Self-fulfilling Volatility, Taylor Rules, and Equilibrium Selection

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What we find

$\exists$A price of risk coming from

$$\underbrace{\text{Aggregate volatility}\uparrow}_{\text{Endogenous}} \Longrightarrow \underbrace{\text{precautionary saving}}_{\text{Endogenous}}\uparrow \Longrightarrow \underbrace{\text{aggregate demand}\downarrow}_{\text{Endogenous}}$$

which occurs in a self-fulfilling way under nominal rigidities (i.e., sticky prices).

1. When households anticipate higher future income volatility, they increase (precautionary) savings while reducing consumption demand.

2. Under sticky prices, it reduces current output (income) as well (recessions). Otherwise, firms will reduce price to restore output.

3. This aggregate volatility is *endogenous*, determined by the aggregate demand dynamics.

Subsequent shocks (positive or negative) lead to stronger income responses. In the subsequent period:

4. A negative shock reinforces precautionary savings, amplifying its contractionary effect

$$\underbrace{\text{Negative realized shock}}_{\text{Exogenous}} \Longrightarrow \underbrace{\text{precautionary saving}}_{\text{Endogenous}}\uparrow \Longrightarrow \underbrace{\text{Contraction}}_{\text{Endogenous}} \uparrow$$

5. A positive shock weakens precautionary savings, amplifying its expansionary effect

$$\underbrace{\text{Positive realized shock}}_{\text{Exogenous}} \Longrightarrow \underbrace{\text{precautionary saving}}_{\text{Endogenous}}\downarrow \Longrightarrow \underbrace{\text{Expansion}}_{\text{Endogenous}} \uparrow$$

Expectations of higher future income (consumption) volatility formed in the initial period become self-fulfilling in equilibrium.

- Again, due to nominal rigidities.
- The channel is missing under log-linearized solutions.

What we find

Takeaway (**Self-fulfilling volatility**)

In macroeconomic models with nominal rigidities, $\exists$global solution where:

- Taylor rules (targeting inflation and output) $\Longrightarrow$ $\exists$self-fulfilling apparition of aggregate volatility.
- Only direct volatility (e.g., risk premium) targeting can restore determinacy, but this rule is *knife-edge*. Thus, the economy is always vulnerable to volatility-driven equilibria.
- Taylor rules might not be as effective in ensuring determinacy as often assumed.

Behavioral frictions and escape clauses: sufficient to eliminate the alternative equilibria

- More robust ways to rule out multiple equilibria than Taylor rules.

A textbook New Keynesian model with rigid price

The representative household's problem (given B_0):

$$\Gamma_t \equiv \max_{\{B_t\}_{t>0},\{C_t,L_t\}_{t\geq 0}} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left[\log C_t - \frac{L_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right] dt$$

subject to

$$\dot{B}_t = i_t B_t - \bar{p} C_t + w_t L_t + D_t$$

where

- B_t denotes holdings of nominal bonds ($B_t = 0$, in equilibrium).
- D_t represents total payouts, including fiscal transfers and profits.
- Rigid price: $p_t = \bar{p}$ for $\forall t$ (i.e., purely demand-determined).

Endogenous volatility

A non-linear Euler equation (in contrast to log-linearized one)

Endogenous drift

$$\mathbb{E}_t\left(\frac{dC_t}{C_t}\right) = (i_t - \rho)dt + \underbrace{\mathrm{Var}_t\left(\frac{dC_t}{C_t}\right)}_{\text{Precautionary premium}}$$

Aggregate volatility↑ $\Longrightarrow$ precautionary saving↑ $\Longrightarrow$ recession (the drift↑)

⋆ The problem is, both **variance** and **drift** are endogenous.

Intra-temporal optimality:

$$\frac{1}{\bar{p}C_t} = \frac{L_t^{\frac{1}{\eta}}}{w_t}$$

Transversality condition:

$$\lim_{t\to\infty} \mathbb{E}_0 \left[e^{-\rho t} \Gamma_t \right] = 0 \tag{1}$$

A textbook New Keynesian model with rigid price

Firm i: face monopolistic competition à la Dixit-Stiglitz with $Y_t^i = A_t L_t^i$ and

$$\frac{dA_t}{A_t} = gdt + \underbrace{\sigma}_{\text{Fundamental risk}} dZ_t$$

- dZ_t: aggregate Brownian motion (i.e., only risk source)
- (g, σ) are exogenous.

Flexible price economy as benchmark: the 'natural' output Y_t^n follows

$$\frac{dY_t^n}{Y_t^n} = \left(r^n - \rho + \sigma^2\right) dt + \sigma dZ_t$$
$$= gdt + \sigma dZ_t = \frac{dA_t}{A_t}$$

where $r^n = \rho + g - \sigma^2$ is the *natural* rate of interest.

Non-linear IS equation

$$\hat{Y}_t = \ln \frac{Y_t}{Y_t^n}, \quad \underbrace{(\underset{\uparrow\ \text{Exogenous}}{\sigma})^2 dt = \mathrm{Var}_t\left(\frac{dY_t^n}{Y_t^n}\right)}_{\text{Benchmark volatility}}, \quad \underbrace{(\sigma + \underset{\uparrow\ \text{Endogenous}}{\sigma_t^s})^2 dt = \mathrm{Var}_t\left(\frac{dY_t}{Y_t}\right)}_{\text{Actual volatility}}$$

Non-linear IS equation

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Exogenous

Endogenous

A non-linear IS equation (in contrast to the textbook linearized one)

$$d\hat{Y}_t = \left(i_t - \underbrace{\left(r^n \overbrace{- \frac{1}{2}(\sigma + \sigma_t^s)^2 + \frac{1}{2}\sigma^2}^{\text{New terms}} \right)}_{\equiv r_t^T} \right) dt + \sigma_t^s \, dZ_t \qquad (2)$$

⋆ Note that r_t^T is the *risk-adjusted* natural rate of interest:

$$r_t^T \equiv r^n - \frac{1}{2} \underbrace{(\sigma + \sigma_t^s)^2}_{\substack{\text{Precautionary} \\ \text{premium}}} + \frac{1}{2}\sigma^2$$

Bounded Equilibria

Equilibrium is defined by the IS equation:

$$d\hat{Y}_t = \left(i_t - \underbrace{\left(r^n \overbrace{- \frac{1}{2}(\sigma + \sigma_t^s)^2 + \frac{1}{2}\sigma^2}^{\text{New terms}} \right)}_{\equiv r_t^T} \right) dt + \sigma_t^s \, dZ_t$$

complemented by a monetary policy rule i_t (to be specified).

⋆ We focus on *bounded* equilibria (Woodford, 2003; Cochrane, 2011), defined by:

$$\lim_{t\to\infty} \mathbb{E}_0 \left| \hat{Y}_t \right| < \infty \tag{3}$$

Big Question

Taylor rule $i_t = r^n + \phi_y \hat{Y}_t$ for $\phi_y > 0 \Longrightarrow$ perfect stabilization?

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Taylor rule $i_t = r^n + \phi_y \hat{Y}_t$ for $\phi_y > 0 \Longrightarrow$ perfect stabilization?

First-order (no volatility feedback): *unique* bounded equilibrium (Cochrane, 2011)

- $\phi_y > 0$ (Taylor principle) $\Longrightarrow$ $\hat{Y}_t = 0$ with $\sigma_t^s = 0$ for $\forall t$

Why? (recap): without the volatility feedback:

$$d\hat{Y}_t = (i_t - r^n)\, dt + \sigma_t^s dZ_t \underbrace{=}_{\substack{\text{Under} \\ \text{Taylor rule}}} \phi_y \hat{Y}_t dt + \sigma_t^s dZ_t$$

Then,

$$\mathbb{E}_t \left(d\hat{Y}_t \right) = \phi_y \hat{Y}_t dt.$$

If $\hat{Y}_t \neq 0$ for some t,

$$\lim_{s\to\infty} \mathbb{E}_t \left(\hat{Y}_s \right) \to \pm\infty$$

- The foundation of modern central banking

Now, with the non-linear effects in (2)

Proposition (Fundamental Indeterminacy: Martingale Equilibrium)

For any $\phi_y > 0$, $\exists$a bounded equilibrium supporting a volatility $\sigma_0^s > 0$ satisfying:

1. $\mathbb{E}_t\left(d\hat{Y}_t\right) = 0$ for $\forall t$ (i.e., local martingale)
2. $\sigma_t^s \xrightarrow{a.s} \sigma_\infty^s = 0$ and $\hat{Y}_t \xrightarrow{a.s} 0$ (i.e., almost sure stabilization)
3. 0^+-possibility divergence or non-uniform integrability given by

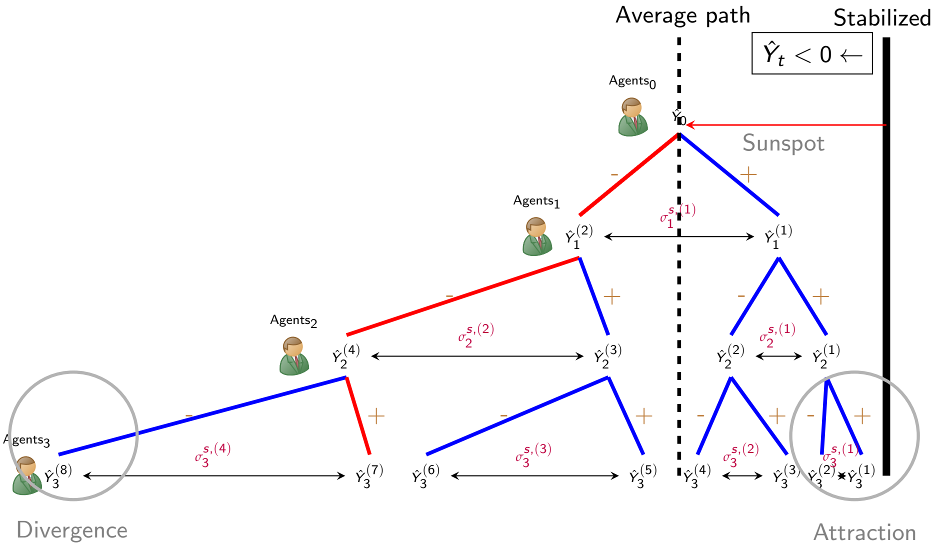
$$\mathbb{E}_0\left(\sup_{t\geq 0}\left(\sigma + \sigma_t^s\right)^2\right) = \infty$$

with

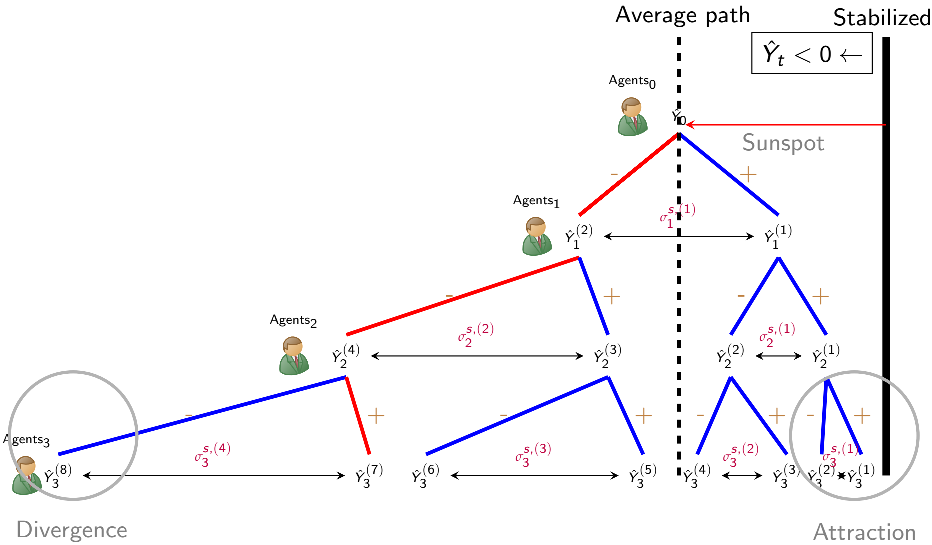
$$\lim_{K\to\infty}\sup_{t\geq 0}\left(\mathbb{E}_0\left(\sigma + \sigma_t^s\right)^2 \mathbb{1}_{\left\{(\sigma+\sigma_t^s)^2\geq K\right\}}\right) > 0$$

Aggregate volatility↑ possible via the intertemporal coordination of agents in their aggregate demand strategy

- Called a "martingale equilibrium" (non-stationary equilibrium)



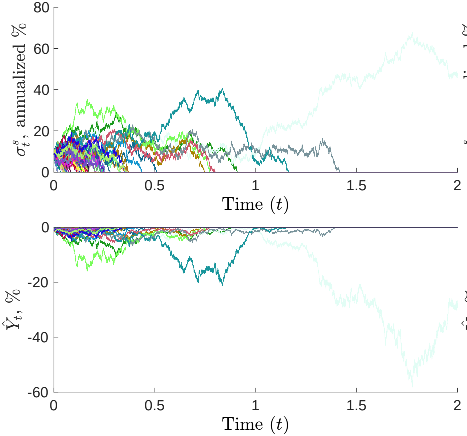
⋆ Stabilized path as an **attractor**: $\sigma_t^s \xrightarrow{a.s} \sigma_\infty^s = 0$ and $\hat{Y}_t \xrightarrow{a.s} 0$



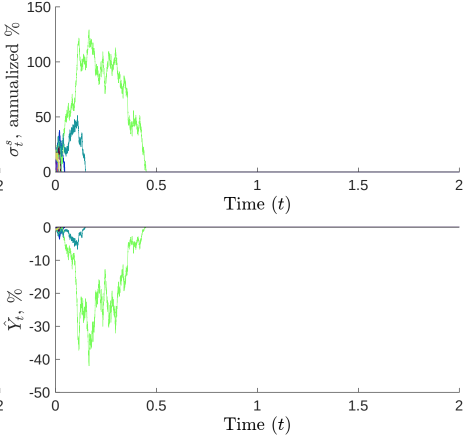
⋆ **But**, diverging with 0^+-probability:

$$\mathbb{E}_0\left(\sup_{t\geq 0}\left(\sigma+\sigma_t^s\right)^2\right)=\infty$$

Simulation results - martingale equilibrium



(a) With Taylor coefficient $\phi_y = 0.11$



(b) With Taylor coefficient $\phi_y = 0.33$

Figure: Simulated paths for the excess volatility $\{\sigma^s_t\}$ and output gap $\{\hat{Y}_t\}$ processes under the martingale equilibrium with $\phi_y = 0.1$ (Panel 1a), and $\phi_y = 0.5$ (Panel 1b). The calibration for the remaining parameters is reported in Online Appendix K.

Stationary equilibria (Ornstein-Uhlenbeck equilibria)

The Ornstein-Uhlenbeck process with endogenous volatility $\{\sigma_t^s\}$

$$d\hat{Y}_t = \left(i_t - \underbrace{\left(r^n \overbrace{- \frac{1}{2}(\sigma + \sigma_t^s)^2 + \frac{1}{2}\sigma^2}^{\text{New terms}} \right)}_{\equiv r_t^T} \right) dt + \sigma_t^s \, dZ_t \tag{4}$$

$$= \underbrace{\theta}_{>0} \cdot \left[\underbrace{\mu}_{\leq 0} - \hat{Y}_t \right] dt + \sigma_t^s dZ_t$$

- μ as an *approximate* average of $\hat{Y}_t$
- θ as a speed of mean reversion
- The Taylor rule $i_t = r^n + \phi_y \hat{Y}_t$ ($\phi_y > 0$) remains unchanged.

Proposition (Fundamental Indeterminacy: Ornstein-Uhlenbeck Equilibria)

For $\theta > 0$, $\mu \leq 0$:

1. The $\{\sigma_t^s\}$ process satisfying (4) is *stable*, and admits a unique *stationary* distribution: with $\sigma \to 0$ and $\mu < 0$, the stationary distribution approaches the generalized gamma distribution, $GGD(a, d, p)$, given by

$$a = \sqrt{\frac{2(\theta + \phi_y)^2}{\theta}}, \quad d = -\frac{2\theta\mu\phi_y}{(\theta + \phi_y)^2}, \quad \text{and} \quad p = 2, \tag{5}$$

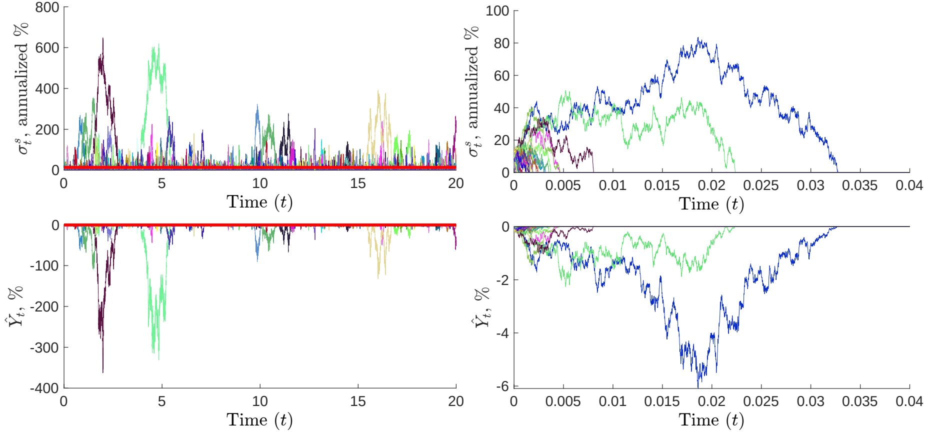
   where a is the scale parameter, and d, p are the shape parameters (power-law and exponential, respectively).

2. For $\theta > 0$ and $\mu = 0$, the $\{\sigma_t^s\}$ process is non-stationary, converging to a degenerate distribution at $\sigma_\infty^s = 0$.

3. The long-run expectations of the output gap $\hat{Y}_t$ and excess variance $(\sigma + \sigma_t^s)^2 - \sigma^2$ are given by

$$\lim_{t\to\infty} \mathbb{E}_0\left[\hat{Y}_t\right] = \mu, \quad \text{and} \quad \lim_{t\to\infty} \mathbb{E}_0\left[(\sigma + \sigma_t^s)^2 - \sigma^2\right] = -2\mu\phi_y.$$

Simulation results - Ornstein-Uhlenbeck equilibrium



(a) $\mu < 0$ (b) $\mu = 0$

Figure: Simulated paths for the excess volatility $\{\sigma_t^s\}$ and output gap $\{\hat{Y}_t\}$ processes under the Ornstein-Uhlenbeck solution with $\theta > 0$ and various μ calibrations. In the left panel, the red line denotes the long-run expected excess volatility, $\sigma^{s,*}$. The calibration for the remaining parameters is reported in Online Appendix K.

- Even with $\sigma_0^s = 0$ (no initial excess volatility) $\Longrightarrow$ stationary $\{\sigma_t^s\}$ (with $\mu < 0$)

A new monetary policy with volatility targeting

New monetary policy:

$$i_t = r^n + \phi_y \hat{Y}_t - \boxed{\frac{1}{2}\left(\underbrace{(\sigma + \sigma_t^s)^2}_{\equiv \mathrm{pp}_t} - \underbrace{\sigma^2}_{\equiv \mathrm{pp}^n}\right)}$$

Aggregate volatility targeting?

- This rule yields $\hat{Y}_t = \sigma_t^s = 0$ as a unique bounded equilibrium, restoring both determinacy and stabilization (off-equilibrium threat).
- The coefficient must be $\frac{1}{2}$ on the excess variance term (knife-edge), which is difficult to satisfy (e.g., multiplicative measurement errors).
- Overall, Taylor rules are not very useful as a device to eliminate multiplicity.

Escape Clause

A new monetary policy with volatility targeting

Equivalent to:

$$\underbrace{\boxed{i_t + \mathrm{pp}_t - \overset{\text{Ito term}}{\frac{1}{2}\mathrm{pp}_t}}}_{= \ \rho + \frac{\mathbb{E}_t(d\log Y_t)}{dt}} = \underbrace{\boxed{r^n + \mathrm{pp}^n - \overset{\text{Ito term}}{\frac{1}{2}\mathrm{pp}^n}}}_{= \ \rho + \frac{\mathbb{E}_t(d\log Y^n_t)}{dt}} + \underbrace{\phi_y \hat{Y}_t}_{\text{Business cycle targeting}}$$

- A % change of (i.e., return on) aggregate output (i.e., demand), not just the policy rate, must follow Taylor rules.

Model with Inflation

Other Takeaways

New Keynesian models are widely used for policy purposes:

- New bounded equilibria with endogenous aggregate volatility processes – implications for policymaking
- Can generate extremely persistent processes for output gap deviations
- How? Strong complementarity in household actions, e.g., paradox of thrift

Welfare costs of the business cycle:

- **First-order costs:** the stationary mean of output gap *can be* below its natural counterpart ($\mu < 0$ in the global solution)
- Monetary policy can generate **first-order gains** by restoring $\mu < 0$ back to zero.

Thank you very much!
(Appendix)

Escape clauses and behavioral frictions

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Our equilibrium construction with self-fulfilling σ_t^s is feasible because households can coordinate their intertemporal consumption choices—potentially over an infinite horizon—based on the history of realized shocks. We show:

- Even a small degree of memory friction renders such consumption coordination infeasible (Angeletos and Lian, 2023)
- Level-k reasoning attenuates households' consideration of the full general equilibrium effects (Farhi and Werning, 2019), rendering the alternative equilibria infeasible.

Fiscal intervention (in the overlapping generations extension of the model) can make the full-stabilization path the unique equilibrium.

Model with Inflation

Model with inflation

Go Back

Nominal rigidities à la Rotemberg (1982)

$$dp_t^i = \pi_t^i p_t^i \, dt,$$

with adjustment cost of inflation rate π_t^i:

$$\Theta(\pi_t^i) = \frac{\tau}{2}(\pi_t^i - \bar{\pi})^2 p_t Y_t,$$

where $\bar{\pi}$ is the central bank's long-run inflation target.

The IS equation then becomes:

$$d\hat{Y}_t = \left[i_t - \pi_t - \left(r^n - \frac{1}{2}(\sigma + \sigma_t^s)^2 + \frac{1}{2}\sigma^2 \right) \right] dt + \sigma_t^s \, dZ_t, \tag{6}$$

Taylor rule:

$$i_t = r^n + \phi_y \hat{Y}_t + \phi_\pi \pi_t \tag{7}$$

- The IS equation (6) and the monetary policy rule (7) form the demand block.

New Keynesian Phillips curve (supply block):

$$\mathbb{E}_t\left(d\pi_t\right) = \left[\left[2(\rho + \pi_t) - i_t - x_t\right]\left(\pi_t - \bar{\pi}\right) + \sigma_t^{\pi}\sqrt{x_t} - \left(\frac{\epsilon - 1}{\tau}\right)\left(e^{\left(\frac{\eta+1}{\eta}\right)\hat{Y}_t} - 1\right)\right] dt \tag{8}$$

where $x_t \equiv (\sigma + \sigma_t^s)^2$.

Volatility of $d\pi_t$
(endogenous)

- The log-linearized Phillips curve:

$$\mathbb{E}_t\left(d\pi_t\right) = \left(\kappa^{\pi}\pi_t - \kappa^{y}\hat{Y}_t\right) dt, \tag{9}$$

  where $\kappa^{\pi} > 0$ and $\kappa^{y} > 0$ are constants.

- With the log-linearized Phillips curve (9), both the martingale equilibrium and Ornstein-Uhlenbeck equilibria emerge in the same analytical way (Section 5.1).

- With the non-linear Phillips curve (8), we conjecture:

$$d\hat{Y}_t = \theta\left[\mu - \hat{Y}_t\right] dt + \sigma_t^s dZ_t, \quad \text{where} \quad \theta \geq 0, \quad \text{and} \quad \mu \leq 0.$$

  with

$$\pi_t \equiv \pi(x_t).$$

  We numerically solve a second-order ordinary differential equation (ODE) for $\pi(\cdot)$.

Numerical results

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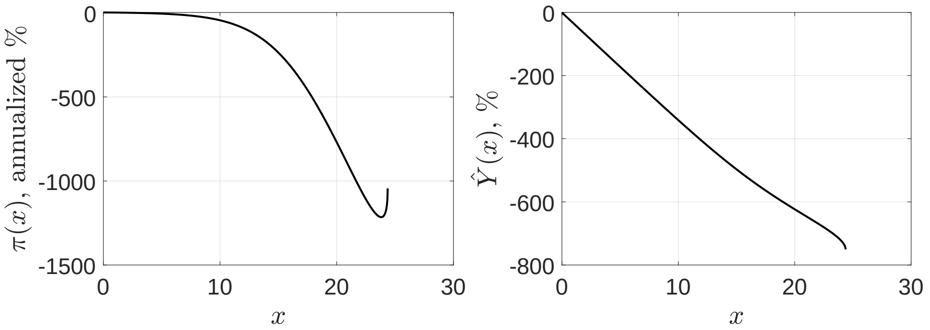

Figure: Mapping from aggregate variance x_t to inflation π_t and output gap $\hat{Y}_t$.

Assume

$$dx_t = \mu_t^x \, dt + \sigma_t^x \, dZ_t.$$

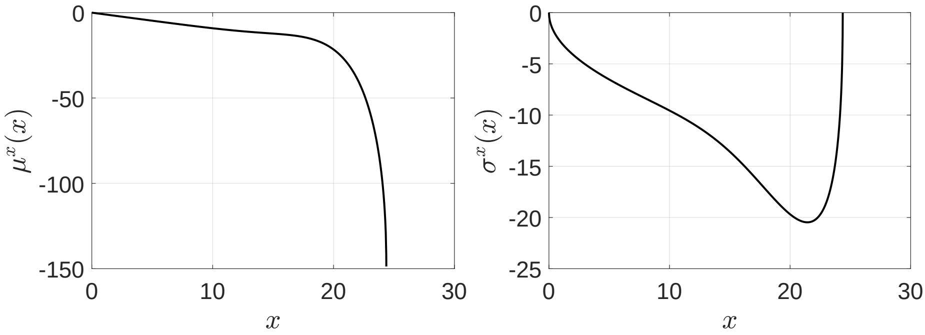

Figure: Drift (μ_t^x) and diffusion (σ_t^x) of the total variance x_t process.

- Bounded below and above (i.e., natural boundary) — the x_t process becomes stationary.

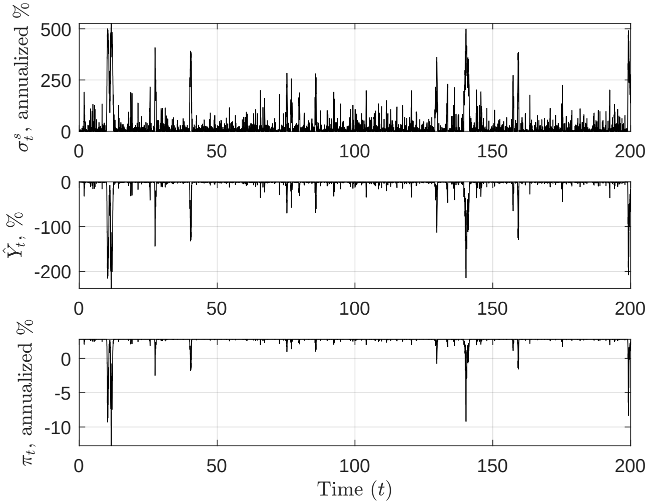

Figure: Simulated path of excess volatility $\{\sigma_t^s\}$, output gap $\{\hat{Y}_t\}$, and inflation $\{\pi_t\}$ under the calibration reported in Online Appendix K.