{Risk-premium}}} dt + \underbrace{(\sigma + \sigma_t^q)}_{\text{Risk term}} dZ_t$$

Equilibrium with rigid prices ($\pi_t = 0, \forall t$)

Flexible price economy as benchmark: the 'natural' consumption of capitalists $C_t^n = \rho A_t Q_t^n$ follows

$$\frac{dC_t^n}{C_t^n} \equiv \frac{d\left(A_t Q_t^n\right)}{A_t Q_t^n} = \left(r^n - \rho + \sigma^2\right) dt + \sigma dZ_t$$
$$= g dt + \sigma dZ_t = \frac{dA_t}{A_t}$$

where $r^n = \rho + g - \sigma^2$ is the 'natural' rate of interest

Define **asset price gap**

$$\hat{Q}_t = \ln \frac{Q_t}{Q_t^n}, \quad \underbrace{0 = \mathrm{Var}_t\left(\frac{dQ_t^n}{Q_t^n}\right)}_{\text{Benchmark volatility}}, \quad \underbrace{\left(\sigma_t^q\right)^2 dt = \mathrm{Var}_t\left(\frac{dQ_t}{Q_t}\right)}_{\text{Actual volatility}}$$

Endogenous

which is proportional to **output gap**

$$\hat{Y}_t = \ln\left(\frac{Y_t}{Y_t^n}\right) \longrightarrow \hat{Y}_t = \underbrace{\zeta}_{>0} \cdot \hat{Q}_t$$

Output and asset price gaps

A non-linear IS equation (in contrast to textbook linearized one)

$$d\hat{Q}_t = \left(i_t - \underbrace{\left(r^n \overbrace{-\frac{1}{2}(\sigma + \sigma_t^q)^2 + \frac{1}{2}\sigma^2}^{\text{New terms}} \right)}_{\equiv r_t^T} \right) dt + \sigma_t^q \, dZ_t \tag{1}$$

$$= \left(i_t - r_t^T \right) dt + \sigma_t^q dZ_t \tag{2}$$

$$\boxed{\sigma_t^q\uparrow \longrightarrow \mathrm{rp}_t\uparrow \longrightarrow \hat{Q}_t\downarrow \longrightarrow \hat{Y}_t\downarrow}$$

What is r_t^T?: a risk-adjusted natural rate of interest ($\sigma_t^q\uparrow\longrightarrow r_t^T\downarrow$)

$$r_t^T \equiv r^n - \frac{1}{2}(\sigma + \sigma_t^q)^2 + \frac{1}{2}\sigma^2$$

$$= r^n - \frac{1}{2}\hat{rp}_t, \quad \hat{rp}_t = \underbrace{\mathrm{rp}_t - \mathrm{rp}_t^n}_{\text{risk-premium gap}}$$

Monetary Policy outside the ZLB

Outside the ZLB: Can we stabilize this economy? Prevent the volatility feedback loop? **Yes!**: Lee and Dordal i Carreras (2024)

- Under a risk-premium targeting rule: $i_t = r_t^T + \phi_q \hat{Q}_t$. With $\phi_q > 0$ (i.e., Taylor principle) $\longrightarrow$ $\hat{Q}_t = 0$ for $\forall t$ (unique equilibrium)

$$d\hat{Q}_t = \left(i_t - r_t^T\right) dt + \sigma_t^q dZ_t \underbrace{=}_{\substack{\text{Under} \\ \text{risk-premium targeting}}} \phi_q \hat{Q}_t dt + \sigma_t^q dZ_t$$

  Then,

$$\mathbb{E}_t \left(d\hat{Q}_t\right) = \phi_q \hat{Q}_t$$

- If $\hat{Q}_t \neq 0$, then $\mathbb{E}_t \left(\hat{Q}_\infty\right)$ blows up $\longrightarrow$ $\hat{Q}_t = 0$ for $\forall t$ as unique equilibrium (Blanchard and Kahn (1980))$\longrightarrow$ $\sigma_t^q = 0$ for $\forall t$ (i.e., zero excess volatility)

- Outside the ZLB, uncertainty can be eliminated by traditional means

At the ZLB, the precautionary feedback loop reappears:

$$d\hat{Q}_t = -r_t^T dt + \sigma_t^q dZ_t = -\left(r^n - \frac{1}{2}(\sigma + \sigma_t^q)^2 + \frac{1}{2}\sigma^2\right) dt + \sigma_t^q dZ_t$$

ZLB from fundamental volatility shock

Thought experiment: fundamental volatility $\sigma\uparrow$: from $\underline{\sigma}$ to $\bar{\sigma}$ on $[0, T]$ (e.g., Werning (2012)) and comes back to $\underline{\sigma}$ with

- $\bar{r} \equiv r^n(\underline{\sigma}) = \rho + g - \underline{\sigma}^2 > 0$: no ZLB before, $t < 0$, or after, $t > T$
- $\underline{r} \equiv r^n(\bar{\sigma}) = \rho + g - \bar{\sigma}^2 < 0$: ZLB binds for $0 \leq t \leq T$

Assume: perfect stabilization (i.e., $\hat{Q}_t = 0$) is achievable outside ZLB, i.e.,

$$i_t = \bar{r} - \frac{1}{2}\hat{rp}_t + \phi_q \hat{Q}_t, \quad \text{with } \phi_q > 0$$

Result: perfect stabilization of risk-premia gap (i.e., excess uncertainty) **<u>inside</u>** the ZLB as well

- **Recursive argument:** Full stabilization at T implies $\hat{Q}_T = 0 \longrightarrow \sigma^q_{T-\mathrm{d}t} = 0$, and so on (so $\hat{rp}_t = 0$ for $\forall t$)

ZLB path (full stabilization after T)

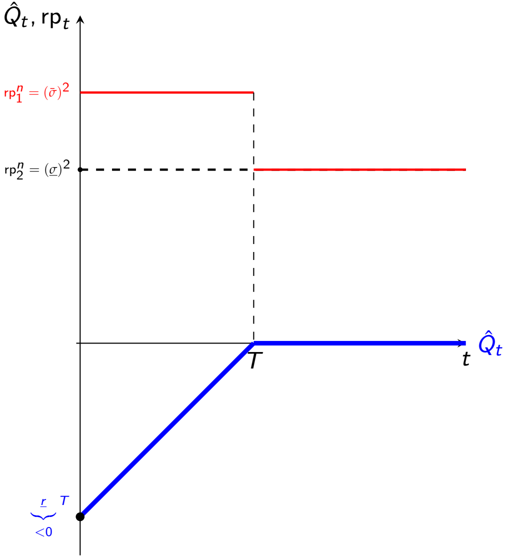


Figure: ZLB dynamics (Benchmark)

Traditional forward guidance

Assume:

- Central bank commits to keep $i_t = 0$ until $\hat{T}^{\text{TFG}} \geq T$ (Odyssean guidance)
- Perfect stabilization (i.e., $\hat{Q}_t = 0$) afterwards, i.e., for $t > \hat{T}^{\text{TFG}}$
- By similar arguments: risk-premium gap stabilization beforehand, $t \leq \hat{T}^{\text{TFG}}$ (no excess volatility while $i_t = 0$)

Problem: Minimize smooth quadratic welfare loss

$$\min_{\hat{T}^{\text{TFG}}} \mathbb{L}^Q\left(\{\hat{Q}\}_{t\geq 0}\right) \equiv \mathbb{E}_0 \int_0^\infty e^{-\rho t}\left(\hat{Q}_t\right)^2 dt$$

$$\text{s.t.}\;\; \hat{Q}_0 = \underbrace{\underline{r}}_{<0} T + \underbrace{\bar{r}}_{>0}\left(\hat{T}^{\text{TFG}} - T\right)$$

- Smoothing the ZLB costs over time (i.e., welfare enhancing)

Traditional forward guidance (keep $\underline{i_t = 0}$ until $\hat{T}^{\text{TFG}} > T$)

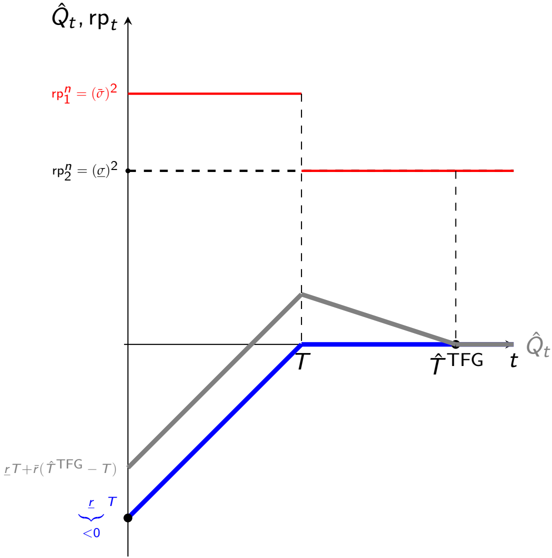


Figure: ZLB dynamics with forward guidance until $\hat{T}^{\text{TFG}} > T$

Alternative forward guidance policies

Big Question

Can we do even **better** than the traditional forward guidance?

Welfare losses are driven by

- High aggregate uncertainty (and risk premium) levels, $(\bar{\sigma} + \sigma_t^q)$, due to high fundamental uncertainty: $\bar{\sigma} > \underline{\sigma}$
- Excess endogenous volatility $\sigma_t^q = 0$ by future stabilization

Can we reduce aggregate uncertainty via $\sigma_t^q < 0$?

- **Yes!** Intuition for $\sigma_t^q < 0$: the real wealth $A_t Q_t$ must respond *less than* proportionally to A_t shocks
- How? Rigid prices $\longrightarrow$ demand-determined production (and hence, wealth)
- Policy challenge: the central bank *must convince* households to behave this way $\longrightarrow$ *higher-order forward guidance*

Higher-order intertemporal stabilization trade-off with commitment

Recall an economic mechanism in the ZLB and forward guidance

$$\boxed{\text{1. Central bank achieves perfect stabilization: } \hat{Q}_t = \hat{rp}_t = 0, \forall t \geq \hat{T}}$$

$$\downarrow$$

$$\boxed{\text{2. } \hat{Q}_{\hat{T}} = 0 \text{ guarantees } \sigma_t^q = \sigma^{q,n} = 0,\ \text{rp}_t = \text{rp}^n \text{ for } t \leq \hat{T}}$$

Still if rp^n is too high, might want to push $\{\sigma_t^q, \text{rp}_t\}$ down for $\hat{Q}_t\uparrow$?

- Thus achieve $\sigma_t^q < \sigma^{q,n} = 0$, $\text{rp}_t < \text{rp}^n \Longrightarrow \hat{Q}_t\uparrow$ at the ZLB

Take contrapositive to the above (necessary condition):

$$\boxed{\neg\text{2. } \sigma_t^q < \sigma^{q,n} = 0,\ \text{rp}_t < \text{rp}^n \text{ for } t \leq \hat{T}}$$

$$\downarrow$$

$$\boxed{\neg\text{1. } \hat{Q}_{\hat{T}} \neq 0. \text{ Central bank commits not to perfectly stabilize the economy after } \hat{T}}$$

- Giving up future financial stability $\Longrightarrow \text{rp}_t\downarrow$ and $\hat{Q}_t\uparrow$ now (at the ZLB)

Higher-order intertemporal stabilization trade-off with commitment

Assume:

- Central bank can commit to keep $i_t = 0$ until $\hat{T}^{HOFG} \geq T$
- No stabilization (i.e., $\hat{Q}_t = \hat{Q}_{\hat{T}^{HOFG}}$) guaranteed afterwards, $t \geq \hat{T}^{HOFG}$
- Pick $\{\sigma_t^q\}$ for $t < \hat{T}^{HOFG}$

Problem: Minimize smooth quadratic welfare loss

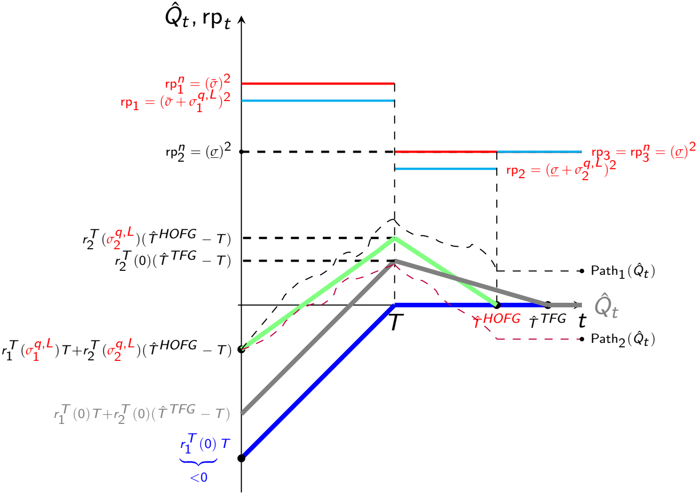
$$\min_{\sigma_1^{q,L},\sigma_2^{q,L},\hat{T}^{HOFG}} \mathbb{L}^Q\left(\{\hat{Q}\}_{t\geq 0}\right) \equiv \mathbb{E}_0 \int_0^\infty e^{-\rho t}\left(\hat{Q}_t\right)^2 dt,$$

$$\text{s.t.} \quad \begin{cases} d\hat{Q}_t = -\underbrace{r_1^T\left(\sigma_1^{q,L}\right)}_{<0} dt + \sigma_1^{q,L} dZ_t, & \text{for } t < T, \\ d\hat{Q}_t = -\underbrace{r_2^T\left(\sigma_2^{q,L}\right)}_{>0} dt + \sigma_2^{q,L} dZ_t, & \text{for } T \leq t < \hat{T}^{HOFG}, \\ d\hat{Q}_t = 0, & \text{for } t \geq \hat{T}^{HOFG}, \end{cases}$$

with

$$\hat{Q}_0 = \underbrace{r_1^T\left(\sigma_1^{q,L}\right)}_{<0} T + \underbrace{r_2^T\left(\sigma_2^{q,L}\right)}_{>0}\left(\hat{T}^{HOFG} - T\right)$$

Central bank picks $\hat{T}^{HOFG}$ and $\{\sigma_t^q\}$



Proposition (Optimal commitment path)

At optimum, $\sigma_1^{q,L} < 0 = \sigma_1^{q,n}$, $\sigma_2^{q,L} < 0 = \sigma_2^{q,n}$, and $\hat{T}^{HOFG} < \hat{T}^{TFG}$

Optimal policy

Proposition (Optimal forward guidance policy)

Optimal higher-order forward guidance always results in an equal or lower expected quadratic loss than the traditional guidance policy

Proof

With $(\sigma_1^{q,L}, \sigma_2^{q,L}, \hat{T}^{\text{HOFG}}) = (0, 0, \hat{T}^{\text{TFG}})$, two solutions coincide

Remarks:

- Alternative higher-order forward guidance policy implementations **are** possible
- This paper shows HOFG dominates TFG in a simple setting

Optimal policy: extension

Extension: higher-order forward guidance policy, but instead of no stabilization, now with stochastic stabilization after $\hat{T}^{\text{HOFG}}$: return to stabilization with νdt probability after $\hat{T}^{\text{HOFG}}$

- Central bank commits to stabilizing the economy with some probability. Expected stabilization after $1/\nu$ quarters
- $\nu = 0$: the above higher-order forward guidance
- $\nu = \infty$: the traditional forward guidance policy

Big discontinuity:

$$\lim_{\nu \to +\infty^-} \mathbb{L}^{Q,*}\left(\{\hat{Q}_t\}_{t\geq 0}, \nu\right) < \underbrace{\mathbb{L}^{Q,*}\left(\{\hat{Q}_t\}_{t\geq 0}, \nu = \infty\right)}_{\text{Traditional forward guidance}}$$

- Slight probability that stabilization might not happen ⟶ HOFG possible

Welfare comparisons

$T = 20$ quarters ZLB spell

Loss function $\mathbb{L}$ as the (conditional) quadratic output loss per quarter:

$$\mathbb{L}^{Y}_{\text{Per-period}} \equiv \rho \int_0^\infty e^{-\rho t} \mathbb{E}_0 \left(\hat{Y}_t^2 \right) \approx \zeta^2 \cdot \rho \int_0^\infty e^{-\rho t} \frac{1}{s} \sum_{i=1}^{s} \left(\hat{Q}_t^{(i)} \right)^2 dt$$

Policy	**No guidance**	**Traditional**	**Higher-Order** (no stochastic stabilization)	**Higher-Order** (*with* stoch. stab., $\nu = 1$)
$\sigma_1^{q,L}$	0	0	-1.27%	-4.13%
$\sigma_2^{q,L}$	0	0	-0.24%	-3.79%
$\hat{T}$	20	25.27	25.09	24.68
$\mathbb{L}^{Y}_{\text{Per-period}}$	7%	1.93%	1.81%	1.69%

- Still, traditional forward guidance too strong: e.g., McKay et al. (2016)
- HOFG with $\nu \to \infty$ but $\nu \neq \infty$ most effective

Bonus: Macroprudential policies

Other policy interventions at the ZLB:

- **Remember**: ZLB as the result of capitalist's unwillingness to bear additional risk when the fundamental volatility is high: $\bar{\sigma} > \underline{\sigma}$

- Two possible fiscal interventions:
    - Subsidy on stock returns (equivalently, tax break on capital income)
    - Fiscal redistribution across agent types with different MPCs

- Effectiveness of policies depends on tax burden distribution

Bonus: Subsidy on stock returns

Subsidy $\tau \geq 0$ on (expected) stock returns, i_t^m

- **Implementation 1:** financed via lump-sum taxation L_t on capitalists

  $$da_t = \left(a_t\left(i_t + \theta_t((1+\tau)i_t^m - i_t)\right) - \bar{p}C_t - L_t\right)dt + \theta_t a_t\left(\bar{\sigma} + \sigma_t^q\right)dZ_t$$

  Solution:

  $$i_t^m \downarrow = \frac{i_t + \left(\bar{\sigma} + \sigma_t^q\right)^2}{1 + \tau \uparrow}$$

  leading to a higher asset price during the ZLB

- **Implementation 2:** financed via lump-sum taxation L_t on hand-to-mouth workers: nullifying the above result

Takeaways

1. Higher-order forward guidance via intertemporal uncertainty management

- Through central bank's equilibrium selection

2. Traditional forward guidance is welfare-enhancing, but can do better

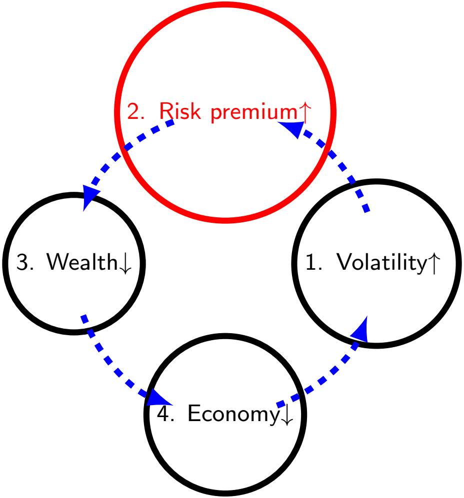
3. Trade-off between current and future financial stability

- Credibly "irresponsible" behaviour in the future

4. Central bank credibility still a necessary condition

Thank you very much!

(Appendix)



2. Risk premium↑

3. Wealth↓

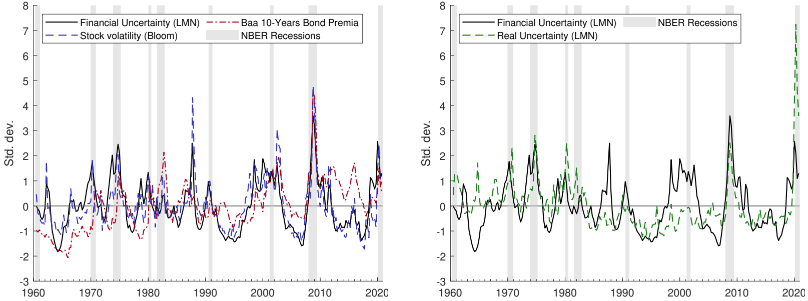
1. Volatility↑

4. Economy↓

- **1** → **2** comes from “non-linearity (not linearizing)”
- **2** → **3** comes from “portfolio decision” of each investor and externality
- **3** → **4** comes from the fact wealth drives aggregate demand
- **4** → **1** where business cycle has its own volatility (self-sustaining)

Go back

Financial volatility measures



(a) Financial Uncertainty series (b) Financial vs. Real Uncertainty

Figure: Common measures of the financial volatility (left) and real vs. financial uncertainty decomposed by Ludvigson et al. (2015) (right)

The correlation between series is remarkably high and they all display positive spikes at the beginning and/or initial months following NBER-dated recessions

- Many of past recessions are, in nature, financial

⏩ Go back

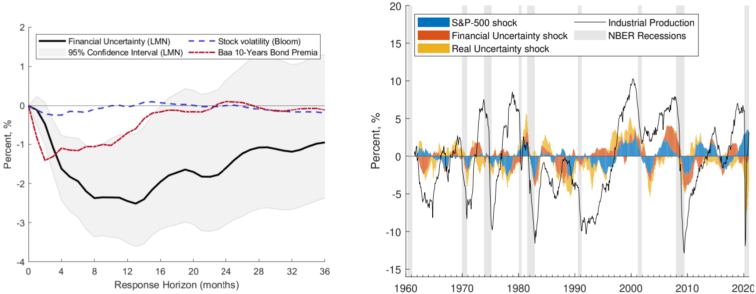
Vector Autoregression (VAR) analysis

In a similar manner to Bloom (2009), Ludvigson et al. (2015):

$$\text{VAR-11 order:} \quad \begin{bmatrix} \log\left(\text{Industrial Production}\right) \\ \log\left(\text{Employment}\right) \\ \log\left(\text{Real Consumption}\right) \\ \log\left(\text{CPI}\right) \\ \log\left(\text{Wages}\right) \\ \text{Hours} \\ \text{Real Uncertainty (LMN)} \\ \text{Fed Funds Rate} \\ \log\left(\text{M2}\right) \\ \log\left(\text{S\&P-500 Index}\right) \\ \text{Financial Uncertainty (LMN)} \end{bmatrix}$$

Financial uncertainty (LMN) is also replaced by the stock price volatility (following Bloom (2009)) and Baa 10-years bond premia

Vector Autoregression (VAR) analysis



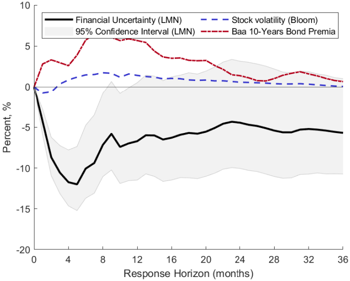
(a) Response: Industrial Production

(b) Industrial Production

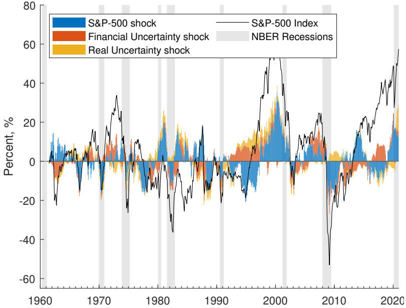
Figure: Impulse-response of IP to one std.dev shock in financial uncertainty measures (left) and the historical decomposition of IP to various attributes (right)

1. IP falls by 2.5% after one standard deviation spike in the Ludvigson et al. (2015)'s financial uncertainty measure
   - Financial uncertainty has been important in driving IP boom-bust patterns
2. Other graphs: IRF and historical decomposition of S&P 500 ⏩ S&P500, and FFR (monetary policy) ⏩ FFR, FEVD ⏩ FEVD

IRF and historical decomposition of S&P500 index



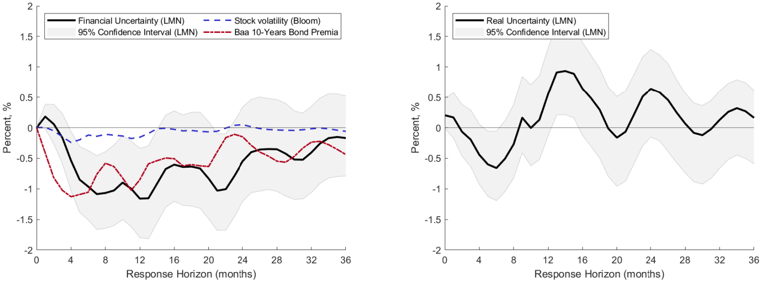
(a) Response: S&P-500 Index



(b) S&P-500 Index

⏩ Go back

IRF of FFR in response to financial and real uncertainty shocks



(a) Shock: Financial Uncertainty

(b) Shock: Real Uncertainty

With 3 different financial uncertainty measures: Ludvigson et al. (2015), Bloom (2009), Baa 10-years bond premia (left)

⏩ Go back

Forecast Error Variance Decomposition (FEVD) of IP, S&P500, FFR

(i) Industrial Production

Horizon	Fin. Uncert. (LMN)	Real Uncert. (LMN)	Stock Vol. (Bloom)	Baa 10-Yr Premia
h=1	0	0.30	0.21	0.12
h=6	1.27	3.37	2.98	1.36
h=12	4.28	4.38	3.16	1.94
h=36	3.24	1.67	1.98	0.64

(ii) S&P-500 Index

Horizon	Fin. Uncert. (LMN)	Real Uncert. (LMN)	Stock Vol. (Bloom)	Baa 10-Yr Premia
h=1	0.11	0.08	0.39	0.06
h=6	3.30	0.25	3.26	0.62
h=12	4.77	0.54	10.03	2.16
h=36	6.50	0.91	12.16	2.40

(iii) Fed Funds Rate

Horizon	Fin. Uncert. (LMN)	Real Uncert. (LMN)	Stock Vol. (Bloom)	Baa 10-Yr Premia
h=1	0.01	0.98	0	0.08
h=6	0.42	0.84	3.11	1.66
h=12	1.47	0.91	4.69	2.30
h=36	2.81	2.05	5.02	3.17

Financial uncertainty shocks explain close to:

- 5% of the fluctuations in both IP and S&P-500 series

Real uncertainty explains:

- Additional 2-4% of movements in industrial activity in the medium run

Go back